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MODEL ANALYSIS THEORY AND CALCULATION FOR THE SOUND PROPAGATION IN A STRAIGHT CIRCULAR DUCT WITH MULTISEGMENTS IN THE PRESENCE OF UNIFORM FLOW

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ABSTRACT
The modal analysis is applied to investigation the sound propagation in a straight circular duct with multi-segmented acoustical treatments. And the prediction model of sound propagation in the presence of uniform flow and shear flow is established. For a three-segmented circular duct (the middle segment is acoustic liner, the others are hard walls) in the present of uniform flow, the influence of the variations of resistance and reactance of liner on the in-duct suppression and sound power level attenuation due to forward travelling pressure and velocity components, is theoretically calculated. Finally, the optimization on operating parameters, resistance and reactance of liners are also presented and numerically calculated.

1 - INTRODUCTION
Although many theoretical analysis methods on duct acoustics have been developed, until now, the modal analysis method has not been used to calculate precisely the sound propagation in a circular duct with multi-segmented acoustical treatments. Because using the modal analysis to establish sound propagation in engine duct can accurately describe the physical mechanism of sound propagation, transmission and reflection, therefore, this article tries to widen the application range of the modal analysis method for the study of sound propagation in a straight circular duct with multi-segmented acoustic liners, in order to establish the prediction model and make numerical calculation.

2 - THEORETICAL MODEL OF SOUND PROPAGATION

2.1 - Prediction model of sound propagation in a circular duct
Convecting wave equation

$$\nabla^2 p - \frac{1}{c_0^2}\frac{D_0^2 p}{Dt} + 2\rho_0 \frac{dU(r)}{dr} R(x,y)\frac{\partial u_r}{\partial z} = 0 \tag{1}$$

For uniform flow, Eq. (1) can be simplified to a Eq. (2)

$$\nabla^2 p = \frac{1}{c^2}\left(\frac{\partial}{\partial t} + V_z \frac{\partial}{\partial z}\right)^2 p \tag{2}$$

In cylindrical coordinate

$$p(r,\theta,z,t) = p_r(r)\, p_\theta(\theta)\, e^{i(k_z z - \omega t)} \tag{3}$$

Sound propagation in the present of uniform flow

$$p_r(r) = J_m(k_r r) \tag{4}$$

$$\frac{\partial p_r}{\partial r} = \left| -ik\beta \left(1 - \frac{k_z}{k} M\right)^2 p_r \right|_{r=r_0} \tag{5}$$

Substituting Eq. (4) into Eq. (5), setting eigenvalue of radial mode $\nu = k_r r_0$, the equation of eigenvalue in form of $F(\nu) = 0$ could be obtained. By solving this equation, the eigenvalue of radial mode can be determined.

Sound propagation in the present of shear flow

$$\frac{d^2 p_\theta}{d\theta^2} = -m^2 p_\theta \tag{6a}$$

$$\frac{d^2 p_r}{dr^2} + \frac{1}{r}\frac{dp_r}{dr} + \frac{2\bar{K}}{\left(1 - M(r)\bar{K}\right)}\frac{dM(r)}{dr}\frac{dp_r}{dr} + \left(k_r^2 - \frac{m^2}{r^2}\right) p_r = 0 \tag{6b}$$

where

$$k_z^2 \left(1 - M(r)^2\right) + 2M(r)\, kk_z + \left(k_r^2 - k^2\right) = 0 \tag{6c}$$

$$\nabla^2 p - \frac{1}{c_0^2}\frac{D_0^2}{Dt^2} + 2\rho_0 c \frac{dM(r)}{dr}\frac{\partial u_r}{\partial z} = 0 \tag{7}$$

By simplifying Eq. (6a) and Eq. (6b) and using the Runge-Kutta integral and Newton-Raphson iterative methods, Eq. (6a)-(6c) can be solved numerically. Substituting the radial eigenvalue into Eq. (4), the eigenfunction of radial mode can be determined. Then in terms of series expansion of eigenfunction, the following generalized solution of sound pressure could be constituted. Where k_{rmn} is model eigenvalue of (m,n) order (divided by r_0), k_{zmn} is axial propagating constant for (m,n) mode. A_{mn} is modal coefficient.

$$p(r,\theta,z,t) = \sum_m \sum_n \left[A_{mn} p_{\theta m}(\theta)\, p_{rn}(k_{r,mn} r)\, e^{ik_{zmn}z}\right] e^{-i\omega t} = p(r,\theta,z)\, e^{-i\omega t} \tag{8}$$

2.2 - Prediction model of sound propagation in a circular duct with multi-liners

Considering the transmission, reflection of forward and backward travelling waves at the interfaces of each segment, system matrix equation can be established

$$\{A\} = [S]^{-1}\{Q\} \tag{9}$$

By solving Eq. (9), mode coefficients at each interface can be obtained. After the model vector A is determined, the sound energy at interfaces of each segment can be solved by calculating the axial sound intensity listed as below.

$$I_z(r,z) = \left(1 + M^2\right) \mathrm{Re}\left[pV_z^*\right] + (M/\rho c)\, pp^* + \rho_0 c M V_z V_z^* \tag{10}$$

$$E = 2\pi \int_0^{r_0} I_z(r,z)\, r dr \tag{11}$$

3 - RESULTS AND ANALYSIS

Effects of resistance R and reactance X on the attenuation Δ and Δf at the condition of different frequency and Mach number (m=1, n=1,2,3,4)

