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THE INTENSITY POTENTIAL APPROACH TO PREDICT SOUND PROPAGATION THROUGH PARTIAL ENCLOSURES

M. Thivant, J.L. Guyader

Laboratoire Vibrations Acoustique, INSA de Lyon, 20 avenue Albert Einstein, 69621, Villeurbanne, France

Tel.: 04-72-43-80-80 / Fax: 04-72-43-87-12 / Email: mthivant@lva.insa-lyon.fr

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ABSTRACT
The prediction of sound propagation from acoustic sources through partial enclosures is of great interest in machinery and vehicle noise control. The use of classical integral equation is possible for low frequencies but quite unrealistic in mid and high frequencies. To overcome this problem, we propose here the Intensity Potential Approach, which describes propagation phenomena and ignores local reactive effects. With this method one can predict the irrotational part of active intensity, propagating in an acoustic medium with obstacles. The method allows us to calculate the sound pressure radiated outside partial enclosures, from internal acoustic power sources. Standard Finite Element Heat Transfer software can be used to solve industrial acoustic problems using this method. An application to trucks engine noise is under development.

1 - INTRODUCTION

The method presented in this paper is aimed to predict sound propagation from acoustic sources to the far field through complex partial enclosures. This issue is frequently encountered for transport and machinery noise prediction. Problems are assumed to be stationary. Classical methods based on Helmoltz's equation or integral formulation are quite accurate in low frequencies and for simple geometry, but they are limited for industrial problems by their computing time and their lack of robustness. To overcome these difficulties, several energy methods have been developed for both vibration and acoustic issues, among which is diffusion equation [1-5]. However difficulties arise, due to the diffusion constant to be used. The present formulation of the diffusion equation differs from the previous ones by the energy variable used: the introduction of the Intensity Potential instead of acoustic energy releases an assumption on the sound field. Moreover, the diffusion equation obtained is directly computable on usual Heat Transfer Finite Element software.

2 - FORMULATION OF DIFFUSION EQUATION

2.1 - State of the art

Energy methods aim to describe sound fields with energy path. The main advantages of these methods are much lower computing time, smoother solutions, and above all better robustness. Classical diffusion equations, dedicated to either structure vibrations or acoustics, deal with total energy E, which is the sum of potential and kinetic energies. Constitutive equations are the local power balance (eq. 1), written here at a point M apart from sources (sources can be taken into account in boundary conditions), and an assumption on the sound field (eq. 2).

$$div\left(\vec{I}\,(M)\right) = -\eta\omega E \tag{1}$$

$$\vec{I} = -Dgr\vec{a}dE \tag{2}$$

where $\vec{I}$ is the active sound intensity, h the damping loss factor, and ω the angular frequency. Equation (1) is valid for any type of wave. But (2) has only been demonstrated for certain types of waves. For damped free waves, D is related to group velocity [2-4], and for diffuse fields it can be linked with the mean free path between obstacles [5]. Equation (2) is analogue to Fourier's law encountered in Heat Transfer problems. It states that energy flows from zones of high energy density to zones of lower density. Combining (1) and (2) leads to the diffusion equation:

$$\Delta E - \frac{\eta\omega}{D} E = 0 \tag{3}$$

2.2 - Intensity potential

According to Helmoltz's theorem, any vector field can be expressed as the sum of the gradient of a scalar potential φ and the curl of a vector potential $\vec{C}$. Thus the active intensity can be written

$$\vec{I} = \vec{I}_\varphi + \vec{I}_C \tag{4}$$

where

$$\vec{I}_\varphi = -gr\vec{a}d\,(\varphi) \tag{5}$$

is called irrotational intensity and

$$\vec{I}_C = r\vec{o}t\left(\vec{C}\right) \tag{6}$$

is called rotational intensity.

The intensity potential φ is defined up to a constant. If no acoustic source is assumed in the far field, this constant can be fixed to zero at infinity.

