inter.noise 2000
The 29th International Congress and Exhibition on Noise Control Engineering
27-30 August 2000, Nice, FRANCE

I-INCE Classification: 7.2

INVERSE FORCE SYNTHESIS: STATE OF THE ART AND FUTURE RESEARCH

M. Blau

TU Dresden, Institut f. Akustik u. Sprachkommunikation, D-01062, Dresden, Germany

Tel.: +49 351 463 3041 / Fax: +49 351 463 7091 / Email: matthias.blau@epost.de

Keywords:
STRUCTURE-BORNE SOUND, EXCITATION FORCES, INVERSE METHODS, FUTURE RESEARCH

ABSTRACT
In analyzing structure-borne noise problems, it is often desired to characterize vibrational sources in conjunction with a given receiving structure. Since the dynamical behavior of the latter is most often described by transfer models relating to excitation forces, sources may be characterized by the forces they excert on the receiving structure under operating conditions. Over the last two decades or so, various techniques to experimentally determine these forces have been proposed. They all have in common that the forces are inferred from the resulting vibration responses on and the transfer characteristics of, the receiving structure. They mostly differ in the way how the transfer characteristics of the receiving structure is obtained: by nonparametric or various parametric models, both of which can be based on measured or synthetic data. Regardless of the transfer model employed, all these techniques suffer from a high sensitivity to uncertainties in the data (measured or calculated), which means that it may become very hard to reliably predict the forces one is interested in. This is true even when no formal inversion of the transfer model is required. Therefore, techniques to quantify errors and to "regularize" the problem have evolved. In the present paper, the advantages and drawbacks of the various methods are discussed. Special attention is paid to issues that are felt to be rather under-represented in discussions found in the literature body, such as the choice of deterministic or stochastic force models. On the basis of this comparison, it is attempted to sketch the directions of possible further research.

1 - INTRODUCTION
In analyzing structure-borne noise problems, it is sometimes desired to characterize vibration sources in conjunction with a given receiving structure. Since the dynamical behavior of the latter is most often described by transfer models relating to excitation forces, sources may be characterized by the forces they exert on the receiving structure under operating conditions.
Over the last two decades or so, various techniques to experimentally determine these forces have been proposed. They all have in common that the forces are inferred from the resulting vibration responses on and the transfer characteristics of, the receiving structure. Since it is easier to consider point forces only (which may either actually be justified or represent the sampling of a distributed load), one has a classical multiple-input/multiple output (MIMO) problem, where the L inputs (excitation forces) and the M outputs (vibration responses) are related via

$$\begin{array}{ccccccc} \mathbf{S}_{yy}(\omega_s) & = & \mathbf{H}_{fy}^{H}(\omega_s) & \cdot & \mathbf{S}_{ff}(\omega_s) & \cdot & \mathbf{H}_{fy}(\omega_s) \\ (M\times M) & & (M\times L) & & (L\times L) & & (L\times M) \end{array} \quad (1)$$

assuming linearity and time invariance. In this equation, $\mathbf{S}_{yy}(\omega_s)$ is the $M\times M$ vibration response spectral density matrix (SDM), $\mathbf{S}_{ff}(\omega_s)$ the $L\times L$ excitation force SDM and $\mathbf{H}_{fy}(\omega_s)$ the $L\times M$ transfer function matrix and ω_s the specific frequency one is interested in. The spectral density matrices are, for each frequency of interest, square Hermitian matrices with auto spectral densities on the diagonals and cross spectral densities elsewhere. The superscript H symbolizes the Hermitian matrix operation (i.e. the complex conjugate transpose). The representation of spectra as power densities (i.e. Fourier transforms of correlation functions) acknowledges the fact that real-world processes can usually not be

modeled by deterministic approaches. (This is sometimes ignored, but it actually is important as it bears consequences for measurement techniques and error handling).
It can be seen from eq. 1 that, if the vibration responses and the transfer characteristics of the receiving structure are known, one can infer the forces, provided $M \geq L$. To this end, one has to get estimates of vibration response and transfer characteristics and to find a way to effectively solve eq. 1 for the force spectra.
Whereas all methods known to the author require the in-situ measurement of the vibration responses (by various means: accelerometers, laser vibrometers, strain gages), they differ in the way how the transfer characteristics of the receiving structure is obtained and by which techniques the solution of eq. 1 is attempted to be made robust.

