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DETERMINATION OF SEA COUPLING LOSS FACTORS BETWEEN CONCRETE/MASONRY PLATES USING FEM AND ESEA

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ABSTRACT
This paper is concerned with the application of Statistical Energy Analysis (SEA) to subsystems that have low local mode counts in prescribed frequency bands. This feature is common to many structural subsystems at low frequencies, e.g. third octave band analysis of vibration transmission between concrete/masonry plates in buildings. For this reason, both deterministic and statistical approaches can be considered in the prediction of sound transmission between concrete/masonry plates. The approach considered in this paper is to use the framework of SEA and incorporate coupling parameters and statistical confidence limits determined from Finite Element Methods (FEM) and Experimental SEA (ESEA) for an ensemble of 'similar' low modal density structures.

1 - INTRODUCTION

Computational [1,2] and physical [3] experiments on beams and plates indicate that the Coupling Loss Factor (CLF) will approximate values that are predicted from wave theory transmission coefficients when the larger of the modal overlap factors for two coupled elements is greater than or equal to unity ($M \geq 1$). Fahy and Mohammed [1] apply an extra condition for plates that there should be at least five modes in the frequency band ($N \geq 5$). When these conditions are not met, predictive SEA can still be used but it must be accepted that errors of unknown magnitude can occur.

In this paper, experimental SEA and FEM are used to determine CLF values between coupled plates with low modal densities and low modal overlap. This approach requires the introduction of the 'ESEA ensemble' as a method of dealing with low modal density subsystems. This gives the opportunity to assess the statistics of the CLF and use the framework of SEA to determine the 'expected range of response' using the 95% confidence interval for the CLF values. Although this paper is only concerned with the output from numerical experiments, the FEM models have previously been validated with measured data from masonry walls that had low modal densities and low modal overlap factors [4].

2 - USING FEM, ESEA, THE ESEA ENSEMBLE AND SEA

With numerical experiments using ESEA, there is the potential to create an ESEA ensemble. Like the SEA ensemble, the ESEA ensemble considers systems that consist of subsystems with 'similar' properties. However, unlike the SEA ensemble, it can include subsystems where the SEA assumption of equipartition of modal energy in a frequency band (i.e. incident energy uniformly distributed over a range of angles) does not apply but ESEA 'weak' coupling still exists. The SEA ensemble considers uncertainty in the description of the modal features. In contrast, the ESEA ensemble is intended for subsystems where there is *limited* knowledge about the modal features but uncertainty as to how the eigenfrequencies will be distributed amongst the frequency bands of interest. The ESEA ensemble can therefore be used for 'similar' sets of structures that have high tolerances on the dimensions and/or material properties. For these structures, a single deterministic analysis is likely to be of limited use.

The following example is used to illustrate a potential application of the ESEA ensemble to buildings. Similar examples could be found for ship, automobile and aerospace engineering. This example concerns the prediction of vibration transmission in third octave bands between adjacent dwellings for a set of 'similar' dwellings. The required output of the study is to find the vibration of the plates in terms of the mean response as well as the expected range of the response. It is the latter output that is expected to be of most interest to the engineer. The walls and floors are composed of rectangular concrete/masonry plates and therefore the subsystem modal density is low and equipartition of modal energy for each subsystem in a chosen frequency band does not occur. However, for the set of 'similar' dwellings there will be variations due to workmanship, material properties and plate dimensions such that any certainty regarding the modes is counteracted by the uncertainty in the prediction of the eigenfrequencies and eigenfunctions at low frequencies. The requirement for third octave bands exacerbates the problem because 'similar' plates in a set of 'similar' dwellings could have zero, one or more eigenfrequencies in the same third octave band.

One approach to this study would be to use predictive SEA with CLF values determined from angular average wave theory [5]. However, errors could occur from incorrect CLF values due to low modal density, low modal overlap [1] and rectangular plates [6]. Also, the output would only be the mean response. This is due to the lack of formal procedures in SEA to determine the expected range of the response for these subsystems. Due to the relatively large plate sizes, variations in material properties and dimensions of the plates in the set of 'similar' dwellings are likely to have negligible effect on the plate energy or the wave theory CLF values in the SEA model. For this reason and the strong modal dependence that can be expected, simple variations to the subsystem properties in the SEA model with wave theory CLF values can not be used to create a realistic range for the response.

