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VIBRATIONS OF FLAT PLATES EXCITED BY LOW MACH NUMBER TURBULENT BOUNDARY LAYERS

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ABSTRACT
Turbulent boundary layer (TBL) excited vibrations in a clamped plate are predicted using random vibration analysis of a finite element model. The predicted vibrations are compared to measurements made at the Ray Herrick Labs at Purdue University using a laser-doppler vibrometer. The plate was flush-mounted into a wind tunnel wall and air was driven past the plate at a Mach number of 0.1. The measured boundary layer displacement thickness and flow velocity were used to estimate the wall pressure fluctuation auto-spectrum using a model developed by Smolyakov and Tkachenko. Two models for estimating the wall pressure cross-spectra were investigated: the Corcos and so-called Modified-Corcos models. Since the ratio of the plate bending wave and convective wavenumbers is well below the convective ridge (where the wavenumber ratio is equal to one), using the original Corcos model leads to finite element vibration predictions that are 10-20 dB higher than those measured. The high predictions are directly due to the original Corcos model significantly overestimating the low-wavenumber excitation of the plate. Using the modified Corcos model however, which better estimates low-wavenumber excitation, leads to excellent agreement between predicted and measured vibrations on the plate.

1 - INTRODUCTION
Turbulent boundary layer (TBL) excitation of plate like structures has been researched extensively over the past 45 years, particularly by the aerospace community. Wall pressure fluctuations excite the structure beneath the TBL causing vibrations and sound radiation. The vibrations cause stress fluctuations within the structural material which can lead to fatigue failure over time. The sound radiation is a concern to passengers inside vehicles.

The coupling of flexural waves in flat plates with TBL wall pressure fluctuations is often analyzed in wavenumber space. Hwang and Maidanik [1] analyzed the response of flat plates with various boundary conditions to TBL pressures. A plot of the wavenumber response of a plate bending mode superposed on a TBL wall pressure wavenumber frequency spectrum is reproduced from [1] in Figure 1. In the plot, the sensitivity of the plate mode to the TBL wall pressure excitation is strongest around its major lobe (at the plate bending wavenumber) where the excitation is weak, and weaker at the minor lobes where the excitation is strong (near the convective wavenumber). Hwang and Maidanik showed that in cases like those in Figure 1 for simply supported and clamped boundary conditions, the low to intermediate wavenumber region of the TBL pressure field is responsible for most of the plate vibration.

Although wavenumber analysis of flow excited flat plate response is useful for examining general trends, in practice most structures are not conducive to such analysis. Instead, the TBL pressure excitation must be integrated over physical space and frequency to determine the structural response. Finite element (FE) models are typically used to simulate the structure, and empirical models for the TBL wall pressures may be applied to the FE models. Using finite elements to simulate complex structures has been well validated. The empirical TBL forcing function models however, have not been well validated in the low to mid wavenumber range.

In this paper, a flat rectangular plate excited by TBL wall pressures is analyzed using a finite element approach. The plate is excited mostly in the low to intermediate wavenumber range. The predicted vibrations are compared to measurements made by Han, Bernhard, and Mongeau [2] of Purdue University.