2 - OVERVIEW OF METHODS

The feature that above all distinguishes the various methods from each other is the way the transfer properties of the receiving structure are obtained. The techniques for solving eq. 1, on the other hand, can roughly be characterized by two refinement steps: first, a solution based solely on the transfer model (either by inverting it formally or by an appropriate formulation that already incorporates the mapping from responses to forces), and second, so-called regularization measures which in general use information of both the transfer model and the operating responses to find an optimal solution. The latter issue will be part of the discussion in section 3.4, whereas the following (certainly not exhaustive) overview will be given on the basis of the different transfer models employed.

2.1 - Measured transfer characteristics

Due to the complexity of real-world structures, most inverse force synthesis methods rely on transfer models based on measured data.
FFT-based transfer models. In the majority of the published work on inverse force synthesis FFT-based transfer functions are used. This means that for each of the FFT frequency points a transfer function matrix is calculated from measured input-output data. This transfer function matrix is then subjected to a pseudoinverse operation to yield a mapping from measured vibration responses to the desired forces. Within this framework, one still has a variety of options: Hammer versus shaker excitation, deterministic versus random test signals, direct versus reciprocal measurements, single versus multiple output measurements, various transfer function estimates, the number and location of the vibration response sensors. Investigations on resulting errors have shown that the following guidelines should be followed: 1.) use shaker excitation, 2.) measure directly, 3.) measure all vibration responses simultaneously and do not change the setup between the transfer function and the operating response measurements, and 4.) use the H_1-estimate. See [1] for details.
In summary, the FFT-based measurement methods are appealing because they potentially yield the most accurate results (at the expense of the highest cost). It has been shown that by optimizing the relevant parameters (especially sensor noise, sensor location) one can get almost as accurate results as if direct measurements were possible, at least for certain frequency regions [2]. Critical factors are transfer function nonlinearities, changes in the transfer behavior between the transfer function measurement and that of the vibration responses (e.g. stiffening by reassembling a multi-point connected source-receiver system). Finally, one can of course not get better than in direct measurements, i.e. the known difficulties measuring lateral forces or moments will continue to be with us, but they will not yield substantially more trouble than in direct measurements.
ARMA and modal transfer models. Once the raw input-output data is available one can proceed further and construct a parametric transfer model such as a modal model [3] or an ARMA model [4], which can then in turn be inverted. The hope behind this is that the known sensitivity to small errors, which is higher at certain frequencies and lower at others, could perhaps be globally lowered by modeling larger frequency regions instead of single frequency points. However, it turned out that this is not the case: the performance regarding error sensitivity does not improve and one will get additional errors by deviations between the actual transfer behavior and the model [5], [4]. The use of these transfer models is therefore not recommended.

2.2 - Calculated transfer characteristics

As experiments are expensive and cumbersome, there is a general trend towards computational methods. To date, these models are limited to simple structures, but this may change in the future.
Finite difference approximation of equations of vibration. One attempt consists of applying a spatial finite difference scheme to solve an appropriate description (by linear differential equations) of the forced vibration of the structure in question (e.g. the inhomogeneous wave equation of a bending

beam, [6]). This approximation yields the force at a given point as a function of the displacements at a number of nearby points. Thus, the resulting formulation already provides an inverted transfer model.
The advantage of this method is that only local (nearby) vibration information is necessary to reconstruct the loading at any given point on the sampling grid. This constitutes an important data reduction and makes a sequential identification of force spectral densities possible, thus reducing the number of simultaneous data acquisition channels. However, this data reduction (which in effect is a first order approximation) will also be a source of errors (in addition to the errors introduced by deviations of the true transfer behavior from the model).
Wavenumber domain solution of equations of vibration. Instead of applying a spatial finite difference scheme one can also use a spatial Fourier transform to solve the equations of forced vibration of the respective structure in the wavenumber domain. Again, a formal description of the forced vibration by linear differential equations is required. An application example is found in [7,8], where Mindlin's plate equations were used to derive a wavenumber domain expression of the forces on the sampling grid, as a function of the normal displacements at all grid points.
This method does not introduce additional errors by finite difference approximations, but requires displacement information at *all* points to be taken into account.
Finite element transfer models. A common problem of the above presented methods is that they require an explicit formulation of the forced vibration by linear differential equations. This is feasible for simple structures such as beams and plates, but will become hard or impossible for more complicated structures. Therefore, finite element transfer models may become a good choice in the near future.
In summary, by using calculated transfer characteristics one gets rid of potentially difficult measurements but will have to cope with additional errors introduced by real-world deviations from idealized models. The use of FE models could be promising in the near future, in particular if one can get them to handle lateral and rotational force components adequately.

