inter.noise 2000
The 29th International Congress and Exhibition on Noise Control Engineering
27-30 August 2000, Nice, FRANCE

I-INCE Classification: 2.6

INFLUENCE OF LONGITUDINAL PARTITIONS ON THE TRANSVERSE MODES PROPAGATION IN CYLINDRICAL DUCTS

J.-L. Peube, C. Prax

Laboratoire d'Etudes Aérodynamiques (Unité Mixte de Recherches 6609), 40 Avenue du Recteur Pineau, 86022, Poitiers Cedex, France

Tel.: 0549453814 / Fax: 0549453663 / Email: jl.peube@lea.univ-poitiers.fr

Keywords:
MODES, PROPAGATION, DUCTS

ABSTRACT
Wave propagation of transverse modes in a cylindrical duct is discussed for the case of square cross sections with internal longitudinal partitions being either protuberances on the inner cylindrical duct surface, or a partial partitioning set in the duct central part. With no effect on longitudinal mode, these partitions leads to lower values for the eigenvalues of Helmholtz equation which determine the cut off frequencies. A numerical solution with finite elements using a control volume technique is given. Especially, the first eigenvalue behavior becomes then singular when the acoustic flowrate is confined into narrow free passages and a Helmholtz resonator model is used for this mode. With no singular cross section obstruction the decreasing of the cut off frequencies of transverse modes is regular.

1 - INTRODUCTION

Wave propagation in a rigid cylindrical duct leads to a eigenvalue problem of Helmholtz equation for complex pressure amplitude $P(x,y)$ in a cross section. The presence of internal cylindrical partitions leads to changes in eigenvalues and cut-off frequencies body which have not been extensively studied, no analytical solution being practicable.

2 - EIGENVALUES PROBLEM

Consider now for example a cylindrical duct W in which we have a cylindrical body limited by surface Γ (figure 1a). C being phase velocity, complex pressure amplitude $P(x,y)$ for modal progressive waves satisfies the equation:

$$\frac{\partial^2 P}{\partial x^2} + \frac{\partial^2 P}{\partial y^2} + \omega^2 \left(\frac{1}{c^2} - \frac{1}{C^2} \right) P = 0 \text{ with } \frac{\partial p}{\partial n} = 0 \text{ on } \Gamma \text{ and } W. \quad (1)$$

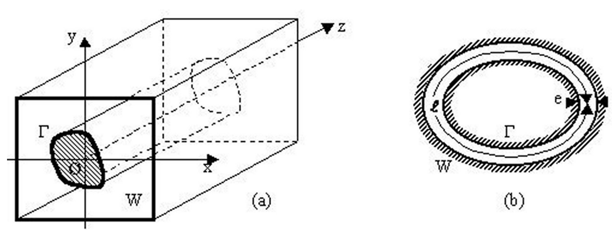

Figure 1: Propagation in cylindrical duct with internal longitudinal partitions.

For propagative waves, pulsation ω is a real number so that cut-off pulsation ω_c is obtained for an infinite value for C. The eigenvalues $\Lambda = \omega \left(\frac{1}{c^2} - \frac{1}{C^2} \right)^{\frac{1}{2}}$ of the problem defined with equation and conditions

(1) are the reduced cut-off pulsations; they are also reduced resonance pulsations of the 2-dimensional cavity with the same cross section as previously. The more the body Γ is cumbersome, lower are the cut-off pulsations. Physical reason of this decrease of cut-off pulsations is that the effective distance on which acoustic waves are travelling is increased, the body forcing the waves to travel around it.

3 - EFFECTS OF SMALL RESTRICTED SECTIONS

In the limit case when internal walls limit a narrow channel of constant thickness e (figure 1b), it is quite obvious that the lower eigenvalues will be obtained for circumferential waves along the perimeter l. For a length l including p wave lengths, eigenvalue is equal to

$$\Lambda = \frac{p\pi}{\ell} \tag{2}$$

This approximation is quite good as it can be seen in table 1, for the case of an annular circular duct with radius ratio equal to 2, external radius of 1 (and so l=0,75p).

	mode 1	mode 2	mode 3	mode 4
Λ formula (2)	1,333	2,667	4,000	5,333
Λ_{anal}	1,355	2,681	3,958	5,333

Table 1: Eigenvalues for circular annular duct; Λ_{anal} from ref. [2].

Same result is obviously valid for a square annular duct, for which we have no analytical value.
Consider now the case of an internal body Γ having a different shape from the duct cross section; if the body size increase, it will appear some narrow localized obstructions which are small zones of high acoustic kinetic energy. So, we have Helmholtz effects which can be studied in the usual way [4].
For the cases of a cross or a 2D diaphragm in a square cross section duct described on figure 2, from [4] we can calculate equivalent acoustic inductance L_a and capacity C_a for these Helmholtz effects:

$$L_a = \frac{2\rho}{\pi L}\ln\frac{2L}{\pi h} \; C_a = \frac{V}{\rho c^2} \tag{3}$$

The form factor h/L, h (or $h/2$) being the restricted section for the cross (or the diaphragm) in the square duct, has the same value for the two considered cases. Taking into accounts the multiple cavities and Helmholtz necks we find the in these two configurations:

$$\Lambda = \left(2\pi/\ln\frac{2L}{\pi h}\right)^{\frac{1}{2}} \tag{4}$$

Results calculated with formula (4) are only valid for h/L small enough (figure 3).

