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DIFFERENT METHODS TO COUPLE A NONLINEAR SOURCE DESCRIPTION IN TIME DOMAIN TO A LINEAR SYSTEM

F. Albertson, H. Bodén

The Marcus Wallenberg Laboratory, KTH, Teknikringen 8, 100 44, Stockholm, Sweden

Tel.: +46-709229421 / Fax: +46-47084004 / Email: fal@fkt.kth.se

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ABSTRACT
In this paper hybrid linear/nonlinear methods are compared concerning accuracy, calculation time and the possibility to perform parametric studies. To this end a model problem consisting of a simple piston restriction system connected to a straight open pipe is studied. In this model, the piston and restriction are considered as the source part, modeled nonlinearly, while the pipe is modeled linearly.

1 - INTRODUCTION
In duct acoustics the fundamental sound generating mechanisms must often be described by nonlinear time domain models, while a linear frequency domain model is sufficient for describing the sound propagation in the connected duct system. This applies both for fluid machines such as IC-engines and compressors and for various wind instruments. Hybrid methods for coupling a nonlinear source description to a linear system description have been proposed by several authors.
The harmonic balance method has previously been used in microwave circuits [1] as well as musical applications [2]. Gupta et al [3] and Mutyala et al [4] suggest other methods that are based on the same type of iteration procedure, but perform the iteration coupling in another way. The methods presented in [1-2], [4] all calculate the coupling between the time and the frequency domains in the frequency domain by an impedance or admittance relation. In [3] the linear and nonlinear parts of the system are coupled by calculating a convolution integral in the time domain. Davies et al [5] as well as Gazengel et al [6] have proposed the use of convolution integrals for solving the complete system without iterations. The convolution is used as a boundary condition to the system of nonlinear differential equations, which is solved by some suitable numerical method.
After this short introduction some different methods for calculating the coupling are presented. The problem used for testing the different methods is given together with governing equations. A comparison of the methods follows, and finally some conclusions are drawn.

2 - PRESENTATION OF METHODS

2.1 - The harmonic balance method
The fundamental idea of the harmonic balance method (HBM) is to decompose the system in two separate subsystems, a linear part and a nonlinear part. The linear part is treated in the frequency domain and the nonlinear part in the time domain. The interface between the subsystems consists of the Fourier transform pair. Harmonic balance is stated when a chosen number of harmonics N satisfy some predefined convergence criteria.
First an appropriate unknown is chosen to be used in the convergence check, which is performed in the frequency domain. Then the equations are rearranged in a so-called convergence loop. This is done with all equations, linear or nonlinear in time domain and linear equations in the frequency domain. We start with an initial value of the chosen unknown, apply the different linear and nonlinear equations according to the defined convergence loop. Finally we have a new value of the chosen unknown. If the difference

between the initial value and the final value of the first N harmonics satisfy the predefined convergence criteria, harmonic balance is reached. Otherwise, an increment of the initial value is calculated using a generalized Euler method, namely the Newton-Raphson method.
The general procedure of HBM can be summarized as below. Note that step 3 and step 4 can be interchanged:

- Choose a convergence unknown $X(f)$
- Give an initial value X_0
- Apply the linear equations in frequency domain
- Apply the nonlinear (and the linear) equations in time domain
- Calculate an incremented initial value X_0^1
- Repeat step 3-5 until convergence

2.2 - Other iteration methods

The iteration method, proposed by Mutyala et al [4], is essentially the same as the HBM, except how the increment of the convergence unknown is calculated. In this method, the final value is used as a new initial value. That is, no increment in the initial value is calculated, since it is merely replaced by the final value of the convergence unknown. Compared to the HBM, in this approach it is more likely to find divergent solutions, because the lack of an increment condition.
In the method of Gupta et al [3], the iterative procedure is similar to the HBM, but all calculations are carried out in the time domain. Initial and final values of the convergence unknown are also compared in the time domain. Here, as well as in [4], the new initial time record in the convergence loop is given by a mere replacement with the final time record of the convergence unknown. By using this convolution integral, it is possible to study transient solutions as well. The price that has to be paid is increased calculation time.

2.3 - Convolution methods

In [5] and [6] convolution methods are presented. In both papers, a method where the convolution integral is used as a boundary condition is proposed.
Davies et al [5] use the method of characteristics in determining the time domain solution. The key concepts of the method are as follows. First the time domain reflection function $R(t)$ from the frequency domain impedance function $Z(f)$ is calculated. This represents the linear part of the system. Then, a volume flow is determined from $R(t)$ using the method of characteristics; a fast Fourier transform is applied and finally the pressure $P(f)$ is calculated in the frequency domain. Note that this is not an iterative method.
In [6] Gazengel et al propose that the impedance condition

$$P(f) = Z(f)\,Q(f)$$

where $P(f)$ is the acoustic pressure, $Z(f)$ the acoustic impedance and $Q(f)$ the volume flow in the frequency domain, is rewritten in the time domain as