By utilizing the theoretical model mentioned above, sound propagation in a three-segmented circular duct (radius $r = 0.254$ m) in present of uniform flow is calculated. The length of three segments is 0.248 m, 0.229 m, 0.253 m respectively. Figs. 1, 2, 3, 4, 5 and 6 are the effects of resistance and reactance of liners with different frequency and Mach number on the in-duct overall sound power level attenuation Δ and sound power level attenuation Δf induced by sound pressure and velocity components of forward travelling wave, and the results are compared with that in reference [3]. The results agree quite well. The solid line and dashed line marked "Δ" are the results of this paper, and the solid line and dashed line marked "Δ" are the results of reference [3]. Fig. 1 shows that in-duct overall sound power level attenuation Δ increases with the increase of resistance, and decreases while reactance increases. Fig. 2 shows that the sensitivity of Δf to reactance is larger than that to resistance, but Δf will increase with the increase of reactance, and decrease when resistance increases. Fig. 3 and Fig. 5 show the changes of Δ with resistance

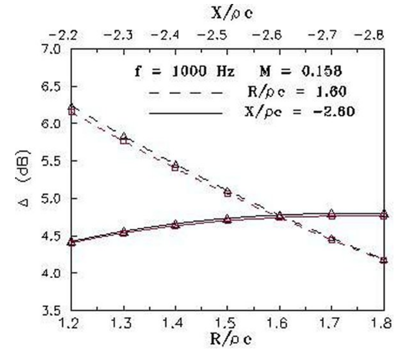

Figure 1: Δ=$F(R,X)$.

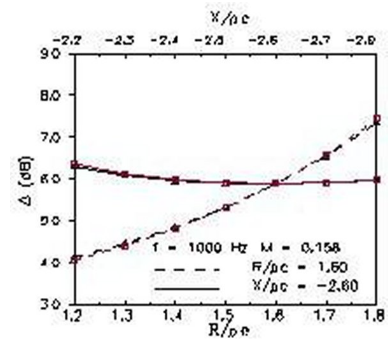

Figure 2: Δf=$F(R,X)$.

(R) and reactance (X), while frequency and Mach number increase. Comparing Fig. 4 with Fig. 2 shows that the sound power level attenuation due to the forward travelling pressure and velocity components increases with the increase of resistance and reactance, as frequency and Mach number increase. The effects of resistance and reactance on Δf show the nonlinear attenuating characteristics. Fig. 7 is the relations of the in-duct sound power level attenuation with source frequency. It demonstrates that the sound power level attenuation varies greatly with different operating condition. Fig. 8 is the contour of the relations between the sound power level attenuation and resistance, reactance. The selection of the suitable impedance of linings is important to improve noise suppression in duct.

The prediction model gives an integrated analysis for the study of sound propagation in an inlet duct of engine nacelle. Combining with experimental modal measurement, the application of the mode analysis theory in sound propagation prediction in ducts with varying cross-section can provide the engineering design criteria and lay theoretical foundation for noise suppression design in an aircraft engine.

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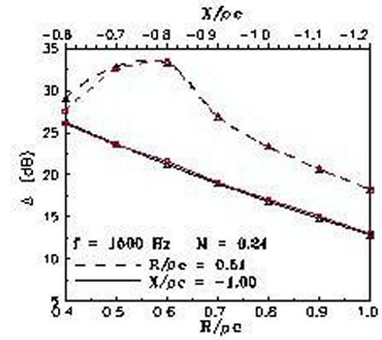

Figure 3: Δ=$F(R,X)$.

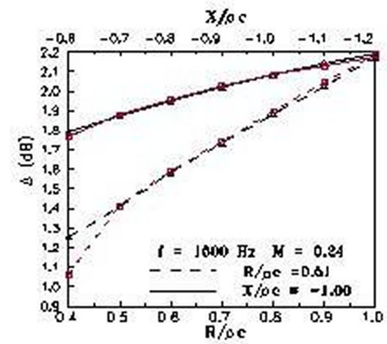

Figure 4: Δf=$F(R,X)$.

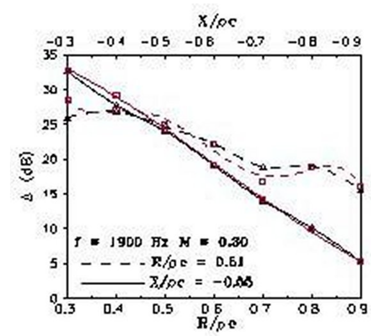

Figure 5: Δ=$F(R,X)$.

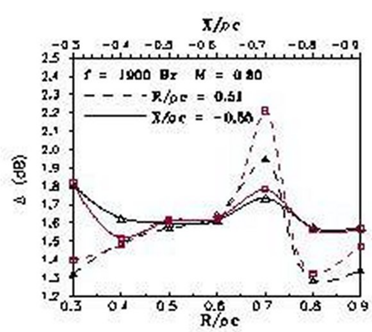

Figure 6: Δf=$F(R,X)$.

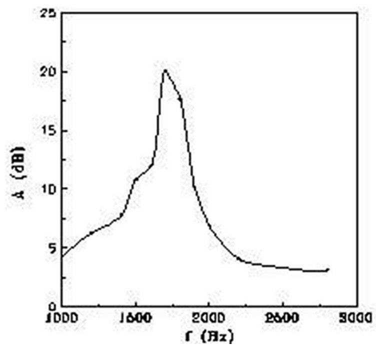

Figure 7: Relations between frequency and in-duct attenuation.

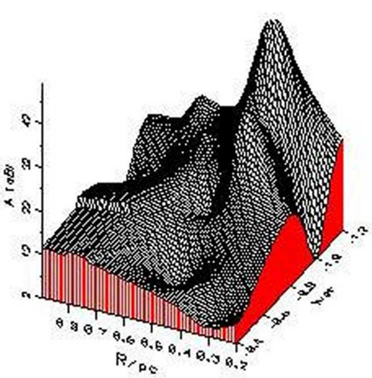

Figure 8: Effects of resistance, reactance on the in-duct attenuation.