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NOISE INDUCED BY TURBULENT FLAMES

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ABSTRACT
A model for the sound spectrum from a turbulent non-premixed flame as observed in the far field is formulated. A closed form description for the sound spectrum is derived for fast and not so fast chemistry. In the former case the sound spectrum can be described in terms of the 1D turbulent spectrum of the mixture fraction at the flame front. The input values for this turbulent spectrum are derived from a steady state CFD flame calculation. The predictions from the sound spectrum model are compared with experiments performed in an acoustically one-dimensional combustion chamber. The comparison between measurements and prediction is good for flames in which the mixed-is-burnt approach is valid.

1 - INTRODUCTION

In boilers and gas turbines, a hot flue gas flow is generated by combustion of a gaseous fuel. The thermal power of a single gas turbine burner is in the range of 1 to 40 MW. If only a small fraction of this thermal power is converted into noise, the amplitude of the pressure fluctuations will be very high and may lead to failure of components. In this paper the processes leading to the so-called thermo-acoustic noise will be investigated.
The focus of the research during the last two decades has mainly been on premixed natural gas flames, both laminar and turbulent (see e.g. Poinsot (1987)). Little research has been performed on turbulent non-premixed flames during the last decade. Bohn and co-workers had some publications on this subject (see e.g. Bohn (1996)). With the development of advanced IGCC coal gasification plants, non-premixed combustors are becoming more widely used. Now it is experienced that these gas turbines fired on (low calorific) coal gas show instabilities as well, unless design precautions are taken.

2 - THERMO-ACOUSTIC SOUND IN A FLAME.

Sound generation by turbulent flames originates from the fluctuating heat release in the flame. The description of this fluctuating heat release in turbulent flames is complicated due to the interaction of turbulence, mixing, combustion and noise. In a turbulent flame the instantaneous density, velocity, pressure, temperature and species concentrations are determined by the transport equations for mass, momentum, enthalpy and species and by the equation of state. Combining the equations for mass and momentum the following equation relates instantaneous changes in pressure and density to fluctuations in velocity:

$$\frac{\partial^2 \rho}{\partial t^2} - \nabla^2 p = \nabla^2 \left[(\rho \underline{u} \underline{u}) - \underline{\tau} \right] \tag{1}$$

The density is a scalar function depending on 2 variables, which are taken here as pressure and entropy: $\rho = \rho(p, s)$. Rewriting the 2$^{\text{nd}}$ order derivative of the density with respect to time, using thermodynamic relations of an ideal gas for the entropy and the equation of state:

$$\frac{\partial}{\partial t}\left[\frac{1}{c^2}\frac{\partial p}{\partial t}\right] - \nabla^2 p = \nabla^2 \left[(\rho \underline{u} \underline{u}) - \underline{\tau} \right] - \frac{\partial}{\partial t}\left[\frac{\gamma - 1}{c^2}\rho T \frac{\partial s}{\partial t}\right] \tag{2}$$

The first rhs term, the Lighthill acoustic source term is relevant in high velocity flows, but in flames this term, and the dissipative 2$^{\text{nd}}$ rhs term, can be neglected with respect to the fluctuating entropy term.

The entropy may change in a flame due to several processes. An expression for the time derivative of the entropy can be derived from the transport equation for the enthalpy (neglecting viscous dissipation and assuming Lewis number of unity):

$$\rho \frac{Dh}{Dt} = \frac{\partial P}{\partial t} + \nabla \left(\frac{\lambda}{c_p} \nabla h \right) - \nabla . \underline{q} \tag{3}$$

Further by thermodynamic definition:

$$dh = Tds + \frac{1}{\rho} dP + \sum \mu_i dm_i \tag{4}$$

The instantaneous transport of species is described by:

$$\frac{Dm_i}{Dt} = \omega_i - \frac{1}{\rho} \nabla . \underline{J}_i \tag{5}$$

Combing eqs. 3 to 5, and neglecting heat conduction transport terms and diffusion transport of species, leads to:

$$\rho T \frac{\partial s}{\partial t} = -\rho T \left(\underline{u} \nabla \right) s + \sum \mu_i \omega_i - \nabla . \underline{q} - \left(\underline{u} \nabla \right) P \tag{6}$$

