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# ASYMPTOTIC FORMULAS FOR ACTIVE AND REACTIVE MUTUAL POWER OF FREE VIBRATIONS OF A CLAMPED ANNULAR PLATE

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**ABSTRACT**
Normalized mutual power of radiation, active and reactive, has been analyzed in this paper. The sound source is an annular plate, clamped to a planar, rigid and infinite baffle. The plate's vibrations are axisymmetric and sinusoidal with respect to time. A Kirchhoff-Love's model of a perfectly elastic plate has been used. First, the integral formulas for the mutual sound power have been presented and then the asymptotic formulas for fast numerical computations of the mutual sound power have been computed, which can be used for acoustically fast waves. The contour integral computation and stationary phase methods have been used. The analysis presented can be used as a basis for analysis of the total power of excited and damped vibrations.

## 1 - INTRODUCTION

A modal analysis has been made to get the normalized mutual sound power, active and reactive, of a clamped annular plate. Free, axisymmetric vibrations sinusoidally varying with respect to time with no friction and damping forces have been considered. A Kirchhof-Love model of a perfectly elastic plate has been used. Results of this analysis can further be used to compute the total sound power of a clamped annular plate.

The frequency equation and its solution have been presented in several works (e.g. [2], [5, 6]). The contour integral computation and stationary phase methods have been used to transform some integral formulas to asymptotic formulas with known approximation error. And those methods are presented in several papers for computation of mutual sound power of a circular plate (e.g. [3, 4]) or sound power of an annular plate [5]. A number of papers treat of several theoretical as well as experimental methods for obtaining the sound power of circular plates (e.g. [1]).

## 2 - INTEGRAL REPRESENTATION

The mutual sound power lost via mode $(0,m)$ and used to overcome resistance produced by the same source via mode $(0,n)$ of free vibrations of a clamped annular plate is

$$\Pi_{nm} = \frac{1}{2}\int_S p_{nm} v_n^* dS,\ \text{ where } \Pi_{nm}^{(\infty)} = \sqrt{\Pi_n^{(\infty)}\Pi_m^{(\infty)}} = \lim_{k\to\infty}\Pi_{nm}(k) = \frac{\rho_0 c}{2}\int_S v_n v_m^* dS \tag{1}$$

$p_{nm}$ − the sound pressure produced by vibrating annular plate via mode $(0,n)$ exerted on the same plate via mode $(0,m)$, $v$ − normal component of the amplitude distribution of the plate's vibration velocity, $v^*$ − conjugate magnitude of $v$. We denote the normalized mutual sound power as $P_{nm} = P_{c,nm} - iP_{b,nm} = \frac{\Pi_{nm}}{\Pi_{nm}^{(\infty)}}$. Those definitions and denotations make possible to express the normalized mutual sound power of a clamped annular plate in the form of integral

$$\begin{aligned} P_{nm} = \quad & 4\beta x_n^2 x_m^2 g_{nm} \int_0^{\frac{\pi}{2}-i\infty} \left\{ x_n \left[ sC_1^{'}(sx_n) J_0(su) - C_1^{'}(x_n) J_0(u) \right] \right. \\ & \left. -u \left[ sC_0^{'}(sx_n) J_1(su) - C_0^{'}(x_n) J_1(u) \right] \right\} \left\{ x_m \left[ sC_1^{'}(sx_m) J_0(su) - C_1^{'}(x_m) J_0(u) \right] \right. \\ & \left. -u \left[ sC_0^{'}(sx_m) J_1(su) - C_0^{'}(x_m) J_1(u) \right] \right\} \frac{ud\vartheta}{(x_n^4 - u^4)(x_m^4 - u^4)} \end{aligned} \tag{2}$$

where

$$s = \frac{r_2}{r_1},\ u = \beta \sin\vartheta,\ \beta = kr_1$$
$$g_{nm} = \frac{1}{\sqrt{s^2 C_0'^2(sx_n) - C_0'^2(x_n)}} \frac{1}{\sqrt{s^2 C_0'^2(sx_m) - C_0'^2(x_m)}}$$

$x_n{=}k_n r_1$ are the successive eigenvalues of the frequency equation, $k_n^4 = \frac{\omega_n^2 \rho h}{B'}$ is a structural wavenumber, $C_i'(sx_n) = J_i(sx_n) - C_n N_i(sx_n)$, $i$=0,1 and $C_n$ is an integral constant presented in [5].