3 - ERROR HANDLING

Virtually everyone who has ever set out to inversely measure excitation forces was sooner or later confronted with the problem of the notoriously high sensitivity of the synthesized forces to small errors in the data. Therefore, it is important to understand how these errors are generated and by which mechanisms they can be minimized.

3.1 - Number of simultaneous response measurement channels

An important requirement concerns the number of simultaneous measurement channels. Although it is in principle possible to obtain all necessary information from 2-channel measurements, it has been shown that this may yield high errors [1]. These errors are due to the statistical nature of the responses: In each consecutive measurement another realization of the process in question is observed.
Therefore, the most reliable approach consists of measuring all responses simultaneously. This has the advantage that all sensors can be left in place (i.e. will be part of the receiving structure), which offers a number of benefits, in particular if the transfer characteristics are measured: the whole response measurement setup would be used for both transfer and operating response measurements, i.e. no calibration, no errors by detaching/reattaching of sensors or sensor mass effects. Also, a random test signal could be used in the transfer measurement which could be designed to be similar to the expected force signal, in which case the effects of transfer nonlinearities would be minimized. The drawback is a high measurement expenditure, even if sensor locations can be chosen such that no or only a weak overdetermination is required.
If one wants a less expensive solution, one could resort to multiple reference methods (e.g. [9]), which have their roots in holography techniques. The idea is to decompose the matrix of response spectral densities in

$$\mathbf{S}_{yy}\left(\omega_s\right) = \mathbf{S}_{ry}^{H}\left(\omega_s\right) \cdot \mathbf{S}_{rr}^{-1}\left(\omega_s\right) \cdot \mathbf{S}_{ry}\left(\omega_s\right) \tag{2}$$

where the subscript r denotes the reference signals (which may be chosen from the responses). The number of references must be equal to or greater than the number of incoherent sources, which should first be identified (e.g. by singular value decomposition techniques). Then, one only has to measure the references simultaneously, whereas the cross spectra between references and responses ($\mathbf{S}_{ry}\left(\omega_s\right)$) can be determined sequentially. The drawback of this technique consists of additional errors introduced by sensor detaching/reattaching and related issues, by the now necessary calibration and by the possible cumulation of noise effects.

3.2 - Estimating error bounds

As errors may easily become so important that no meaningful result whatsoever is obtained even with accurately (according to "normal" standards) measured data, it is important to have an idea on how large they are for a given measurement. To date, error prediction models exist for methods based on measured transfer functions only [2], [10]. They cover the following error types (assuming standard measurement techniques): 1.) measurement noise induced bias on response spectra, 2.) random errors on response spectra, 3.) random errors on transfer functions, 4.) leakage. The influence of these errors on the synthesized force spectra can be predicted on the basis of known or measured information. However, in their present form, these models predict rather conservative error bounds.

Tighter bounds can be obtained by using estimates of measurement noise induced errors and random transfer function errors together with Monte-Carlo or tolerance matrix methods. First experiments have shown that these methods can yield very accurate problem-specific error bounds, but require more information and are computationally more expensive than the original ones. Also, they cannot be transformed to a similar problem such as another excitation situation (which can easily occur e.g. if the vibration source is replaced by another one). Still, precise error predictions will be needed, e.g. for choosing optimal regularization parameters (see discussion in section 3.4), which is why these methods could be a worthy subject of future work.

3.3 - Selecting response measurement points

A critical decision in inverse force synthesis is the choice of the points at which the operating response shall be measured. Usually, it is desired to make this decision *before* the operating response measurement. Therefore, one needs selection criteria which do not depend on the actual excitation situation. In [11], the quantity

$$\Delta(\omega) \cdot \sqrt{M}/\omega \cdot \|\mathbf{H}_{fy}(\omega)\|_F^2 \rightarrow \min \tag{3}$$

was proposed to serve this purpose. In eq. 3, $\|\cdot\|_F$ is the Frobenius norm, and $\Delta(\omega)$ is a frequency dependent factor characterizing the amplification (in the F-norm sense) of noise induced bias on response spectra to errors on synthesized force spectra. $\Delta(\omega)$ may either be estimated by the square of the transfer function matrix condition number (quick but coarse) or from Monte-Carlo simulations with the given transfer function matrix (accurate but expensive).