Hence we see that in a flame, where multiple species are mixed, noise can be generated by fluctuations in convective entropy transport, chemical reaction, heat addition and flow work. At combustion interfaces the difference between the Gibbs potential of reactants and products is large, and hence in a flame the 2^{nd} rhs term is the main source term driving pressure fluctuations in case of fluctuating combustion. The wave equation for the pressure is then reduced to:

$$\frac{\partial}{\partial t} \left[\frac{1}{c^2} \frac{\partial p}{\partial t} \right] - \nabla^2 p = -\frac{\partial}{\partial t} \left[\frac{\gamma - 1}{c^2} \sum_{i=1}^{n} \mu_i \omega_i \right] = -\frac{\partial}{\partial t} \left[\frac{\gamma - 1}{c^2} q \left(\underline{x}, t \right) \right] \tag{7}$$

In a *turbulent* flame the rate of production of a product species, and related heat release, has a strong spatial dependence and is temporally "randomly" fluctuating by nature. These fluctuations are the origin of the thermo-acoustic noise production, but are not easily calculated. The present study focuses on the modeling of the transient and averaged monopole source term in eq. (7) which is local in the flame field. Processes that have to be described are mixing, turbulence and chemical delay times. The modeled averaged source term is integrated over the flame volume and leads to an analytical expression for the noise generated in the far field. Input parameters are calculated with a Computational Fluid Dynamics simulation. A solution for the sound pressure at the far field location $\underline{x}$ (where the sound field is 1D) can be derived, assuming constant density and speed of sound, using the 1 D Green's function:

$$G \left(\underline{x}, t \left(\underline{x_0} \right), \tau \right) = \frac{c_0}{S} H \left(t - \tau - \frac{\underline{x} - \underline{x_0}}{c_0} \right) \tag{8}$$

Multiplying eq. 7 with eq. 8 and partially integrating over time gives for the sound pressure at location $\underline{x}$:

$$p \left(\underline{x}, t \right) = -\frac{c_0}{S} \int \int \int \frac{\gamma - 1}{c^2} q \left(\underline{x}, t - \frac{\underline{x} - \underline{y}}{c} \right) d\underline{y} \tag{9}$$

In a turbulent flame the heat release source term will be heavily fluctuating in space and time with a distribution determined by the turbulent structures. Hence for practical applications the instantaneous pressure will not be of interest. In view of the availability of signal processing techniques the autocorrelation function in the spectral domain will be of great value to predict and compare with measurements. The autospectrum is given by the Fourier transform of the autocorrelation:

$$p^2 \left(\underline{x}, \omega \right) = \int_{\tau} p \left(\underline{x}, 0 \right) . p \left(\underline{x}, \tau \right) e^{i \omega \tau} d\tau \tag{10}$$

Combining with eq. (9) this can be written as:

$$p^2 \left(\underline{x}, \omega \right) = \left(\frac{c_0}{S} \right)^2 \int_{\tau} \left(\frac{\gamma - 1}{c^2} \right)^2 \int_{\underline{y}_1} q \left(\underline{y}_1, - \left| \frac{\underline{x} - \underline{y}_1}{c} \right| \right) d\underline{y}_1 . \int_{\underline{y}_2} q \left(\underline{y}_2, \tau - \left| \frac{\underline{x} - \underline{y}_2}{c} \right| \right) d\underline{y}_2 . e^{i \omega \tau} d\tau \tag{11}$$

The integral in eq. 11 can be evaluated in a similar way as the evaluation of the integral for the sound of a turbulent jet (Crighton et al. P 327). The coordinate $\underline{y}_2$ is written as $\underline{y}_2 = \underline{y}_1 + \Delta \underline{y}$. At large distances from the acoustic source only the axial components introducing acoustical time delay are important. A new time variable is introduced as $\Delta t* = \Delta t + \Delta x / c_0$. The factor $(\gamma - 1)/c^2$ is taken constant over the variation *Dx*. In the far field, $\underline{x} >> \underline{y}_i$, the 1 D autospectrum does not depend on $\underline{x}$, and the equation above can be reduced to:

$$p^2(\underline{x}, \omega) = \left(\frac{c_0}{S}\right)^2 \int_{y_1} \left(\frac{\gamma - 1}{c^2}\right)^2 \int_{\Delta t^*} \cdot \int_{\Delta y} qq\left(\underline{y}_1, \Delta \underline{y}, \Delta t^*\right) e^{i\omega \Delta t^*} e^{-i\omega \frac{\Delta x}{c_0}} d\Delta \underline{x} . d\Delta t^* d\underline{y}_1 \tag{12}$$