FEM ESEA is used here to determine CLF values that could be used in predictive SEA. The ESEA ensemble represents coupled plate junctions in the set of 'similar' dwellings by taking account of the variation in material properties and/or dimensions. To use this approach, SEA must be appropriate for the system under study. This can be indicated during the FEM ESEA analysis by the ability to determine positive CLF and ILF values and also by well-conditioned energy matrices. The advantage of this approach using numerical experiments is that both the mean and variance of the ensemble average CLF can be found without including the effect of sampling errors in the plate energy. These can be significant with physical experiments. Therefore, the mean response for the ESEA ensemble can be obtained and there is an opportunity to calculate the expected range of the response. The latter can be found by ascribing statistical confidence limits to the CLF values such that each CLF can take two values corresponding to the lower and upper confidence limits. These can be used in a series of SEA models including *all* possible permutations of the confidence limits for *all* the CLF values. The number of permutations for the SEA model is equal to 2^{n} where n is the number of CLF values that have lower and upper confidence limits. Although the number of permutations soon increases with many coupled subsystems, matrix solutions are sufficiently fast that this approach will be feasible for small numbers of subsystems. It may also be useful with SEA models where the majority of CLF values are single values determined from wave theory and only a small number of CLF values have confidence limits. The final step is to define the expected range of the response by the minimum and maximum subsystem energy ratios.

In this paper, the uncertainty in the ensemble input data is restricted to the plate length that is perpendicular to the junction. Uncertainty is introduced into the ESEA ensemble using the Monte Carlo technique to generate an ensemble of 'similar' test constructions. The Monte Carlo technique considered for this purpose is based upon random number generation and statistical distributions to simulate a system described by the ESEA ensemble average. Assuming that the variation of the plate dimensions can be described by the normal distribution, these input variables are drawn as random numbers from normal distributions, $N(\mu, \sigma)$ with mean μ and standard deviation σ.

3 - NUMERICAL EXPERIMENTS

The test construction used in the numerical experiments is a typical T-junction of concrete/masonry plates that occurs in buildings. Plate 1 represents the separating wall (x_1=4.0m, y_1=2.4m, z_1=0.215m, ρ_1=2000kgm^{-3}, $c_{\mathrm{L},1}$=3200ms^{-1}, ν =0.2) and plates 2 and 3 represent the flanking walls (x_2=3.5m, x_3=3.0m, y_2=y_3=2.4m, z_2=z_3=0.1m, ρ_2=ρ_3=600kgm^{-3}, $c_{\mathrm{L},2}$=$c_{\mathrm{L},3}$=1900ms^{-1}, ν=0.2).

ANSYS software was used to generate the FEM data with the SHELL63 element (dimensions $<\lambda_{\mathrm{B}}/6$) and 'rain-on-the-roof' excitation. All test constructions had simply supported plate boundaries and a simply supported junction line such that only bending wave motion was considered. The dissipative loss factor, $\eta_{\mathrm{d(FEM)}}=f^{-0.5}$ was used to simulate the total loss factor that would be encountered for walls that were fully connected in complete buildings. The ESEA ensemble was created through the use of random numbers drawn from a normal distribution to vary the wall dimension perpendicular to the junction for all plates. $N(\mu, \sigma)$ used the x dimensions as the mean value μ with a standard deviation σ=0.25m. This ensemble contained thirty members. Single frequency data were generated to create third octave band data in the range 50Hz − 1kHz. To reduce computation times, the number of single frequencies in each band depended upon the modal overlap factor (calculated using $\eta_{\mathrm{d(FEM)}}$ and the local mode count for bending modes on plates with all boundaries simply supported). For M<0.5, 2Hz steps were used to calculate the frequencies for each band. For M ≥0.5, three frequencies were used in each band where one frequency was the band center frequency with the other two frequencies equally spaced over the third octave bandwidth.

