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THE TOTAL POWER OF FORCED AND DAMPED VIBRATIONS OF A CLAMPED ANNULAR PLATE

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ABSTRACT
The sound radiation of a clamped annular plate has been analyzed in this paper. The plate is clamped to a planar, rigid baffle and is excited to vibrations under external pressure. The processes of steady state vibrations, also harmonic with respect to time, have been investigated. The analysis is basis on the complex mutual sound power of free vibrations of a plate discussed elsewhere. Such factors as forcing, internal damping of plate material and interactions of the air column above the plate have been coupled into the equation of the plate's motion. As a result an inhomogeneous equation of the plate's motion in the form of algebraic equation system has been obtained. Its solution provides the normalized, total sound power. Some frequency characteristics have been obtained for the sound power related to the power of a model plate as well as for the mutual power of different vibration modes.

1 - INTRODUCTION
The sound radiation of a thin annular plate is analyzed in this paper. The full theoretical analysis of sound radiation by a circular plate was taken in [6]. The vibration analysis is presented in several publications for circular as well as annular plates (cf. [3], [5], [8]). A number of papers treat of the total sound power determination and its experimental verifications (cf. [1,2]).

2 - THE EQUATION SYSTEM AND ITS SOLUTION
The plate of radii $r_2 > r > r_1$ is clamped into a planar rigid baffle and is excited to vibrations with the external axisymmetric pressure $\mathrm{Re}\left\{f(r)\exp(-i\omega t)\right\}$. The internal friction force of the plate and the damping force from the air column above the plate have been taken into account. We assume that the transverse deflections of the plate are as small as the plate's vibrations are linear. That is why we use the rheologic Kelvin-Voigt's model of a viscously elastic plate, which gives the plate's equation of motion

$$B'\nabla^4\eta(r,t) + \rho h\frac{\partial^2\eta(r,t)}{\partial t^2} = f(r,t) - R'\frac{\partial}{\partial t}\left\{\nabla^4\eta(r,t)\right\} - p(r,t) \qquad (1)$$

where $\eta(r) = \sum_{n=0}^{+\infty} c_n\xi_n(r)\,,\ c_n \in C$ is a transverse deflection of the plate and

$$\xi_n(r) = \sqrt{\frac{1}{2}\frac{s^2-1}{C_0'^2(sx_n) - C_0'^2(x_n)}}\frac{\eta_n(r)}{A_n} = \sqrt{\frac{1}{2}\frac{s^2-1}{C_0'^2(sx_n) - C_0'^2(x_n)}}$$
$$\times\left\{J_0(k_nr) + B_nI_0(k_nr) - C_nN_0(k_nr) - D_nK_0(k_nr)\right\}$$

$$A_n(r) = \sqrt{\frac{1}{2}\frac{s^2-1}{C_0'^2(sx_n) - C_0'^2(x_n)}}\ \ [\mathrm{m}]$$

$$k_n^4 = \omega_n^2 \rho h / B'$$

$p(r,t) = \rho_0 \dfrac{\partial \phi(r,t)}{\partial t}$ is the sound pressure and the following plate's parameters are x_n − eigenvalue of the frequency equation, ω_n − n-th eigenfrequency, $s = \dfrac{r_2}{r_1}$, $B' = \dfrac{Eh^3}{12(1-\nu^2)}$ − bending stiffness, R' − lossiness, h − thickness, ν −Poisson's ratio, ρ − density. Vibrating plate radiates acoustic waves into the hemisphere $z \geq 0$. Several transformations of equation (1) lead to its form of equation system

$$\begin{gathered} \bar{c}_m \left(\frac{k_m^4}{k_B^4} - 1 \right) - i\varepsilon_0 \sum_{n=0}^{+\infty} \bar{c}_n P_{nm} = \bar{f}_m, \qquad \bar{c}_m = c_m \frac{\rho h \omega^2}{f_{\max}}, \\ \bar{f}_m = f_m \frac{\rho h \omega^2}{f_{\max}}, \qquad \omega_n = \left(\frac{x_n}{r_1} \right)^2 \sqrt{\frac{B'}{\rho h}} \end{gathered} \tag{2}$$

where $k_B^4 = \omega^2 \rho h / B$, $B = B'(1 - i\varepsilon')$, $\varepsilon' = \omega R' / B'$,

$$f_m = \frac{2 f_{\max}}{\rho h \omega^2 (s^2 - 1) r_1^2} \int_a^b \xi_m(r)\, r dr$$