This can be written in terms of the Fourier transform of qq as:

$$p^2(\omega) = \left(\frac{c}{S}\right)^2 \int \left(\frac{\gamma - 1}{c^2}\right)^2 F^{\omega, \underline{k}} \left\{ \bar{q}\bar{q} \left(\underline{y}, \omega, \underline{k} = \left(-\frac{\omega}{c}, 0, 0\right)\right)\right\} d\underline{y} \tag{13}$$

with: $\underline{k} = (k_1, k_2, k_3) = \left(-\frac{\omega}{c}, 0, 0\right)$ and $\underline{y}$ the integration coordinate in the flame.

From eq. (13) it can be concluded that only the energy in the heat release spectrum at the acoustic wave number $k = \omega / c$ contributes to the sound spectrum at the coinciding frequency. This means that thermo-acoustic noise is generated by heat release structures in the flame of identical length scale as the acoustic waves. Hence *fortunately* only a small part of the heat release in the flame will be converted into noise, as most turbulent structures in the flame are much smaller than the wave lengths. Another remark on eq. (13) is, that the heat release in the flame is treated 3 dimensional, but the acoustics 1 dimensional. This is valid as the fluid dynamics length scales $\approx U/\omega$ in a turbulent flame are much smaller than the acoustics lengths scales $\approx c/\omega$ i.e. low Mach number flow: $U/c \ll 1$. The noise generated by a turbulent flame can now be predicted if Eq. (13) can be evaluated. To this end the heat release spectrum has to be specified for the turbulent flame. This is considered in the next sections.

3 - EFFECT OF TURBULENCE ON COMBUSTION.

Combustion rates depend on both species concentrations and temperature and they fluctuate all in a turbulent flame. Of concern in a turbulent reacting flow are those reactions that are very sensitive to temperature fluctuations. This property inhibits in a turbulent flow direct calculation of the mean source terms of these chemical reactions with a Reynolds Averaged Navier Stokes method. However, often it can be observed that the chemical time scale is small as compared to the turbulent mixing length (in a diffusion flame) or advection length (in a premixed flame). In that case the specific reaction can be regarded as being in a local equilibrium. Using this property, the heavily fluctuating source terms can be calculated implicitly by the introduction of reaction progress variables for the relevant fuel conversion steps (for example "r" for hydrogen conversion). The reaction rate is then described by the mixture fraction f (fuel mixing) and the reaction progress variable r (chemical reaction). The mixture fraction f equals 1 in pure fuel and 0 in pure air and is determined by the transport equation:

$$\nabla\left(\bar{\rho}\tilde{\underline{u}}\tilde{f}\right) - \nabla\left(D\nabla\tilde{f}\right) = 0 \tag{14}$$

The diffusion coefficient D represents the sum of molecular diffusion and turbulent diffusion. For the variance of f:

$$g = \left\langle (f")^2 \right\rangle$$

the following equation can be derived:

$$\nabla\left(\bar{\rho}\tilde{\underline{u}}g\right) - \nabla\left(D\nabla g\right) = \frac{1}{2}Cg_1\bar{\rho}\chi - Cg_2\bar{\rho}\alpha \tag{15}$$

with rate of scalar dissipation $\chi = 2D\left(\frac{\partial f}{\partial xi}\right)^2$ and mixing rate $\alpha = \frac{\tilde{\varepsilon}}{k}g$.