## 3 - ASYMPTOTIC FORMULAS

Using the contour integral computing and stationary phase methods we can transform integral (2) to the asymptotic formulas for active (3) and reactive (4) mutual power

$$\begin{aligned} P_{c,nm} = \quad & \frac{(\delta_n a_{nm} - \delta_m a_{mn}) \delta_n^2 \delta_m^2}{\beta(\delta_n^4 - \delta_m^4)} \left\{ \frac{1}{\delta_n^2} \left( \frac{1}{\sqrt{1-\delta_n^2}} - \frac{1}{\sqrt{1+\delta_n^2}} \right) + \frac{1}{\delta_m^2} \left( \frac{1}{\sqrt{1-\delta_m^2}} - \frac{1}{\sqrt{1+\delta_m^2}} \right) \right\} \\ & + \frac{q_{nm}}{\beta\sqrt{\beta}} \sqrt{s} \left[ \alpha_1 \cos w_2 + \alpha_2 \sin w_2 + \sqrt{\frac{2}{s-1}} (\gamma_1 - \gamma_4) \cos w_3 + \sqrt{\frac{2}{s+1}} (\gamma_3 - \gamma_2) \cos w_4 \right] \\ & + \frac{q_{nm}}{\beta\sqrt{\beta}} (\beta_1 \cos w_1 + \beta_2 \sin w_1) + O\left( \delta_n^2 \delta_m^2 \beta^{-3/2} \right) \end{aligned} \tag{3}$$

$$\begin{aligned} P_{b,nm} = \quad & \frac{2(\pi\beta)^{-1}}{\delta_n^4 - \delta_m^4} \left\{ \delta_m^2 \left[ d_1 \frac{\mathrm{arcsin}\delta_n}{\sqrt{1-\delta_n^2}} - d_2 \frac{\mathrm{arcsinh}\delta_n}{\sqrt{1+\delta_n^2}} \right] - \delta_n^2 \left[ d_3 \frac{\mathrm{arcsin}\delta_m}{\sqrt{1-\delta_m^2}} - d_4 \frac{\mathrm{arcsinh}\delta_m}{\sqrt{1+\delta_m^2}} \right] \right\} \\ & + \frac{q_{nm}}{\beta\sqrt{\beta}} \sqrt{s} \left[ \alpha_1 \sin w_2 + \alpha_2 \cos w_2 - \sqrt{\frac{2}{s-1}} (\gamma_1 \sin w_3 + \gamma_4 \cos w_3) - \sqrt{\frac{2}{s+1}} (\gamma_3 \sin w_4 + \gamma_2 \cos w_4) \right] \\ & + \frac{q_{nm}}{\beta\sqrt{\beta}} (-\beta_1 \sin w_1 + \beta_2 \cos w_1) + O\left( \delta_n^2 \delta_m^2 \beta^{-3/2} \right) \end{aligned} \tag{4}$$

which consist of elementary functions and expressions only with no integration elements, and