$$p(t) = r_p(t) * [p(t) + Z_C q(t)] + Z_C q(t)$$

Here Z_C denotes the characteristic impedance and $r_p(t)$ the reflection coefficient in time domain. This time domain pressure is then used in an appropriate numerical scheme for solving the complete system, such as an Adams scheme.
In [6] much effort is spent on various correction methods to create a digital/discrete pipe system which has approximately the same features as the analogue/continuous one.
Here, the equations are arranged in a nonlinear differential equation system, where the convolution integral is used as a boundary condition. The memory of the system, that is the convolution length, is set to one period. It is therefore a method that can be directly compared with the HBM. An Adams-Bashforth algorithm is used as a predictor and an Adams-Moulton algorithm is used as a corrector. The differential equation system is solved with constant step length.

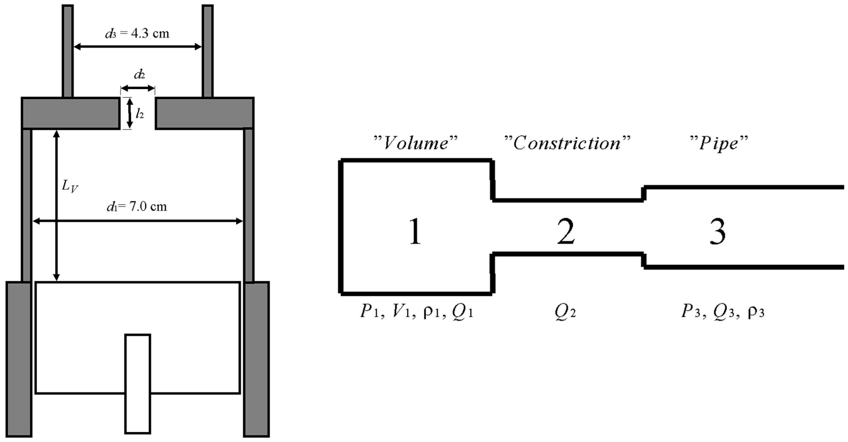

Figure 1: In the left-hand side, the piston-restriction system is shown; in the right-hand side, a schematic sketch of the system is shown with the unknowns.

3 - MODEL USED FOR COMPARISON

For comparison purposes, a simple piston-restriction system was studied as a model problem. The equations are derived in great detail in [7], so only a brief summary is provided here. The used system and a simple sketch including the unknowns of the system are depicted in Fig. 1.

As seen in Fig. 1, the system is divided into three separate parts: volume 1, which is the oscillating volume source, volume 2, the restriction, and volume 3, the pipe system. Volume 1 and 2 are considered as the source part, and are described by a nonlinear differential equation system in time domain. In volume 3, which is after the flow has fully developed, linear equations are considered to hold. A standard impedance/admittance relation gives this linear equation. Supposing conservation of momentum and volume flow in junctions, that the system is adiabatic and that the restriction acts like a stiff mass plug gives the following equation system.

$$P_1 - P_3 = \begin{cases} \frac{\rho_2}{2} Q_3^2 \left(\frac{1}{S_2^2} - \frac{1}{S_1^2} \right) + \frac{dQ_3}{dt} \frac{\rho_2}{S_2} (\delta_1 + l_2) & , Q_3 > 0 \\ -\frac{\rho_2}{2} Q_3^2 \left(\frac{1}{S_2^2} - \frac{1}{S_3^2} \right) + \frac{dQ_3}{dt} \frac{\rho_2}{S_2} (\delta_3 + l_2) & , Q_3 < 0 \end{cases}$$

$$\frac{d}{dt} (V_1 \rho_1) = -\rho_1 Q_3$$

$$P_1 = P_0 \left(\frac{\rho_1}{\rho_0} \right)^{1.4}$$

$$Q_3(\omega) = Y_3(\omega) P_3(\omega)$$

All subscripts denote the parts of the system according to Fig. 1. Note that only the last equation is given in frequency domain, while the others are given in time domain. Here, P denotes pressure, ρ density, Q volume flow, S cross sectional area, V the oscillating volume caused by the moving piston and finally Y the pipe admittance.

4 - COMPARISON OF METHODS

The HBM was found to be a simple tool for parametric studies. In Fig. 2 the pressure in volume 3, see Fig. 1, is shown for three different diameters of this pipe. The rest of the system parameters are held constant. The possibility to define the initial value gives short calculation times when a parameter is changed. A former solution from a similar system can be used as initial value in the calculations, and a small change in a parameter value yields in some sense a small change in the solution too.

For the used convolution method, it is found that the calculation time is essentially longer then for the HBM. The method is also more sensitive to changes in parameters, since a constant step length is used. A change that creates larger oscillations also implies a shorter step length to find a convergent solution.