The mean chemical reaction progress is determined by the transport equation:

$$\nabla\left(\bar{\rho}\tilde{\underline{u}}\tilde{r} - (D\nabla\tilde{r})\right) = \left(\overline{\frac{1}{W}\frac{\partial^2 W}{\partial f^2}}\right)\frac{1}{2}\tilde{r}C_{g_2}\bar{\rho}\tilde{\alpha}g + \bar{S}_r \tag{16}$$

The function $W = W(f)$ depends on f and describes the effect of mixing on the flame. In equilibrium r equals unity and the rhs source terms vanish. Due to mixing of fuel and air the 1$^{\text{st}}$ rhs term decreases r, but this is corrected by the 2nd rhs term describing chemical reaction. When all chemical time scales are small, r is always very close to unity and the combustion controlled by mixing only. All concentrations depend uniquely on the values of f and r. Averaging over a probability density function renders the mean source terms and concentrations (see for details Kok & Louis 1998).

4 - THE HEAT RELEASE SPECTRUM IN A TURBULENT FLAME.

In the case of a turbulent flame, combustion is determined by mixing and/or by chemistry, depending on burner type, fuel composition, gas inlet temperature and absolute pressure. Hence the combustion process and consequential heat release is taken to depend on mixing, via the mixture fraction variable f, and chemistry via a reaction progress variable r. The instantaneous species concentrations y_i are then determined by:

$$y_i = y_i\left(f\left(t\right), r\left(t\right)\right) \tag{17}$$

Each species, with mass fraction y_i, is transported in space and time according to:

$$\rho\frac{dy_i}{dt} - \nabla\left(\rho D \nabla y_i\right) = \omega_i\left(f\left(t\right), r\left(t\right)\right) \tag{18}$$

Combining these two equations gives:

$$\omega_i = -\rho D \left.\frac{\partial^2 y_i}{\partial f^2}\right|_r \nabla f . \nabla f + \left.\frac{\partial y_i}{\partial r}\right|_f \left(\rho\frac{dr}{dt} - \nabla\left(\rho D \nabla r\right)\right) - \rho D \left.\frac{\partial^2 y_i}{\partial r^2}\right|_f \nabla r . \nabla r \tag{19}$$

In case of chemical equilibrium r is constant and equal to its equilibrium value of unity in the entire flame domain. In that case the heat release rate simplifies to:

$$\sum \mu_i \omega_i = -\rho D \sum \mu_i \left.\frac{\partial^2 y_i}{\partial f^2}\right|_{r=1} \nabla f . \nabla f \tag{20}$$

In a diffusion flame with chemical equilibrium (r=1) the mass fractions are linear functions of the mixture fraction, with a discontinuity in $\partial y_i / \partial f|_{r=1}$ at the stoichiometric point (Bilger (1980)). Hence the second order derivative is (if r=1) a Dirac delta function with only a contribution at the stoichiometric point. The heat release term can then be written for an ideal gas with constant c_p and uniform inlet temperature as:

$$\frac{\gamma - 1}{c^2} q\left(\underline{x}, t\right) = -\frac{\delta\left(f - f_{stoich}\right)}{f_{stoich}\left(1 - f_{stoich}\right)} \rho D \left(\nabla f . \nabla f\right) \tag{21}$$

Substituting this in eq. (13) leads to the following expression for the autospectrum:

$$p^2\left(\omega\right) = \left(\frac{c}{S}\right)^2 \int \left(\frac{\delta\left(f - f_{stoich}\right)}{f_{stoich}\left(1 - f_{stoich}\right)}\right)^2 F^{\omega, \underline{k}} \left\{\rho D \overline{\left(\nabla f . \nabla f\right)}^2 \left(\underline{y}, \omega, \underline{k} = \left(-\frac{\omega}{c}, 0, 0\right)\right)\right\} d\underline{y} \tag{22}$$

It can be observed that the spectrum above will have a quadrupole behavior, like a turbulent jet (Crighton et al. 1992). The quadrupole character of a diffusion flame is weak as only gradients perpendicular to the flame sheet will be significant. This observation explains the measured quadrupole behavior of a diffusion flame by Ohiwa et al. (1993).

