inter.noise 2000
The 29th International Congress and Exhibition on Noise Control Engineering
27-30 August 2000, Nice, FRANCE

I-INCE Classification: 7.2

OPTIMIZATION OF A FREE FIELD TECHNIQUE FOR THE MEASUREMENT OF THE REFLECTION COEFFICIENT AT REAL ANGLES OF INCIDENCE

G. Jansens*, J.F. Allard, W. Lauriks*, G. Vermeir***

* Catholic University of Leuven, Laboratory for Acoustics and Thermal Physics, Laboratory for Building Physics, Celestijnenlaan 200 D, B-3001, Heverlee, Belgium

** Université du Maine, Laboratoire d'Acoustique, Avenue Olivier Messiaen, BP535, 72017, Le Mans Cedex, France

Tel.: (32) 16/32 71 28 / Fax: (32) 16/ 32 79 84 / Email: gert.jansens@fys.kuleuven.ac.be

Keywords:
ABSORPTION, IMPEDANCE, FREE FIELD, IN-SITU

ABSTRACT
Often the reflected component of the sound field above a reflecting surface is considered coming from a 'mirror source' with corrected source strength. This 'mirror source' description of the sound field above an absorbing surface is not accurate enough if this source is located close to the surface. In this paper a more precise representation of the sound field above an absorbing plane will be presented. This should allow measurements closer to the surface, which could be interesting for the in-situ determination of reflection coefficients. Improvements and drawbacks will be demonstrated on the basis of simulations. Practical problems related to measurements will be discussed and comparisons with other measurement techniques will be presented.

1 - INTRODUCTION
One of the problems related to in-situ measurements are the finite dimensions of the sample and the presence of neighboring walls, which cause disturbing reflections. One option is to apply a time window to the measured data and 'cutting' these reflections away. This can only be done when the signal can clearly be distinguished from the 'noise', or in other words when the delay of these reflections is sufficiently large. Therefore it is convenient to locate the source as near as possible to the surface.
Most in situ techniques are based on a kind of 'mirror source' model, which in his turn assumes a plane or spherical wave reflection at the surface, or they make some kind of assumption about the sample under investigation [1]. The sound field of a monopole source near a reflecting surface however cannot be described by plane waves, nor by spherical waves. The exact representation of such a sound field is a plane wave expansion: both direct and reflected field can be considered as an infinite sum of plane waves and inhomogeneous waves, with the corresponding plane wave reflection coefficients for the reflected field. It can be shown [2] that this exact representation can be simplified in the case of a sound source at a sufficiently large distance from the material and near normal incidence, which results in a correction term in comparison with the mirror source model. This simplification turns out to be valid for much smaller source-material distances then present models and makes no assumptions about the nature of the material (local/extended reacting).
A specific measurement method corresponds with each of the above representations of the sound field: a Nearfield Holographic technique as proposed by Tamura [3] will illustrate the plane wave expansion model, the two-microphone technique will illustrate the general mirror source model. It will be shown that the sound pressure at three positions above the material is sufficient to determine the reflection coefficient according to the corrected mirror source model. It appears however that this method is very sensitive to noise (source height, temperature and measurement noise), which makes it very difficult to obtain reliable results.

2 - CORRECTION TO THE MIRROR SOURCE MODEL

A two dimensional spatial inverse Fourier transform links the sound pressure field in a point (x,y,z) to the components of the wave number spectrum:

$$p\left(x,y,z\right)=\frac{1}{2\pi}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}P\left(k_x,k_y,z\right)e^{i(k_x x+k_y y)}dk_x dk_y \tag{1}$$

In the case of a monopole source in the origin, this can be transformed into

$$\frac{e^{ikr}}{r}=\frac{i}{2\pi}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\frac{e^{ik_z|z|}}{k_z}e^{i(k_x x+k_y y)}dk_x dk_y \tag{2}$$

where

- $k_z^2\equiv k^2-\left(k_x^2+k_y^2\right)$,
- k wave number in air,
- $\vec{r}\left(x,y,z\right)$ vector from point source to observation point.