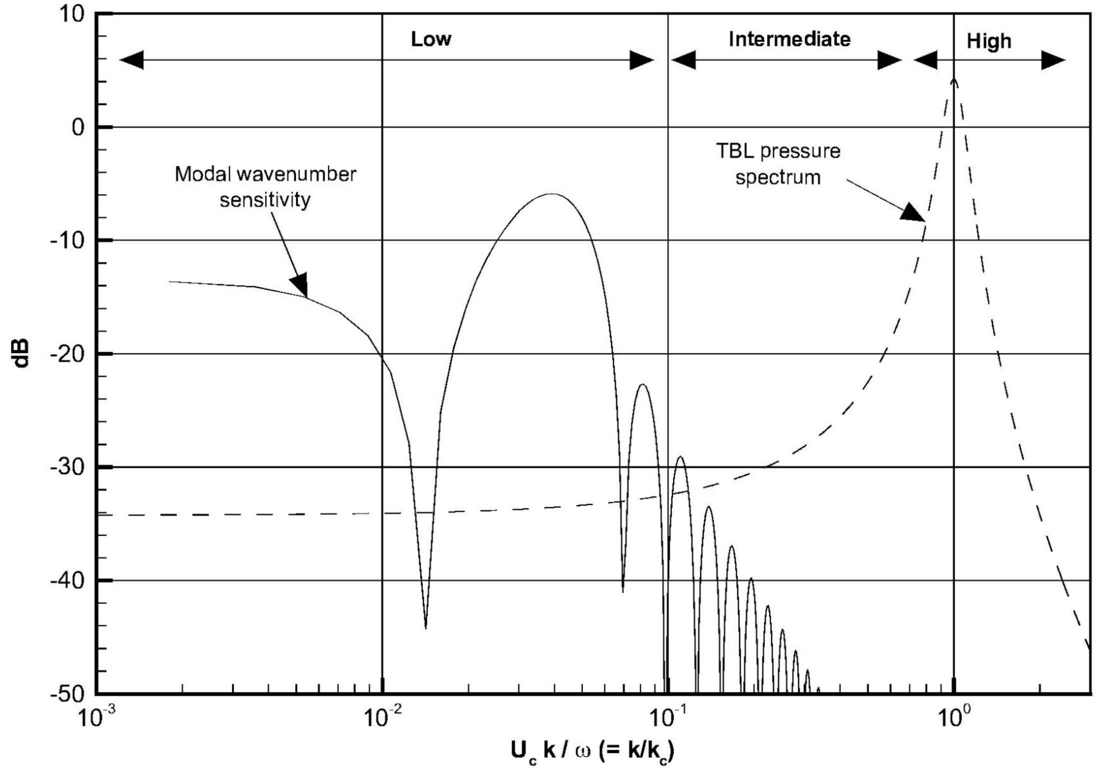

Figure 1: Coupling of TBL wall pressures and flexural plate mode.

Hopefully, the results will support using the finite element approach and empirical TBL wall pressure models to analyze more complicated structures excited by slow moving flow fields.

2 - FLOW EXCITED PLATE VIBRATION MEASUREMENTS

Han, Bernhard and Mongeau [2] measured the vibrations of a flat rectangular plate excited by TBL wall pressures to investigate energy flow analysis methods for flow excited plates. The plate is made of steel and is 47 cm long, 37 cm wide, and 16 cm thick and flush mounted into the floor of a wind tunnel test section with screws to simulate clamped boundary conditions along the plate edges. For more details on the test setup, refer to [2]. The air flows along the length (47 cm) of the plate at a free stream velocity U_o of 35.8 m/s (a Mach Number of 0.1). Plate normal velocities were measured at several locations between 0 and 600 Hz using a Scanning Laser Doppler Vibrometer (SLDV). The boundary layer displacement thickness δ^* was also inferred from hot wire anemometer measurements as 2.7 mm at a U_o of 35.8 m/s. Since the TBL behavior does not vary significantly over the plate, the wall pressure field was considered spatially homogeneous.

3 - MODELING AND ANALYSIS

3.1 - Turbulent boundary layer forcing function models

Most current models for TBL wall pressure fluctuations are empirical and based on a wide range of experimental measurements. Assuming a spatially homogeneous, temporally stationary excitation field, Corcos [3] postulated that the wall pressure cross spectral model may be separable in terms of the similarity variables $\omega\xi_1/U_c$ and $\omega\xi_3/U_c$:

$$\Gamma\left(\xi_1, \xi_3, \omega\right) = \phi_{pp}\left(\omega\right) A\left(\omega\xi_1/U_c\right) B\left(\omega\xi_3/U_c\right) \qquad (1)$$

where $\phi_{pp}\left(\omega\right)$ is the pressure autospectrum, A and B are decay functions in the flow and cross-flow directions, and ξ_1 and ξ_3 are spatial separations in the flow and cross-flow directions.

Smolyakov and Tkachenko [4] derived a pressure autospectrum model that agrees very well with measured data:

$$\phi_{pp}(\omega) \approx \left(\frac{\tau_w \delta^*}{U_o}\right)\left(\frac{5.1}{1+0.44\left(\omega\delta^*/U_o\right)^{7/3}}\right) \tag{2}$$

where τ_w is the wall shear stress which can be estimated for TBL flow with zero pressure gradient using the empirical relations $\mathrm{Re}_\delta \approx 8U_o\delta^*/\nu$ and $\tau_w \approx 0.0225\rho U_o/\mathrm{Re}_\delta$, where Re_δ is the boundary layer thickness Reynolds number, ν is the kinematic viscosity, and ρ is the fluid density.
The Smolyakov/Tkachenko model for Han's flat plate flow is compared to prior measurements by Farabee [5] and Bull and Thomas [6] for comparable flow in air in Figure 2. The agreement is very good. Also shown in the plot is the frequency range of interest for the plate. The wall pressure levels are nearly constant for the frequency range in these studies (0-600 Hz).