For the majority of industrial acoustic problems, atmospheric dissipation can be neglected in comparison with dissipation due to absorbing material. Hence the local power balance (1) is reduced to

$$div\left(\vec{I}\,(M)\right) = 0 \tag{7}$$

Eqs. (4) and (7) lead to

$$div\left(\vec{I}_\varphi\,(M)\right) + div\left(\vec{I}_C\,(M)\right) = 0$$

Taking into account $div\,(gr\vec{a}d\,(\varphi)) = \Delta\varphi$ and $div\left(r\vec{o}t\left(\vec{C}\right)\right) = 0$, one gets Laplace equation for the intensity potential:

$$\Delta\varphi\,(M) = 0 \tag{8}$$

Unlike equation (3), equation (8) is strictly equivalent to thermal diffusion equation. Indeed, in usual heat transfer problems, dissipation only arises on the boundaries. Therefore there is no dissipation term in thermal diffusion equation. This latter point is important from a practical point of view, since (8) can be solved directly by usual finite elements thermal software. On the other hand, it should be noticed that no assumption has been made so far on the structure of the acoustical field. Therefore, computation of equation (8), together with adapted boundary conditions, leads to the exact solution in terms of intensity potential φ and irrotational intensity $\vec{I}_\varphi$.

2.3 - Rotational intensity

The method proposed here ignores the rotational part of active intensity. Even though equation (8) is not an approximation but the exact equation governing φ and $\vec{I}_\varphi$ one can wonder whether $\vec{I}_C$ can be neglected for expressing the boundary conditions, and for assessing pressure values.

Let us first write the power balance for a closed surface s_f surrounding a source:

$$\pi_s = \oint_{s_f} \vec{I}.\vec{n}ds \tag{9}$$

where π_s is the source power.

Using Stockes theorem and equation (4) one gets:

$$\oint_{s_f} \vec{I}.\vec{n}ds = \int_V div\left(\vec{I}_\varphi\right).dV + \int_V div\left(\vec{I}_C\right).dV \tag{10}$$

As a curl, $\vec{I}_C$ has a zero-divergence. Thus, the second term of eq. (10) is zero. This means that the power related to the rotational part of active intensity is zero, even for a volume containing a source. On the contrary, the power related to the irrotational intensity is fully representative of the power injected by the source:

$$div\left(gr\vec{a}d\left(\varphi\right)\right) = \Delta\varphi \;\Rightarrow\; \pi_s = \oint_{s_f} \vec{I}_\varphi.\vec{n}ds = -\int_V \Delta\varphi dV \tag{11}$$

Therefore it seems quite reasonable to describe sources power by the only irrotational part of active intensity on a surface s_i, as done in table 1.
In reference [6-7], acoustic fields structures are studied by decomposing active intensity $\vec{I}$ into $\vec{I}_C$ and $\vec{I}_\varphi$. It is shown that rotational intensity only changes local energy paths, and that global energy transfer between two points of space can be described by the only potential φ. As $\vec{I}_C$ is a curl, its flux lines (tangent to $\vec{I}_C$ at each point) are necessarily closed on themselves. Thus the corresponding energy is trapped in these flux lines, and is not transmitted to the far field. Actually, rotational intensity appears when two or more waves interfere, and vanishes in the free field. Computing $\vec{I}_\varphi$ is therefore self-sufficient to evaluate the acoustic energy transmitted to the far field. Moreover the solution will be more robust, since it is not affected by local interference and varies smoothly in space.

2.4 - Link between intensity and pressure

Unlike the irrotational intensity, the pressure level is much affected by interference. Pressure variations are linked to rotational part of active intensity and also to reactive intensity. Therefore calculating $\vec{I}_\varphi$ will generally not be sufficient to fully determine the sound field. However it will describe the energy transfer from one part to another.
We might also get the sound pressure level in the far free field, using the common relation:

$$P = \rho_0 c\left|\vec{I}\right| = \rho_0 c\left|\vec{I}_\varphi\right| \tag{12}$$

as $\vec{I}_{\vec{C}} = \vec{0}$ in the free field.
This relation is equivalent to Lp=L_I, taking adequate reference levels, such as I_0=10^{-12}W/m^2 and P_0=2 10^{-5}Pa.