The integrand on the right hand side of equation (2) represents a plane wave in case of $k_z^2>0$, and an inhomogeneous wave in case of $k_z^2<0$.
The reflected field can be described very similar to equation (2), where each plane wave is multiplied with its corresponding plane wave reflection coefficient. Hence the sound field of a monopole source above an absorbing surface can be written as:

$$p\sim\frac{e^{ik|\vec{r}-\vec{r_b}|}}{|\vec{r}-\vec{r_b}|}+\frac{i}{2\pi}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}R\frac{e^{ik_z|z+z_b|}}{k_z}e^{i(k_x x+k_y y)}dk_x dk_y \tag{3}$$

Only when the reflection coefficient can be considered as independent of the angle of incidence, equation (3) can be transformed into the well-known form

$$p\sim\frac{e^{ik|\vec{r}-\vec{r_b}|}}{|\vec{r}-\vec{r_b}|}+R\frac{e^{ik|\vec{r}-\vec{r_{b'}}|}}{|\vec{r}-\vec{r_{b'}}|} \tag{4}$$

It can be shown however that equation (4) is identical to the plane wave solution if the source is not too close to the receiver. This means that it is a good approximation of the sound field above an absorbing surface, if source and receiver are sufficiently far apart.
Brekhovskikh and Godin showed that the exact representation of equation (4) can be simplified, in case of $kR>>1$ and nearly normal incidence:

$$p=p_0\left\{\frac{e^{ikR_2}}{R_2}+\frac{e^{ikR_1}}{R_1}\left[R\left(\sin\theta\right)-\frac{iN}{kR_1}\right]\right\} \tag{5}$$

where N is a function of sinθ, not of R_1. The terms in higher orders of $1/kR$ are neglected.

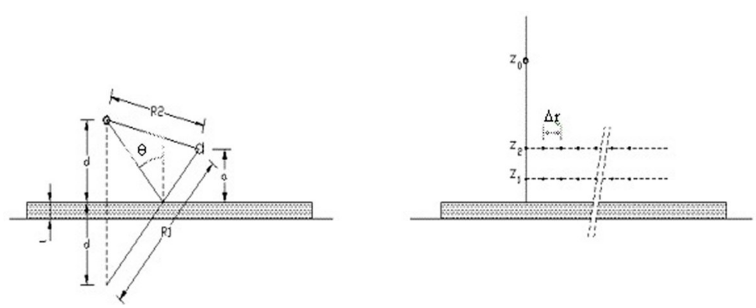

Figure 1: Measurement set-up and conventions for two and three-microphone techniques (left); measurement set-up and conventions for NAH technique (right).

With a monopole source at a distance d and a microphone at a distance a from the reflecting layer, one obtains at normal incidence $R_1=d-a, R_2=d+a$. If we repeat this measurement at three positions a_1, a_2, a_3 we have three equations:

$$p_i = p_0 \frac{e^{ik(d-a_i)}}{(d-a_i)} + p_0 \frac{e^{ik(d+a_i)}}{(d+a_i)} \left[R + \frac{A}{k\,(d+a_i)} \right],\ i = 1,2,3 \tag{6}$$

from which

$$R = \frac{Rp_0}{p_0} = -\frac{\begin{vmatrix} p_1 & e^{ika_1}/\,(d+a_1)^2 & e^{-ika_1}/\,(d-a_1) \\ p_2 & e^{ika_2}/\,(d+a_2)^2 & e^{-ika_2}/\,(d-a_2) \\ p_3 & e^{ika_3}/\,(d+a_3)^2 & e^{-ika_3}/\,(d-a_3) \end{vmatrix}}{\begin{vmatrix} p_1 & e^{ika_1}/\,(d+a_1)^2 & e^{ika_1}/\,(d+a_1) \\ p_2 & e^{ika_2}/\,(d+a_2)^2 & e^{ika_2}/\,(d+a_2) \\ p_3 & e^{ika_3}/\,(d+a_3)^2 & e^{ika_3}/\,(d+a_3) \end{vmatrix}} \tag{7}$$

According to equation (7), the measurement of the sound pressure at three positions above an absorbing layer is sufficient in order to determine the reflection coefficient at normal incidence. There are no assumptions concerning the kind of material (locally or extended reacting). It will be shown in the next section that this model remains valid at smaller source-receiver distances than the model of equation (4).