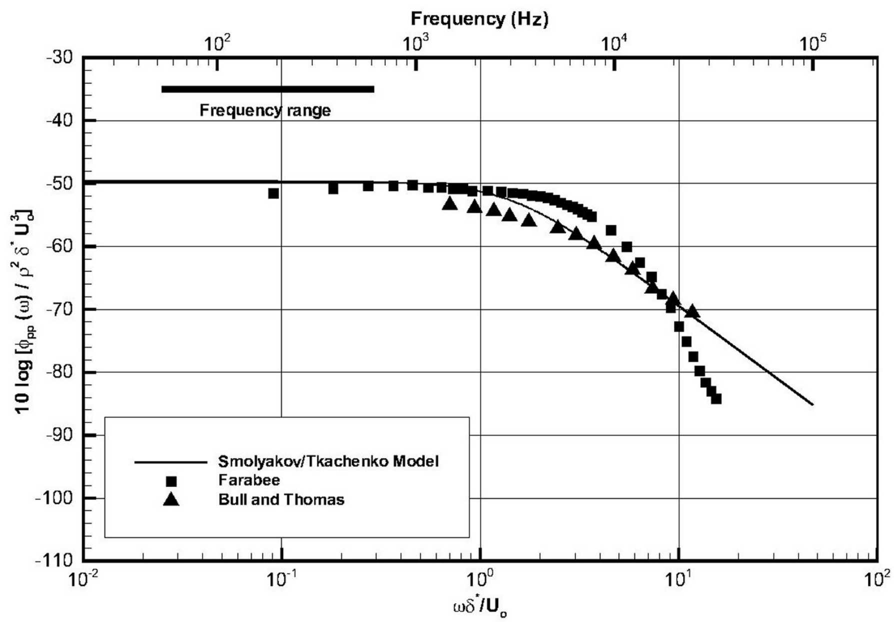

Figure 2: TBL wall pressure autospectrum model vs. measured data.

For the decay terms A and B Corcos uses:

$$A\left(\omega\xi_1/U_c\right) = e^{-\alpha_1|\omega\xi_1/U_c|} \quad \text{and} \quad B\left(\omega\xi_3/U_c\right) = e^{-\alpha_3|\omega\xi_3/U_c|} \tag{3}$$

where α_1 and α_3 are decay factors. Based on Han's measurements, values of 0.11 and 0.70 are used here. Unfortunately, the Corcos model has been shown to significantly overestimate the low to intermediate wavenumber content in TBL wall pressures. A simple modification to the original Corcos model reduces low wavenumber content and redistributes it into the convective wavenumber region ($U_c k/\omega$=1). The so-called Modified Corcos model [7] modifies A to be:

$$A\left(\omega\xi_1/U_c\right) = \left(1+\alpha_1\left|\omega\xi_1/U_c\right| e^{-\alpha_1|\omega\xi_1/U_c|}\right) \tag{4}$$

Finally, the convection velocity U_c is needed to use the forcing function models. Bull [8] derived the empirical relation:

$$U_c/U_o \approx 0.59 + 0.39e^{-0.89\omega\delta^*/U_o} \tag{5}$$

In this study U_c is roughly 85% of U_o for frequencies between 0 and 600 Hz.
Both the Corcos and Modified Corcos models may be easily transformed into wavenumber space and are shown as a function of wavenumber in Figure 3. Also shown in Figure 3 is a box bracketing the major wavenumber sensitivity region of the plate modes between 0 and 600 Hz. The plate is excited mostly by the low to intermediate region of the wall pressure wavenumber spectrum.