3 - APPLICATION TO PARTIAL ENCLOSURES

3.1 - Physical problem

Consider an acoustic source S_i radiating in a non-dissipating medium (air) delimited by complex boundaries. As shown in §2.2, the intensity potential φ is governed by Laplace equation (8) in the whole volume considered. Solving this second order equation requires one to define boundary conditions in terms of φ and $\partial\varphi/\partial n$. They derive from the value of the normal component of active intensity on the boundary surfaces, with the approximation:

$$I_n \approx I_{\varphi_n} = -\frac{\partial\varphi}{\partial n} \tag{13}$$

where $\vec{n}$ is the unit vector normal to boundaries, pointing outward (from the air to the obstacle).

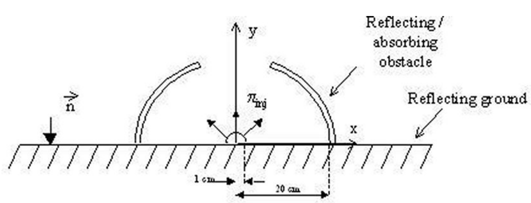

Figure 1: Physical problem.

Physical phenomenon	Normal component of active intensity, pointing to the boundary	Boundary condition
Power π_s distributed on surface s	$$I_n = -\frac{\pi_S}{s} \qquad (14)$$	$$\frac{\partial\varphi}{\partial n} = \frac{\pi_S}{s} \qquad (15)$$
Absorption	$$I_n = \alpha\left\{I_n\right\}_{free-field} \qquad (16)$$	$$\frac{\partial\varphi}{\partial n} = \alpha\left\{\frac{\partial\varphi}{\partial n}\right\}_{free-field} \qquad (17)$$
Far free field radiation (written on a R-radius hemisphere)	$$I_n(R) = \frac{1}{R}\varphi(R) \qquad (18)$$	$$\frac{\partial\varphi}{\partial n} = -\frac{1}{R}\varphi(R) \qquad (19)$$

Table 1: Boundary conditions.

The source is modeled by its intensity I_φ=W/m (L_I=150 dB ref. 10^{-12} W/m^2), applied on a hemisphere (radius 1cm). Several configurations have been tested for the obstacle. Here is presented the result for a perfectly reflecting obstacle (α=0), with a 30° aperture around y-axis. The boundary condition is I_n=0. For absorbing obstacle, the calculation would require two steps: determination of the incidence intensity with a free field calculation (without obstacle), then final calculation with the boundary condition (16). A far free field radiation condition has been introduced in order to limit the calculation to an R-radius hemisphere. It derives from two assumptions: at a distance R much longer than obstacles dimensions, the acoustic field behaves locally as spherical waves propagated in the free field (no wave back), and φ=0 at infinity (φ is an energy). Under these assumptions, one can explicit $\vec{I_\varphi}$ and φ:

$$I_r(R) = \frac{1}{2}Re\left(P^*U\right) = \frac{\rho_0 c k^2}{32\pi^2}\left|Q\right|^2\frac{1}{R^2} \qquad (20)$$

where ρ_0 is air density, c the speed of sound, $k = \omega/c$ the wave number, and Q the voluminal rate of flow of the equivalent spherical source.

$$\vec{I_\varphi} = -gr\vec{a}d\left(\varphi\right) \;\Rightarrow\; \varphi(R) = \frac{\rho_0 c k^2}{32\pi^2}\left|Q\right|^2\frac{1}{R} \qquad (21)$$

One eventually gets eq. (18).

3.2 - Results

Figure 3 shows curves of constant irrotational intensity, drawn every 3dB. They were obtained with Ideas Finite Element Heat Transfer solver.