The subject of research is now the evaluation of the integral in eq. (22). For a detailed derivation is referred to Klein & Kok (1999). In that reference is shown that the sound spectrum is described by:

$$p^2\left(\omega\right) = \left(\frac{c_0}{S}\right)^2 \int \left(\frac{\delta\left(f - f_{stoich}\right)}{f_{stoich}\left(1 - f_{stoich}\right)}\right) \left(\rho D B\right)^2 \frac{l_{cor,y_2}}{2U} E_{ff}^{1D^2} \left(\frac{\omega}{2U}\right) d\underline{y} \tag{23}$$

The constant B is determined by Parseval's theorem (Crighton et al.(1992)), using the scalar dissipation perpendicular to the flame sheet:

$$\frac{1}{\pi}\rho D B \int_0^\infty E_{ff}^{1D}\left(k\right) dk = \frac{1}{2}\rho \varepsilon_{ff}^{y_3} \cong \frac{1}{2}\rho\frac{\overline{ff}}{\tau_{turb}} \tag{24}$$

For the 1D turbulent energy spectrum E_{ff} is taken (Hinze (1975)):

$$E_{ff}^{1D}\left(k_0\right) = \int_{k_0}^{\infty} \frac{x^3}{\left(1+x^2\right)^{17/6}} \exp\left[-2x^2\left(\frac{k_e}{k_{kol}}\right)^2\right] dx \qquad (25)$$

Here k_{kol} is the Kolmogorov wave number and k_e the integral wave number. The correlation length $l_{cor,y2}$ is taken equal to the circumference of the flame front. The turbulent intensity, the integral wave number and the local mean flow velocity have to be calculated to determine the turbulent spectrum. These values, together with the value of the mixture fraction, are derived for all locations in the flame from a steady CFD combustion calculation with the k-epsilon turbulence model and a local equilibrium PDF combustion model. The $\underline{y}$-integrand is determined by a Riemann sum over all local flame contributions (see for results Klein & Kok 1999). In the inertial range the model gives good results in a flame with fast chemistry like a hydrogen flame or a natural gas flame with high inlet temperature.

5 - NOISE GENERATED BY A TURBULENT FLAME – EXPERIMENTS.

Experiments were performed in an, acoustically 1D, combustion chamber. Acoustic dampers were attached to the combustion chamber inlets and exit (see Klein & Kok 1999). The results from the experiments for two different powers with identical fuel (38% CO, 5% H_2, 57% N_2) are compared with the model calculations (figure 1). Increasing the power from 16.6 kW to 33.2 kW increases the sound level by a factor 10. A good comparison in shape and amplitude with measurements is observed.

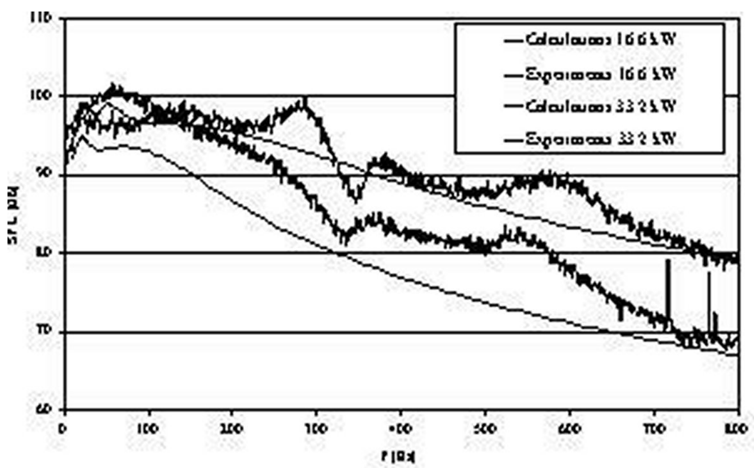

Figure 1: The comparison of the measured and calculated sound spectra of two flames with identical fuel composition but a different power.

6 - CONCLUSION.

An integral expression has been derived for the noise generated by a turbulent flame. In case of a mixing controlled flame the integral has been evaluated using a mixture fraction variable combustion model and an assumed turbulence spectrum. The effect of flame shape is taken into account by CFD simulation data input. The model predictions for the noise spectrum of a non premixed coal gas flame compare quite well with experimental data.

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