$$\alpha_1 = C_0^{'}(sx_n) C_0^{'}(sx_m) - \delta_n \delta_m C_1^{'}(sx_n) C_1^{'}(sx_m)$$
$$\alpha_2 = \delta_m C_1^{'}(sx_m) C_0^{'}(sx_n) + \delta_n C_1^{'}(sx_n) C_0^{'}(sx_m)$$
$$\alpha_{\pm 3} = C_1^{'}(sx_n) C_0^{'}(x_m) \pm C_1^{'}(x_n) C_0^{'}(sx_m)$$
$$\alpha_{\pm 4} = C_1^{'}(x_m) C_0^{'}(sx_n) \pm C_1^{'}(sx_m) C_0^{'}(x_n)$$
$$\alpha_5 = C_1^{'}(sx_n) C_1^{'}(x_m) + C_1^{'}(sx_m) C_1^{'}(x_n)$$
$$\alpha_6 = C_0^{'}(x_m) C_0^{'}(sx_n) + C_0^{'}(sx_m) C_0^{'}(x_n)$$
$$\alpha_7 = sC_1^{'}(sx_n) C_0^{'}(x_m) - C_1^{'}(x_n) C_0^{'}(x_m)$$
$$\gamma_1 = \delta_n \alpha_{-3} - \delta_m \alpha_{-4},\ \gamma_2 = \delta_n \alpha_{+3} + \delta_m \alpha_{+4},\ \gamma_3 = \delta_n \delta_m \alpha_5 - \alpha_6,\ \gamma_4 = \delta_n \delta_m \alpha_5 + \alpha_6$$
$$\beta_1 = C_0^{'}(x_n) C_0^{'}(x_m) - \delta_n \delta_m C_1^{'}(x_n) C_1^{'}(x_m),\ \beta_2 = \delta_m C_1^{'}(x_m) C_0^{'}(x_n) + \delta_n C_1^{'}(x_n) C_0^{'}(x_m)$$
$$a_{nm} = g_{nm} \alpha_7,\ a_{mn} = g_{mn} \alpha_7$$
$$w_1 = 2\beta + \frac{\pi}{4},\ w_2 = 2s\beta + \frac{\pi}{4},\ w_3 = (s-1)\beta + \frac{\pi}{4},\ w_4 = (s+1)\beta + \frac{\pi}{4}$$
$$q_{nm} = \frac{2\delta_n^2 \delta_m^2 g_{nm}}{\sqrt{\pi}(1-\delta_n^4)(1-\delta_m^4)}$$
$$d_1 = a_{nm}^{'} \delta_m + a_{nm}^{''} \delta_n,\ d_2 = a_{nm}^{'} \delta_m - a_{nm}^{''} \delta_n,\ d_3 = a_{nm}^{'} \delta_n + a_{nm}^{''} \delta_m,\ d_4 = a_{nm}^{'} \delta_n - a_{nm}^{''} \delta_m$$
$$a_{nm}^{'} = a_{mn}^{'} = g_{nm} \left[ sC_1^{'}(sx_n) C_1^{'}(x_m) + C_1^{'}(x_n) C_1^{'}(x_m) \right]$$
$$a_{nm}^{''} = a_{mn}^{''} = g_{nm} \left[ sC_0^{'}(sx_n) C_0^{'}(x_m) + C_0^{'}(x_n) C_0^{'}(x_m) \right]$$

A number of asymptotic equations must be used to obtain formulas (3) and (4). The consequence of this is that the formulas are valid for wave acoustically fast only, i.e. when acoustic wavenumber is greater than structural wavenumber. But the asymptotic formulas make possible fast and accurate numerical computations of mutual power active and reactive and can be the basis of analysis of the total sound power of a clamped annular plate. Additionally, the formulas make possible separation and further analysis of oscillating and non-oscillating parts of mutual sound power. This cannot be done using integral or any other formulas.

**4 - CONCLUSIONS**

In the case of mutual sound power of mode numbers even and odd the non-oscillating part disappear and the oscillating part has amplitude of a considerable value Fig. 1. In the case of both odd as well as both even mode numbers the non-oscillating part is clear and the amplitude of oscillating part is a little smaller Fig. 2. Considered mutual interactions between free vibration modes proceed in surroundings of a liquid or gaseous medium (e.g. air). An air column above vibrating plate absorbs the sound power in some terms of time and develops the sound power in the rest terms of time. In the case of vacuum or a very weak air the mutual interactions, considered herein, disappear or became much weaker than in the case of more thick gas. The air column has been used only as a transmission medium for the mutual interactions between some vibration modes but no damping influence has been considered using a Kirchhoff-Love model of a perfectly elastic plate.

The modal analysis and formulas for the active and reactive sound power presented herein can be the basis of the total power computing. To make a correct design of an acoustic device the active sound power is not enough because of the energy aspect of the device. It does not matter if the active power is small or big. If the reactive power is to high the sound device can be easily destroyed. This is the reason why the reactive power is so important and has also been presented.