One can notice that unlike Helmoltz's equation, Laplace equation (8) remains the same whatever the frequency. The intensity potential approach only cares about frequency dependency of absorption coefficients and of power sources. That's why the method tends to overestimate diffraction at the aperture for high frequencies. One would then expect more directivity along the y-axis. This is due to the omission of the rotational part of the active intensity. Actually, exact boundary condition on the reflecting obstacle would be $I_n = I_{\varphi_n} + I_{\vec{C}_n} = 0$ rather than $I_\varphi = 0$. Neglecting $\vec{I_C}$ in a region where it might be high seems to have much effect on diffraction.

3.3 - Far field condition

A test has been performed in order to evaluate the minimum distance R where the far free field condition should be applied. For the first calculation we took R=1m, which is 5 times longer than the obstacles radius. Then we changed R to 10 meters, i.e. 50 times the obstacles radius. The amplitude of intensity on the x-axis behind the obstacle (in the shadow zone) is drawn on figures 4 and 5. These curves show a very little variation (0,7dB at r=1m) whereas R has been multiplied by 10. This indicates that R=1m is already a good position for the far free field condition.

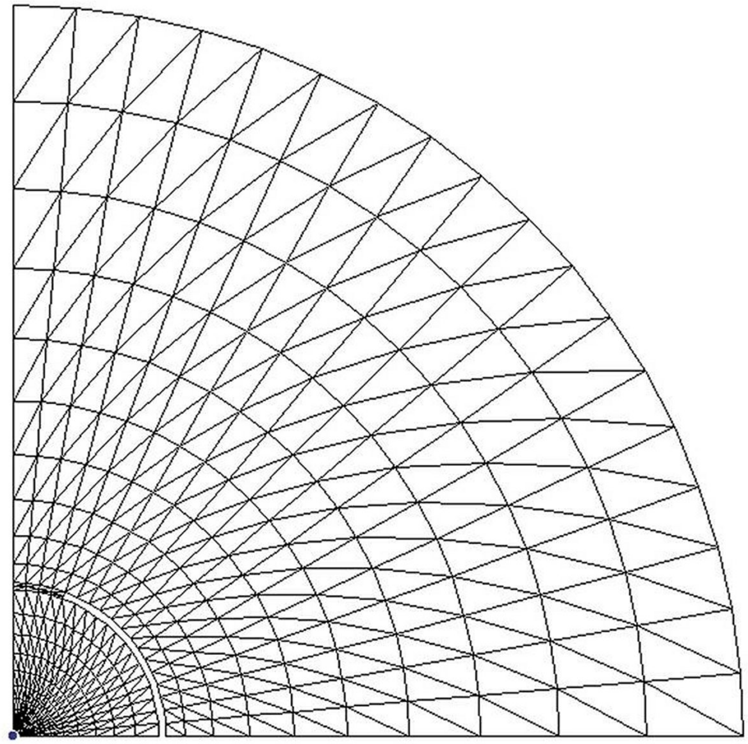

Figure 2: Finite Element Mesh.

4 - CONCLUSION

A diffusion equation has been established for acoustical cases. It is happening to be Laplace equation for the scalar intensity potential φ. Solving this equation together with energy boundary conditions leads to the irrotational component of the active intensity. The computation can be done for complex geometry, using a Finite Element Heat Transfer solver. One can then determine the pressure level away from acoustic sources and obstacles, since it is related to the intensity level in the free field. Both experimental validation and comparison with FE and Ray Tracing methods are under development. Industrial application to trucks engine noise is also planned.