$$\begin{aligned} f_m = \; & \sqrt{\frac{2}{s^2-1}} \sqrt{\frac{1}{s^2 C_0'^2(s x_m) - C_0'^2(x_m)}} \frac{f_{\max}}{\rho h \omega^2 x_m} \\ & \times \left\{ s \frac{b}{r_2} \left[J_1\left(s x_m \frac{b}{r_2} \right) - C_m N_1 \left(s x_m \frac{b}{r_2} \right) \right] - \frac{a}{r_1} \left[J_1 \left(x_m \frac{a}{r_1} \right) - C_m N_1 \left(x_m \frac{a}{r_1} \right) \right] \right. \\ & \left. + s \frac{b}{r_2} \left[B_m I_1 \left(s x_m \frac{b}{r_2} \right) + D_m K_1 \left(s x_m \frac{b}{r_2} \right) \right] - \frac{a}{r_1} \left[B_m I_1 \left(x_m \frac{a}{r_1} \right) + D_m K_1 \left(x_m \frac{a}{r_1} \right) \right] \right\} \end{aligned}$$

$$f(r) = \begin{cases} f_{\max} & \text{if } r_1 < a < r < b < r_2 \\ 0 & \text{if } r_1 < r < a \text{ and } b < r < r_2 \end{cases}$$

P_{nm} is the mutual sound power in the form of the Hankel's representation taken from the modal analysis (cf. [4], [7]). The complex factors c_n have been computed by the solution of the equation system (2). The total sound power is

$$\Pi = \pi \rho_0 c \omega^2 r_1^2 \frac{s^2 - 1}{2} \sum_{n=0}^{+\infty} \sum_{m=0}^{+\infty} c_n c_m^* P_{nm}, \qquad \Pi' = \pi r_1^2 \rho_0 c \left(\frac{f_{\max}}{\rho h \omega_0} \right)^2 \tag{3}$$

Normalizing the sound power (3) we get

$$P'(\bar{\omega}) = \frac{\Pi}{\Pi'} = \frac{s^2 - 1}{2\bar{\omega}^2} \sum_{n=0}^{+\infty} \sum_{m=0}^{+\infty} \bar{c}_n \bar{c}_m^* P_{nm}(\bar{\omega}) \tag{4}$$

The total power P has been related to the total power P' of a model plate. This makes possible producing some frequency characteristics independent of the plate's material parameters.

3 - CONCLUSIONS AND RESULTS DISCUSSION

The frequency characteristics present the sound power as a function of the frequency of the exciting force factor ω, related to the plate's fundamental resonant frequency ω_0, i.e. $\bar{\omega} = \omega / \omega_0$. Some plate material parameters have been introduced $\bar{\varepsilon}_0 = \dfrac{\rho_0 c}{\rho h \omega_0}$, $\bar{\varepsilon}' = \dfrac{\omega_0 R'}{B'}$, where c is the sound propagation velocity in air, and $\varepsilon_0 = \bar{\varepsilon}_0 / \bar{\omega}$, $\varepsilon' = \bar{\varepsilon}' \bar{\omega}$.

All the figures show that the sound power reaches the local maximums for the forcing frequencies equal to the plate's successive eigenfrequencies − the greatest one for the fundamental eigenfrequency. If the plate's shape is close to an annulus, i.e. $s \to 1$, some regular oscillations appear in the frequency domain for the sound power modulus. Also successive eigenfrequencies appear for higher frequencies. If the plate's shape is close to a circle, i.e. $s \to \infty$, the oscillations disappear, the eigenfrequencies appear

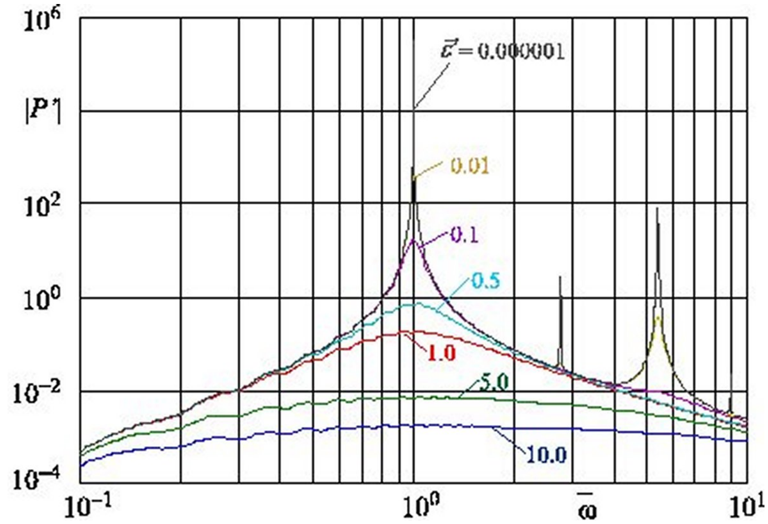

Figure 1: The normalized total sound power modulus of an annular plate $|P'|$, where s=1.2, h=1e-3m.