4 - NUMERICAL PROCEDURE AND RESULTS

To solve the Helmholtz equation (1), the control-volume based finite-element method (CVFEM) [1] has been employed. The main advantage of this method is to conjugate the flexibility of finite element mesh and an integration technique based on a local balance equation. The results show more accuracy than for other classical finite element methods.
The cavity is first discretized into three nodes triangular finite elements mesh. The grid points at which the value of pressure field has to be computed are located at the corners of the elements. A linear interpolation is used to approximate P within each element. Polygonal control volumes are then defined around each node by joining the centroid and the middles of sides of the triangular elements next to this node. Integration of basic conservative equations of acoustics on every control volume gives discretized equations at each node.
All boundaries in the cavity are rigid walls. Neumann conditions are then used, normal pressure gradient (i.e. normal acoustical velocity) is zero at wall. This condition is implicitly respected by the control volume balance at boundary.
Partition-walls are created by doubling nodes and modifying elements connectivities.
One finally obtains a classical linear homogeneous algebraic system. The generalized eigenvalue problem is solved by using a QZ algorithm [3].
Square cavity with either a diaphragm or a central cross-shaped partition-wall are studied. We used a 21x21 regular grid and some double grid points to create partition-walls.

The first seven eigenvalues have been calculated for various values of form factor h/L. Results for three of them are presented in table 2 with corresponding values for the square duct without partitioning (1st column). As it can be seen for the cases where partitioning has no effect on a mode (partition-walls are then streamlines), numerical errors increase with the order of the mode and when the restricted free-section becomes low. Apart numerical errors, increasing size of partitioning will decrease corresponding eigenvalues of each configuration. Because of symmetry some eigenvalues are the same for the two configurations.

		h/L=0.6	h/L=0.6	h/L=0.4	h/L=0.4	h/L=0.2	h/L=0.2
	square	cross	diaphragm	cross	diaphragm	cross	diaphragm
mode 1	3,142	2,812	2,810	2,417	2,418	1,966	1,970
mode 2	3,142	2,814	3,145	2,421	3,145	1,969	3,184
mode 3	4,443	4,367	3,999	3,929	3,575	3,107	3,238
mode 4	6,283	6,297	6,297	6,297	6,297	6,307	6,298
mode 5	6,283	6,305	6,304	6,304	6,301	6,309	6,301
mode 6	7,025	6,689	6,689	6,400	6,401	6,310	6,313
mode 7	7,025	6,693	7,064	6,410	7,063	6,314	7,066

Table 2: Eigenvalues for different form factors.

First eigenmode shown in figure 2 for form factor of 0.4 behaves Helmholtz resonator like. When the acoustic flowrate and kinetic energy are confined to a restricted free-section, numerical results in figure 3 tends to agree with the previous Helmholtz resonator model.

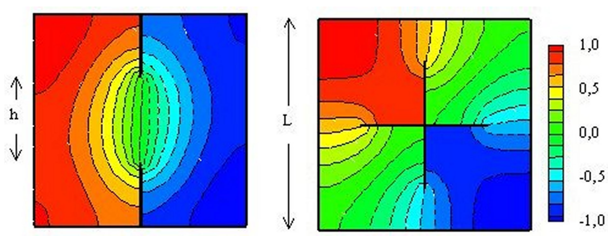

Figure 2: First eigenmode for diaphragm and cross partition in duct central part (form factor h/L=0.4, total restricted section h, eigenvalue L=2.42).

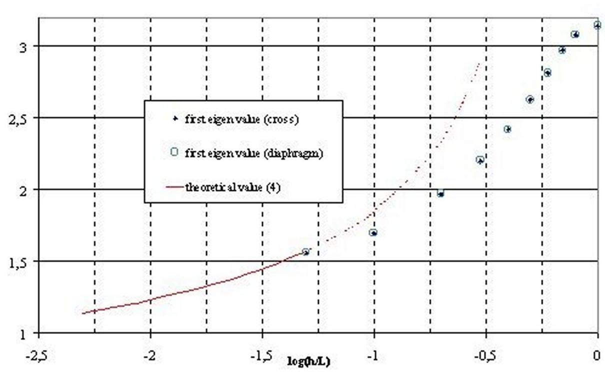

Figure 3: First eigenvalue as a function of form factor h/L.

5 - CONCLUSION

The effect of insertion of partition-walls inside a cylindrical duct is to decrease the cut-off frequencies in a regular way, except for the first modes for which some Helmholtz singular effects may happen.

REFERENCES

1. **O.Vaillant, C. Prax, H. Sadat**, Mise en oeuvre d'une méthode de volumes de contrôle à maillages non structurés pour le calcul de modes propres acoustiques, *Revue européenne des éléments finis*, Vol. 8 (3), pp. 251-271, 1999

2. **J.F. Bridge, S.W. Angrist**, An extended table of roots of J'n(x), Y'n(beta.x) -J'n(beta.x), Y'n(x)=0, *Math. Comput.*, Vol. 16, pp. 198-204, 1962

3. **J.H. Wilkinson**, Kronecker's canonical form and the QZ algorithm, *Lin. Alg. Appl.*, Vol. 28, pp. 285-303, 1979

4. **P.M. Morse, K.U. Ingard**, *Theoretical Acoustics*, McGraw-Hill, pp. 486-490, 1968