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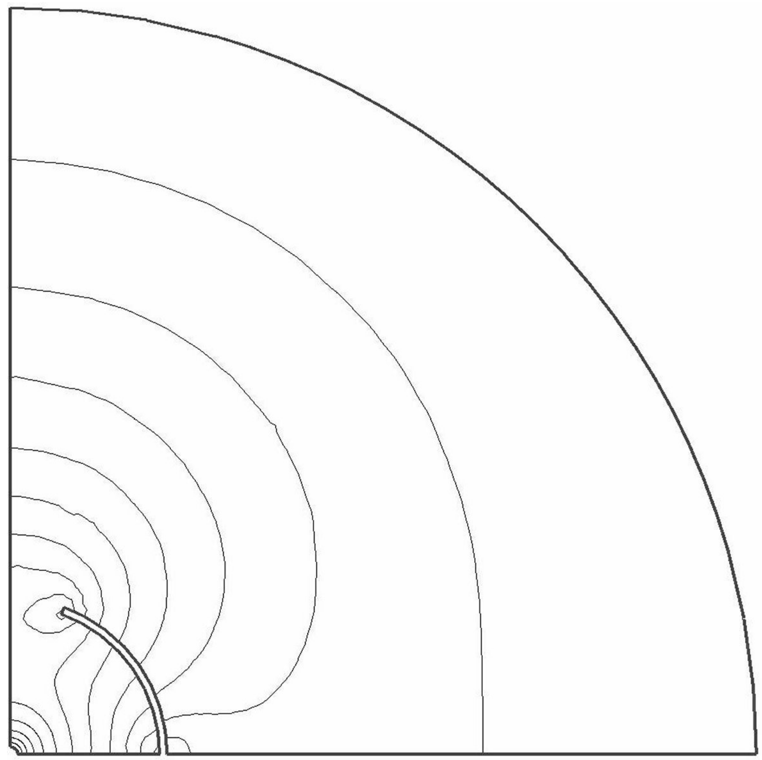

Figure 3: Curves of constant I_φ.

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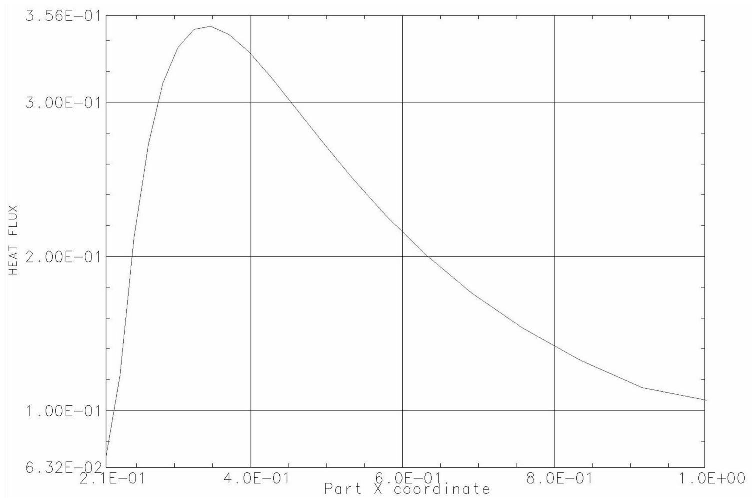

Figure 4: Intensity behind obstacles for a far free field condition at R=1m.

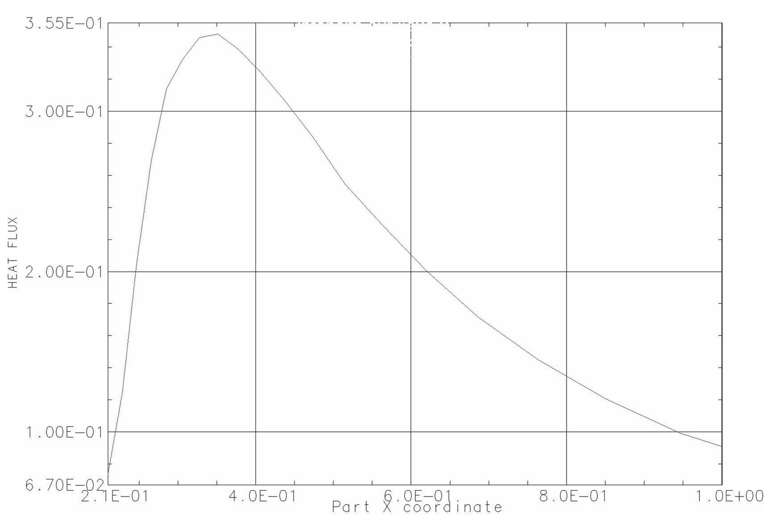

Figure 5: Intensity behind obstacles for a far free field condition at R=10m.