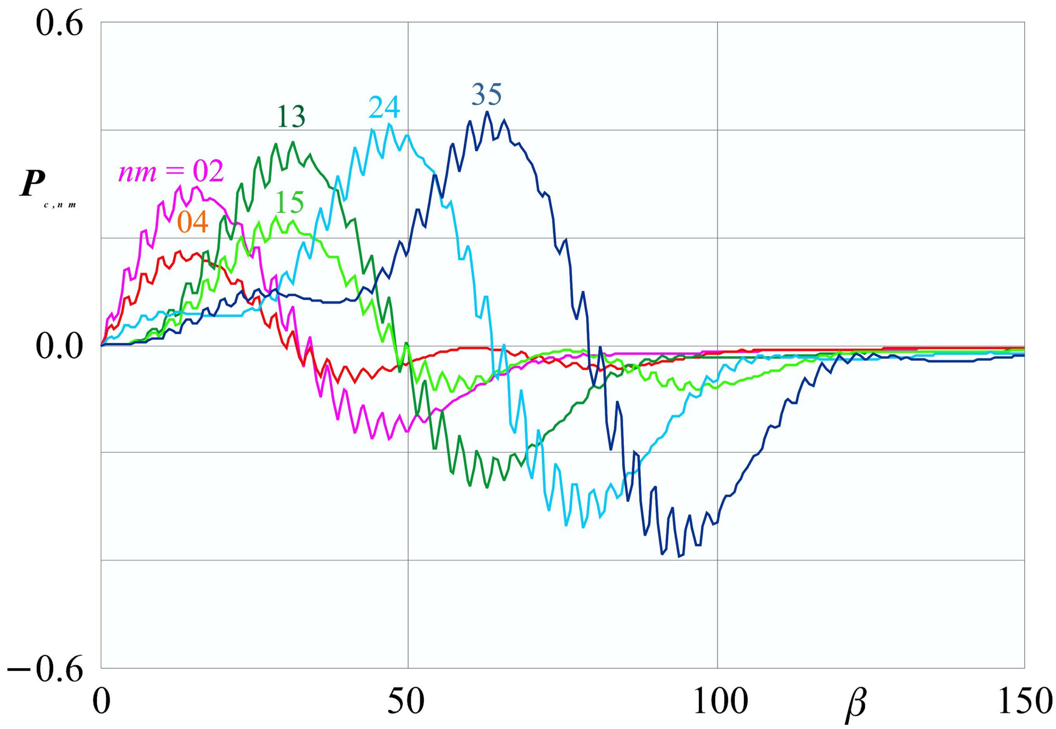


**Figure 1:** The normalized mutual active power of vibrating annular plate $P_{c,nm}$, where $s$=1.2, $nm$=02,04,13,15,24,35.

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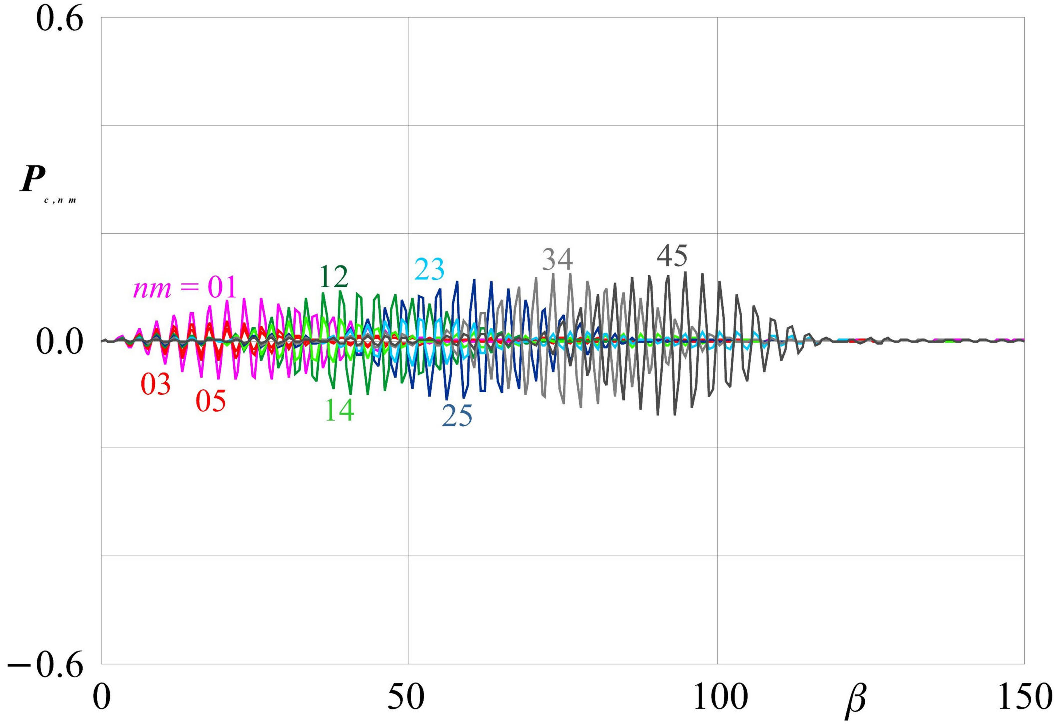


**Figure 2:** The normalized mutual active power of vibrating annular plate $P_{c,nm}$, where $s$=1.2, $nm$=01,03,05,12,14,23,25,34,45.


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