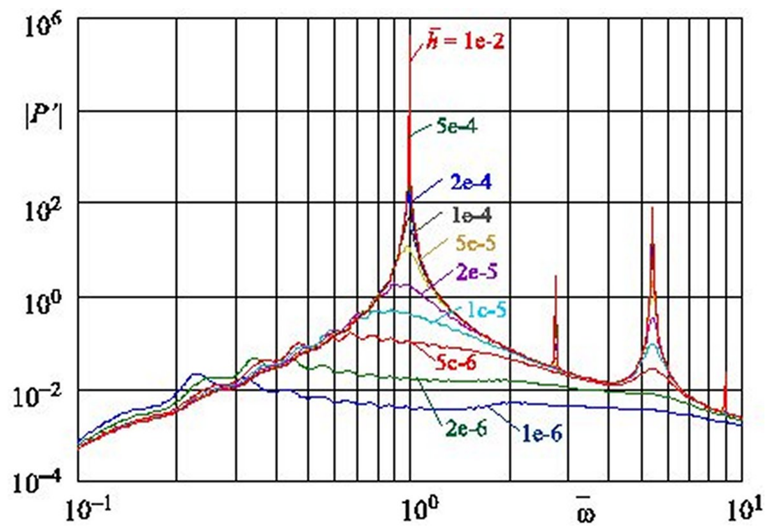

Figure 2: The normalized total sound power modulus of an annular plate $|P'|$, where s=1.2, $\bar{\varepsilon}'$=1e-6, the plate's thickness h is the curve parameter.

for lower frequencies, and the sound radiation became analogous to the radiation of a circular plate of identical size.

The parameter $\bar{\varepsilon}_0$ describes the medium density related to the plate material density. The greater is its value the stronger the plate's vibrations are damped by the medium and the greater are the reciprocal interactions of the plate's successive vibration modes. The parameter $\bar{\varepsilon}'$ describes the plate's internal friction related to the plate's bending stiffness (cf. Fig. 1) and the parameter $\bar{h} \equiv \dfrac{h}{\sqrt{S_0}} = \dfrac{h}{r_1\sqrt{\pi\,(s^2-1)}}$ is the plate's thickness related to the plate's area S_0 (cf. Fig. 2). The smaller is $\bar{\varepsilon}'$ value and the greater is $\bar{h}$ value the greater values of the sound power modulus are reached for the successive eigenfrequencies. Some sample values of the mutual power separated from the total power (4) are shown on Fig. 3. It is visible that values of the sound power of identical indexes have the greatest fraction in the total power. The fraction of values of the mutual power is much smaller.

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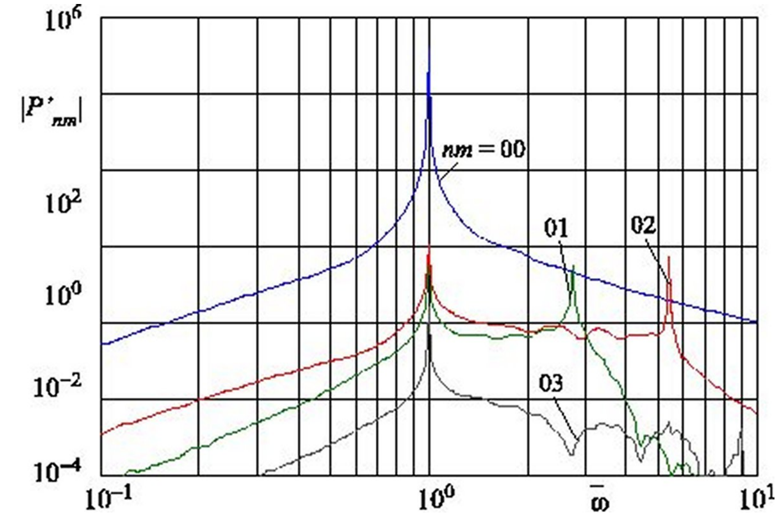

Figure 3: The mutual sound power modulus $|P'_{nm}|$, where s=2, h=1e-3m, nm=00, 01, 02, 03.

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