3 - SIMULATIONS

In case of a cylindrical symmetry of the sound field, the reflected field in equation (3) can be rewritten as follows [4]

$$p_r = i \int_0^{\infty} R\,(k_r)\, e^{ik_z(z+z_0)} J_0\,(k_r r)\, \frac{k_r dk_r}{k_z} \tag{8}$$

where

- $z{=}a_i,\ r_0{=}d,\ r{=}0,$
- $R = \dfrac{Z/\cos\theta - Z_s}{Z/\cos\theta + Z_s}$
- $Z_s = i \dfrac{Z_1}{\cos\theta_1 \phi} \mathrm{cotg}\,(k_1 l \cos\theta_1)$

The characteristic impedance Z_1 and the wave number k_1 of the layer are calculated according to the equivalent fluidum model [5], [6]. This model assumes that the frame of the porous material can be considered rigid which justifies the description of the sample as a fluid with a modified density and a modified compressibility. The parameters that are used in the calculations, are those determined on the sample used in the measurements. They are summarized in table 1. The correspondence of the equivalent fluidum model with measurements in the Kundt's Tube is very satisfying.

thickness	density	tortuosity	porosity	airflow resistivity	viscous characteristic length	thermal characteristic length
l [m]	ρ [kg/m^3]	α_∞	ϕ	σ [Ns/m^4]	Λ [μm]	Λ' [μm]
0.02	25	1.1	0.99	3700	150	300

Table 1: Parameters of the material under investigation.

With the help of the first term in equation (3) for the direct field and equation (8) for the reflected field, the sound pressure above the reflecting surface is calculated. These pressures are used in equation (7) to calculate the reflection coefficient according to the 'three-microphone technique'. This result is subsequently compared with the results of the theoretical values as calculated with the equivalent fluidum model, and also with the results of the two-microphone technique (here only the pressures of the two microphones the nearest to the surface are used).

One can clearly see that the two-microphone technique does not give accurate results with source-receiver distances smaller than about 20 cm. Important (> 10 %) deviations from the analytical model can be noticed at low and high frequencies. Moreover the distance at which the two-microphone technique becomes inapplicable depends on the properties of the material. The results of the three-microphone technique however are very convincing in the middle and higher frequencies. For the lower frequencies

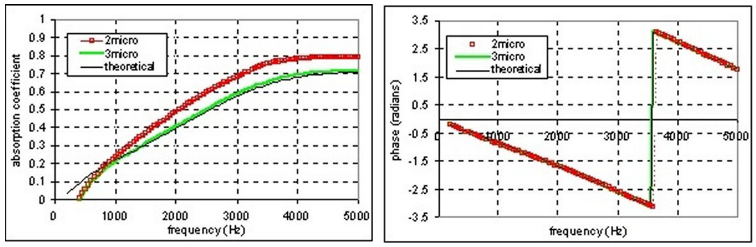

Figure 2: Absorption coefficient and phase of the reflection coefficient calculated from simulated sound pressures (d=20 cm, a_i= 0.5, 5.5 and 10.5 cm).

(< 750 till 1000 Hz depending on the material) the condition of kR being sufficiently large is being violated (for the source-receiver distances used in this paper at least) and no improvements are achieved as compared with the two-microphone technique. Larger source-receiver distances are required in order to get accurate results below these frequencies.

4 - MEASUREMENTS

The technique, which is most closely related to the plane wave expansion model of the sound field, is a NAH measurement developed originally by Tamura [3]. This measurement method scans the sound field in two planes above an absorbing material. These sound pressures allow the calculation of the reflection coefficient, not only at normal incidence, but also at oblique incidence and even for inhomogeneous waves.