3.2 - Finite element analysis

Structural shapes other than simple rectangles and circles are not usually conducive to analysis in wavenumber space. Therefore, the plate vibrations are analyzed using frequency response analysis of

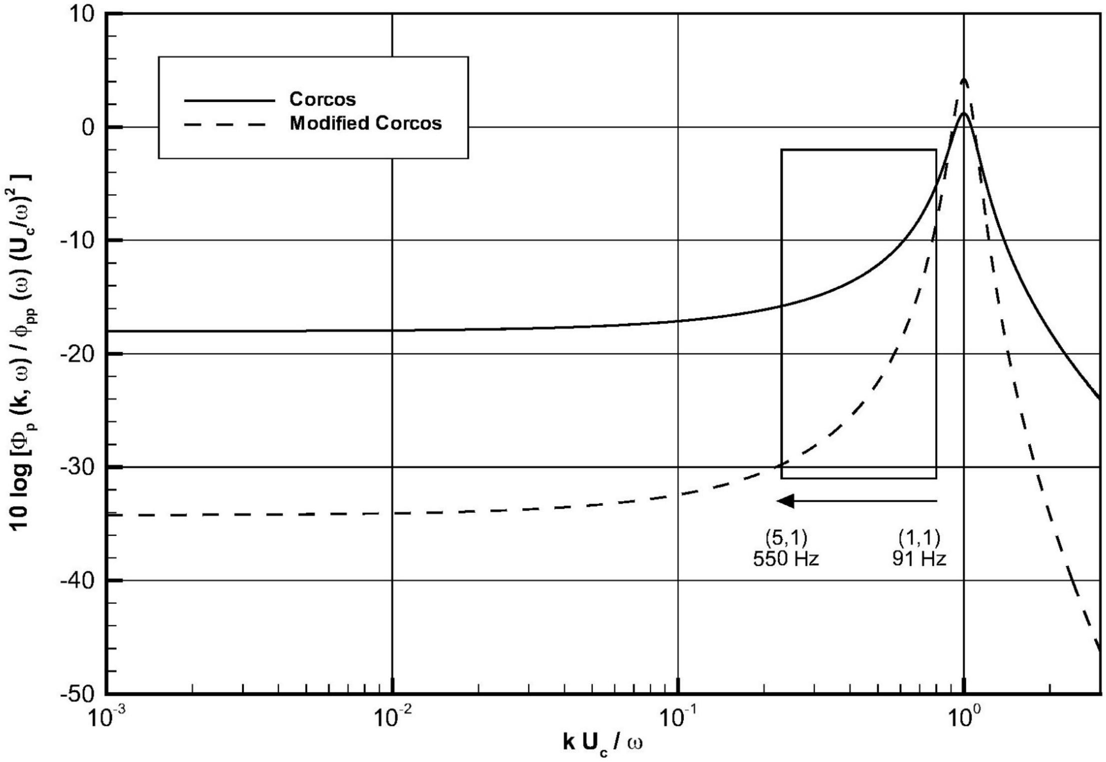

Figure 3: TBL wall pressure cross spectrum models vs. plate modal wavenumbers.

a finite element model in physical space so the analysis method will be applicable to general structures. An efficient random analysis approach developed by Hipol [9] was applied to compute the vibration autospectra:

$$G_{xx}(\omega) = \sum_{j}^{q} \sum_{i}^{q} H_{ix}(\omega)\,\Gamma_{ij}(\omega)\,H_{jx}(\omega)^{*T} \qquad (6)$$

where $G_{xx}(\omega)$ is the plate velocity autospectrum at location x, $H_{ix}(\omega)$ are the frequency response function (FRF) mobility matrices generated by the FE model, and $\Gamma_{ij}(\omega)$ is the TBL wall pressure cross spectrum applied to all i and j points.

The FE model must resolve both flexural waves in the plate and the excitation field under the TBL. At low flow speeds, the element sizes are dictated by the convective wavelength. Based on a maximum analysis frequency of 600 Hz and an approximate U_c of 30 m/s (about 85% of U_o), the minimum convective wavelength (U_c/f) is 5 cm. Therefore, to maintain at least 8 elements for each convective wavelength, 75 elements were used in the flow direction. Elements in the cross flow direction need only resolve the plate flexural waves, and 26 were used based on a convergence study of the plate resonance frequencies. A material loss factor of 0.005 for frequencies above 250 Hz was assumed based on Han's measurements, with the loss factors of the lowest order modes varying linearly from 0.016 to 0.005 between 80 and 250 Hz.