The modal overlap factors and mode counts for the T-junction plates are shown in Figure 1 for all thirty ensemble members. These indicate the frequency ranges in which the modal overlap factor, M<1 and the local mode count, N<5. It also highlights the role of the ESEA ensemble as trying to ensure that with low modal density subsystems, there are members of the ensemble that have a local eigenfrequency that falls within each third octave band of interest. In practice, the use of a global mode approach such as FEM means that it is the global eigenfrequencies that are of interest. However, this figure is intended to provide an overview of the ensemble from the local mode viewpoint of SEA.

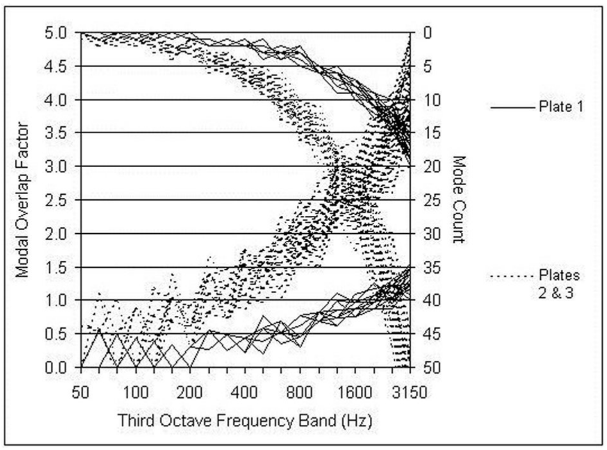

Figure 1: Ensemble data for the T-junction; modal overlap factor (lower curves) and mode count (upper curves).

4 - STATISTICAL DISTRIBUTIONS OF FEM ESEA CLF DATA

The aim of this section is to assess the normality of the *linear* CLF using normal quantile plots. Example normal quantile plots are shown on Figure 2 for both linear and logarithmic η_{12} values for the fourteen third octave bands in the range, 50Hz − 1kHz. From Fahy and Mohammed [1], it is expected that non-normal distributions will occur for the linear CLF when M<1. In these example plots, all third octave bands have M<1 for at least one of the subsystems i and j that are involved in η_{ij}. The linear CLF data indicates that when M<1, the distributions can have significant right skew. Therefore a logarithmic transformation can be applied to determine if the linear CLF ensemble could be described as a lognormal distribution. The transformation to logarithmic CLF values gives rise to sufficiently straight lines that the linear CLF can be described as a lognormal distribution.

The evidence in this section (albeit limited) indicates that there is potential in the use of the ESEA ensemble for subsystems with low modal density and low modal overlap. The ability to determine statistical parameters that describe the lognormal distribution of the linear CLF facilitates its inclusion in predictive SEA using the matrix solution. The next step is to obtain the expected range of $D_{v,ij}$ by using the FEM ESEA CLF 95% confidence limits in predictive SEA.

5 - PREDICTIVE SEA USING FEM ESEA CLF DATA

The required output from the SEA model is the velocity level difference, $D_{v,ij}$ in decibels where i is the source subsystem and j is the receiving subsystem. This section compares the FEM ensemble $D_{v,ij}$ data with ensemble average, minimum and maximum $D_{v,ij}$ data determined from predictive SEA using the FEM ESEA CLF. Example $D_{v,ij}$ data are shown on Figure 3. Comparison of predictive SEA using the FEM ESEA CLF with the FEM ensemble members allows a check on the ESEA matrix inversion and the assumption of a lognormal distribution for the linear CLF.