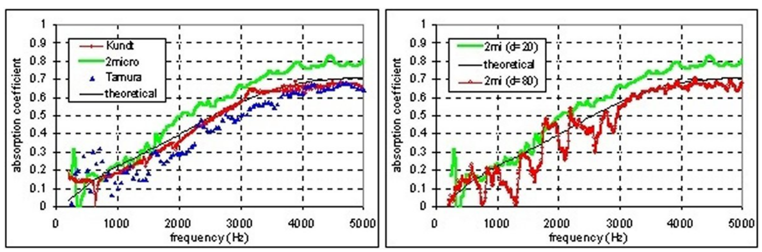

Figure 3: Left: Absorption coefficient according to different measurement techniques: Tamura (d=8.7 cm, z_1=0.3 cm, z_2=0.8 cm, Δr=1 cm, n=50), two-microphone technique (d=20 cm, a_1=0.7 cm, a_2=2.7 cm), Kundt's tube; right: Influence of source height with the two-microphone technique (d=20, 80 cm).

The two-microphone technique is only valid in the case of sufficiently large source-receiver distances. The results of measurements at two source-receiver distances are presented ($d \approx$ 80, 20 cm) and compared with measurements in the Kundt's tube. As expected the small source-receiver distance in the second case, gives an overestimation of the absorption coefficient for the middle and higher frequencies.
It turns out to be very difficult to obtain good experimental results with this new three-microphone technique. This technique is very noise sensitive, which can be illustrated by adding a small amount of noise to the calculated sound pressures in the simulations. The results of the three-microphone technique are completely disturbed, while the two-microphone technique seems to be much more robust. Small errors on source height and temperature (or sound speed) can be considered as a kind of noise and have a dramatic influence on the three-microphone technique.

5 - CONCLUSIONS

In this paper a correction to the classic 'mirror source' model is proposed, which makes it possible to describe the sound field above a reflecting surface at much smaller source-receiver distances. This could be interesting for in-situ techniques, for the windowing of disturbing reflections would be easier. According to this corrected model, the sound pressure at three positions above the absorbing layer is sufficient to determine the reflection coefficient of this layer. Especially at middle and high frequencies important improvements can be expected.
However this correction appears to be very sensitive to noise, whereas the two-microphone technique turns out to be rather robust.

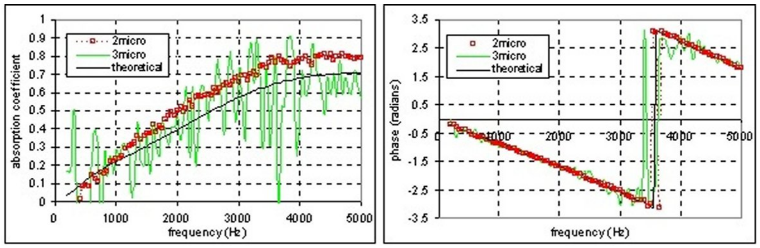

Figure 4: Same as Fig. 2, but a small amount (about 10 %) of noise is added to the calculated sound pressures.

REFERENCES

1. **J.F. Allard, Y. Champoux**, In situ two microphone technique for the measurement of the acoustic surface impedance of materials, *Noise Con. Eng. Journ.*, Vol. Jan./Feb.pp. 15-23, 1989

2. **L.M. Brekhovskikh, O.A. Godin**, *Acoustics of layered media II*, Springer-Verlag, Berlin, 1992

3. **M. Tamura**, Spatial Fourier transform method of measuring reflection coefficients at oblique incidence. I: Theory and numerical examples, *J. Acoust. Soc. Am.*, Vol. 88 (5), pp. 2259-2264, 1990

4. **L.J. Ziomek**, *Fundamentals of Acoustic Field Theory and Space-Time Signal Processing*, CRC Press, Florida, 1995

5. **Y. Champoux, J.F. Allard**, Dynamic tortuosity and bulk modulus in air-saturated porous media, *J. Applied Phys.*, Vol. 70 (4), pp. 1975-1979, 1991

6. **Ph. Leclaire, L. Kelders, W. Lauriks, M. Melon, N. Brown and B. Castagnede**, Determination of the viscous and thermal characteristic lengths of plastic foams by ultrasonic measurements in helium and air, *J. Applied Phys.*, Vol. 80, pp. 2009-2012, 1996