3.4 - Regularization

Because of the high sensitivity of inverse methods to small errors in measured data, so-called regularization methods have been developed. The idea is to slightly change the transfer model such that the solution to eq. 1 is less dependent on small changes in the measured responses. In other words, one adds a small error (to the transfer model) and hopes that this is compensated for by a larger reduction in the sensitivity to the other errors. Obviously, this is a risky business because it works only if the "right" changes are applied to the transfer model: otherwise, errors may drastically increase. Of course, there are procedures to get optimal regularization, but at least the common ones cannot be applied to inverse force synthesis in a straightforward manner. This shall be illustrated in the following.

Singular value truncation / low-pass spatial filtering. One common method consists of performing a singular value decomposition of the transfer function matrix, and to base the transfer model inversion on the largest singular values only. By doing so, one decreases the rank of the transfer function matrix and thus the number of incoherent forces that could be identified. An identical effect is achieved by applying a low-pass filter to inversely synthesized forces on a spatial grid [12]. The problem now is to determine a threshold for the singular value discarding or a cut-off wavenumber for the spatial low-pass filter, respectively. If the threshold is too low (or the cut-off wavenumber too high), then no regularization effect is observed, if it is too high (or the filter cutoff wavenumber too low), then the resulting errors may be even higher than without regularization [13]. To date, there is no robust guideline to choose this parameter. Therefore, this technique should be applied with caution.

Tikhonov regularization. The second popular approach is the so-called Tikhonov regularization, which is also based on the singular value decomposition of the transfer function matrix. Here, instead of ignoring the smallest singular values, all singular values are modified according to

$$\sigma_R(\omega_s) = \sigma(\omega_s) + \beta(\omega_s)/\sigma(\omega_s) \tag{4}$$

where $\sigma_R(\omega_s)$ is the regularized singular value, $\sigma(\omega_s)$ the original one and $\beta(\omega_s)$ the regularization parameter. It is seen from eq. 4 that the relative change of the singular values is larger for small singular values and for high β's. The big question now is how to choose $\beta(\omega_s)$ in practical measurements.

The approach that is usually proposed (e.g. in inverse air-borne noise problems, [14]) is the so-called cross validation technique of which several variants exist. These cross validation techniques compute cost functions which become minimal for a given value of $\beta(\omega_s)$. The problem is that the validation consists of some sort of comparing observed responses to those predicted if the chosen transfer model is fed with different synthesized forces. This works well for problems where the difference between the two is actually caused by the sort of error the inversion is sensitive to (this will in most cases be errors related to responses, either in the response measurement or the transfer model [1], [10]). As soon as the transfer model not only consists of simple delays (such as in free-field conditions), but of resonant systems, the cross validation will sense errors that have an excitation-related structure (because they depend on the actual realization of the excitation forces) whereas the important error types such as measurement noise on the responses are masked. In other words: one tries to modify a problem according to criteria on which the problem does not depend. Therefore, this approach is less than optimal for inverse force synthesis.

However, the principle of Tikhonov regularization is certainly worth being considered. Future work is required to develop methods for choosing $\beta(\omega_s)$. This will necessarily involve the consideration of different error types. One way could be to use the above mentioned Monte-Carlo / tolerance matrix methods (section 3.2) to obtain precise error estimates on which the choice of an optimal regularization parameter could be based.

4 - CONCLUSION

It is sometimes desirable to characterize vibration sources in conjunction with a receiving structure, in which case the forces acting at the interfaces between source and receiving structure are of interest. These forces can be determined experimentally, by inverting a transfer model (which is required to be linear and time invariant) between force inputs and vibration responses on the receiving structure.

To date, these transfer models are established either by measurement or by calculation (linear differential equations, for simple structures only). Future work should extend calculations to more complicated structures, e.g. by using finite element methods.

A major concern is the high sensitivity of the synthesized forces to small errors in measured data. For the case of measured transfer functions, (conservative) error bound prediction formulae are available. Future work should build onto these models, in particular towards three goals: First, they should be extended to incorporate calculated transfer models. Second, they should be extended to less restrictive requirements (e.g. to multiple reference methods instead of the simultaneous measurement of all responses) and third, towards more accuracy for the most important error types. The latter could be achieved by problem-dependent predictions via Monte-Carlo or tolerance matrix methods.

A way to minimize the sensitivity of the synthesized forces to small errors in measured data is to apply regularization techniques. Candidate techniques include singular value truncation and Tikhonov regularization. If optimal regularization parameters were available, these methods could decrease the otherwise high force errors. To date, reliable methods to choose optimal regularization parameters in inverse force problems do not yet exist. Future work should therefore focus on this issue. A perhaps promising way could be to use high-accuracy error predictions (which will probably be available in the very near future) as criteria for choosing the Tikhonov regularization parameter $\beta(\omega_s)$.

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