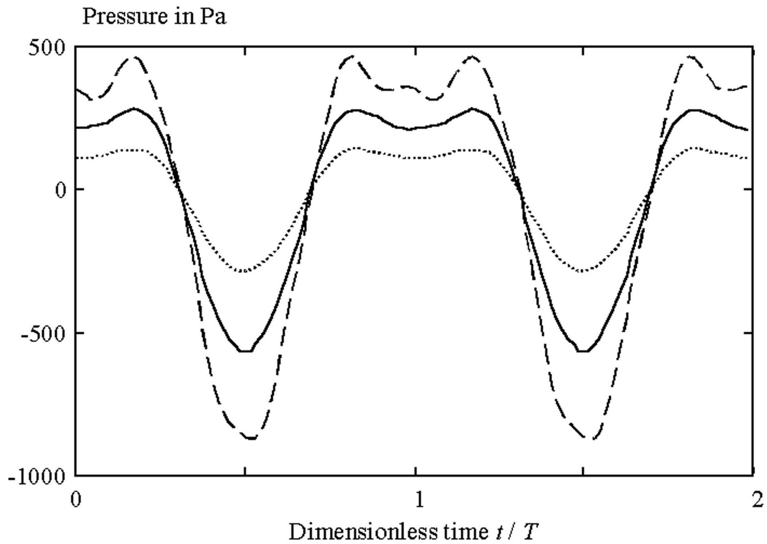

Figure 2: The pressure in volume 3 for three different diameters of the pipe.

Every calculation starts at time zero, and a transient solution is present during the first oscillations. This time domain method thus has to stabilize before the steady-state solutions can be extracted and the solution has to be calculated for a longer time period. Concerning parametric studies, this is of course a drawback. Every parameter change implies a recalculation from time zero.

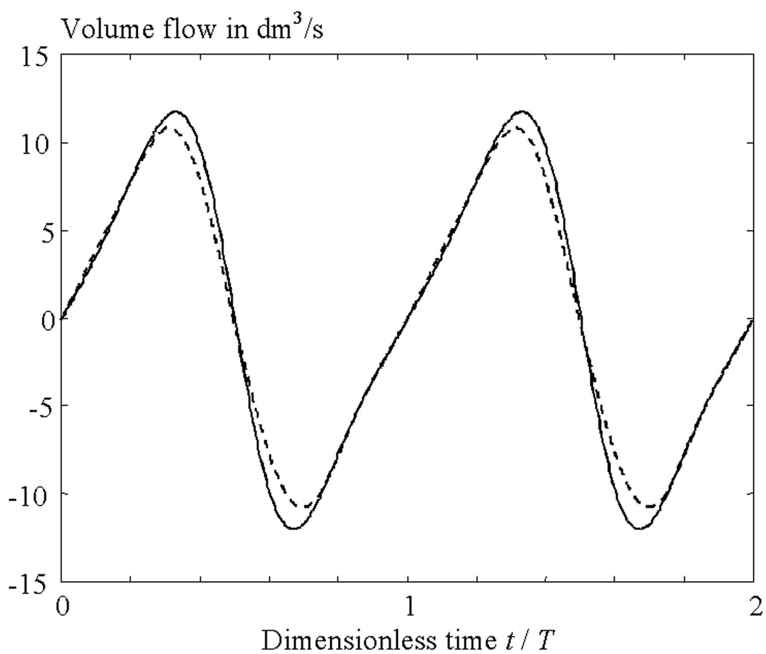

Figure 3: Comparison of the volume flow from the HBM solution (solid) and a solution found by the convolution method (dashed); note that two periods are plotted.

In comparison with the HBM Fig. 3 shows the volume flow for a certain parameter configuration. There are many possibilities for the slight difference between the solutions. For example, the step length in the time domain method has an influence on the results. In both methods some simplifications are done. In HBM as well as the convolution method, the impedance calculation in frequency domain is truncated to a finite number of harmonics. A time window is applies in the convolution method.
No convergent solution was found for this model problem using the method that was proposed by Mutyala et al [4]. They however got solutions on another compressor system using this method, so it should be possible to get convergent solutions. The reason for this discrepancy still has to be elucidated.

5 - CONCLUSIONS

The harmonic balance method was found to be a useful tool for prediction of the steady-state periodic regime. It is fast and it provides a possibility to perform parametric studies with a low computational cost. The drawback is that only steady-state periodic solutions can be found. Fractional orders of harmonic frequencies can be very important in nonlinear applications, especially when the driving force is not a single frequency. Any rational order of the fundamental frequency can though be included by a simple change of the frequency variable in the Fourier series. Nonlinear phenomenon in the steady-state periodic regimes are thus efficiently studied via the HBM.

The convolution methods have an important advantage compared to HBM. Since the convolution integral is calculated, we are not restricted to periodic steady-state regimes in the solutions. But, since each time record has to be calculated starting from time zero, the preceding solutions can not be used to perform efficient parametric studies. This method is however the best choice in applications where transient solutions are of great interest.

The iteration method of Mutyala [4] has not performed any convergent solution for this model problem. When a solution from the HBM is inserted in this iteration method, it ends up with the same solution. But already with a small change in some of the system parameters, this method creates a divergent solution for the model problem.

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