4 - COMPARISON OF RESULTS

Predicted velocities at a point 15 cm from the plate leading edge and 12 cm from its side are compared to Han's measured data in Figure 4. Predictions were made using the Corcos and Modified Corcos cross spectrum models. The plate mode orders are indicated at the resonance peaks in the plot. Predictions made using the original Corcos model clearly overestimate the measured vibrations, whereas using the modified Corcos model leads to excellent agreement between predicted and measured vibrations. Both TBL models predict similar response at the lowest order modes, which is consistent with Figure 3, which shows that the maximum sensitivities of the low order modes are close to the convective ridge region. The response of the modes above 150 Hz, however, are clearly most strongly influenced by the low to mid wavenumber region of the TBL excitation which differs significantly between the two models.

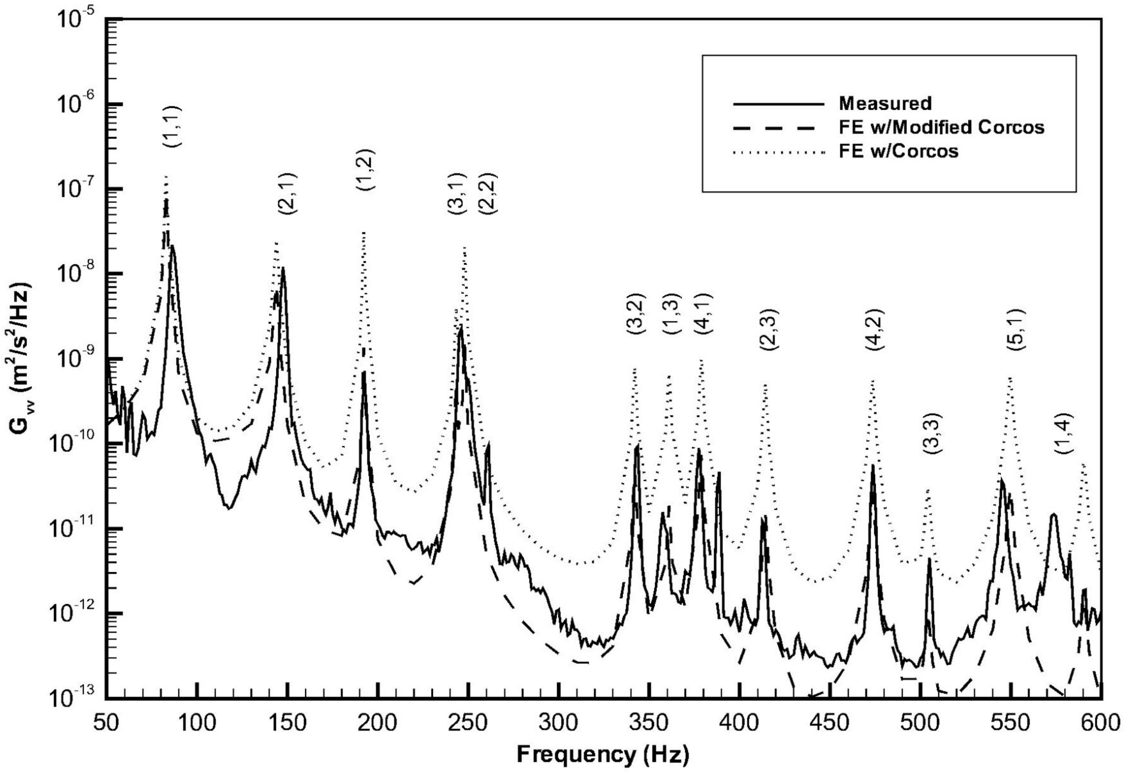

Figure 4: FE vs. measured velocity autospectra at x=15 cm, y=12 cm, U_o=35.8 m/s.

5 - CONCLUSIONS

The flow excitation (Modified Corcos model) and FE models used in this study predict TBL excited plate vibrations very well. However, great care was taken to ensure that enough finite elements were used to resolve the excitation field models in the flow direction. The slower the flow, the smaller the convective wavelength, and the more elements are needed. For large, complex structures, the number of elements required to resolve the excitation field can quickly become infeasible. Hwang [7], however, has suggested a simple model for use when very low wavenumber excitations are dominant. Hwang's model represents only the 'wavenumber-white' region of the TBL cross-spectrum, ignoring the high-wavenumber convective ridge region. To validate the model, however, more experimental data is needed.
Also, although using the Modified Corcos model led to excellent agreement between measured and predicted plate vibrations, other TBL wall pressure models have been suggested by many investigators. Evaluating the performance of other TBL wall pressure models on the plate modeled here may be enlightening.

ACKNOWLEDGEMENTS

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