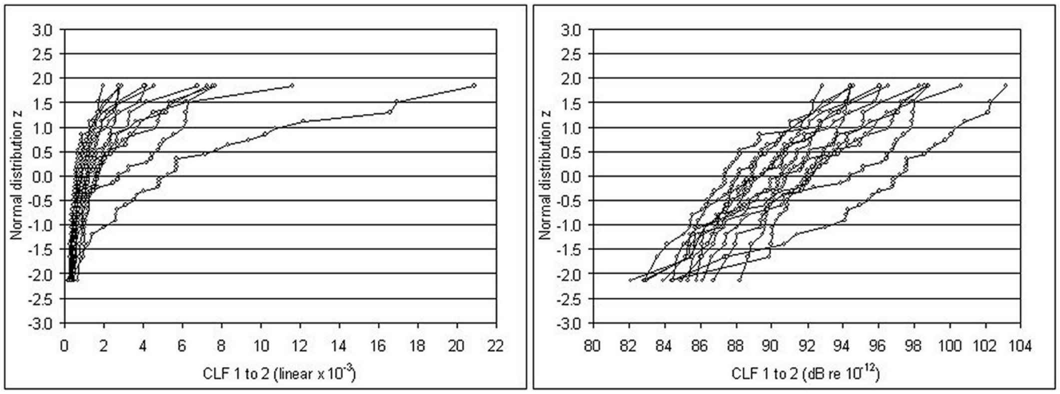

Figure 2: T-junction; simply supported junction line; normal quantile plots for linear η_{12} (left) and h_{12} in dB (right); fourteen third octave bands, 50Hz -1kHz; ensemble members: 30.

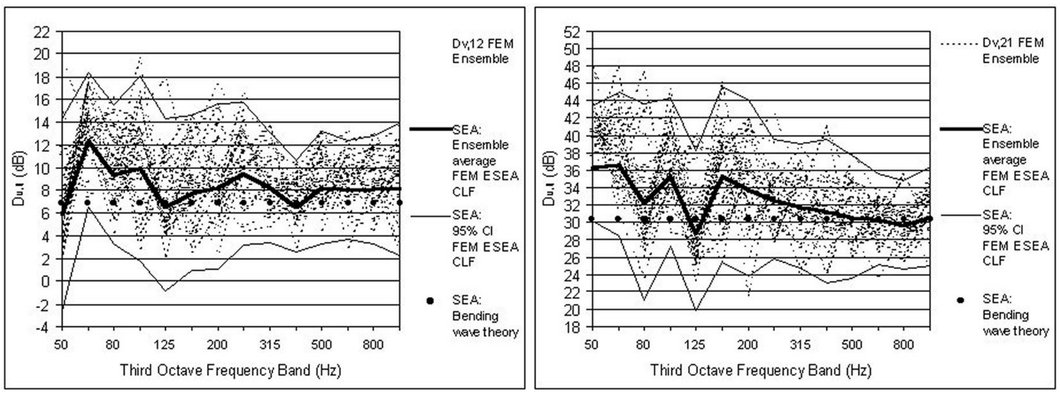

Figure 3: T-junction; simply supported junction line; vibration level differences $D_{v,12}$ (left) and $D_{v,21}$ (right); FEM ensemble with average, minimum and maximum SEA predictions using the ensemble average and 95% confidence interval FEM ESEA CLF data; ensemble members: 30.

$D_{v,ij}$ data determined using SEA wave theory are also included on the graphs. Although $D_{v,ij}$ from SEA wave theory is a reasonable approximation for the ensemble average $D_{v,ij}$ when M<1 and N<5, the range of expected values can be similar or greater in magnitude to the ensemble average value. This provides the motivation for determining the expected range of values. The minimum and maximum $D_{v,ij}$ data determined from all permutations of the FEM ESEA CLF 95% confidence intervals are seen to provide a satisfactory estimate of the expected range. In general, the minimum $D_{v,ij}$ value tends to be a slight underestimate and can be considered to err on the side of caution.

6 - CONCLUDING DISCUSSION

For the T-junction in this paper and the L-junction in a previous paper [7], there were no significant problems with negative ILF or CLF values from the ESEA matrix inversions despite the relatively high matrix condition numbers that occurred when the modal density and modal overlap was low. The only negative CLF values occurred for transmission across the straight section of the T-junction. However, in this case at least 70% of the ensemble members gave positive CLF values. This is a benefit of the ESEA ensemble approach over that of a single deterministic analysis from which the latter might lead to the conclusion that SEA is not appropriate. When the majority of the ESEA ensemble members have positive CLF values, this provides sufficient motivation to attempt an SEA model. However, future work could consider matrix-fitting procedures [8] to try and avoid negative CLF values. The statistical distribution of the linear FEM ESEA CLF was shown to approximate a lognormal distribution for engineering purposes. This allows statistical confidence intervals for the CLF to be used in both SEA path analysis and in the full matrix solution. Numerical experiments were used to assess the definition and application of the FEM ESEA ensemble with the full matrix solution. They provide some evidence that this approach can be of use in describing the large variation in response that occurs between 'similar' plate systems in which the plates have low modal density and low modal overlap.

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