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OPTIMAL INTERPOLATION OF BAD OR NON-EXISTING MEASUREMENT POINTS IN PLANAR ACOUSTICAL HOLOGRAPHY

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ABSTRACT

The present paper deals with the following problems in planar Near-field Acoustical Holography. In some cases it is very difficult or even impossible to measure the required complete regular rectangular measurement grid. This may be due to obstacles or hazardous measurement conditions, such as high temperatures or moving parts. Another type of problem is a bad signal discovered only after a measurement has been completed. It may be then impossible or at least very expensive to re-take the measurement. A general solution to the problem is to use an interpolation procedure capable of interpolating the complex sound pressure between an arbitrary set of measurement positions in a plane. The paper describes a new such interpolation procedure, which is optimized to the characteristics of the sound pressure distribution over a plane at some distance from the sound sources. Some computer simulations are presented, showing that over the major part of the frequency range, the new method provides very small interpolation errors. The paper also presents results from an actual measurement.

1 - INTRODUCTION

Some basic concepts of Near-field Acoustical Holography need to be stated first. Figure 1 illustrates the geometry of the measurement problem. The sound pressure is measured over a plane $z = z_0 > 0$ in the near-field region of a sound source. All parts of the source are assumed to be in the half space $z < 0$, and the half space $z \geq 0$ is assumed to be source free and homogeneous.

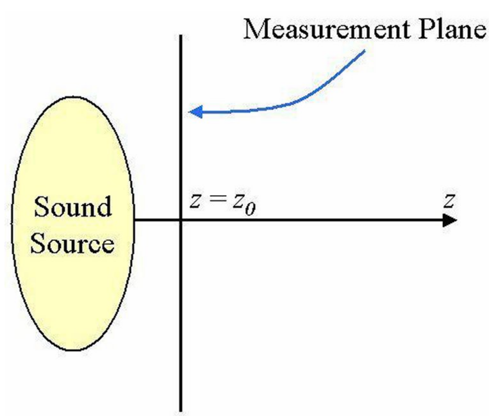

Figure 1: Measurement geometry.

The complex sound pressure field $p(\mathbf{r}) = p(x, y, z)$ at a given angular frequency ω fulfils the homogeneous wave equation in the half space $z \geq 0$:

$$\nabla^2 p + k^2 p = 0, \qquad z \geq 0 \tag{1}$$

$k \equiv \dfrac{\omega}{c}$ being the wave number.
For any given z-coordinate, we now introduce the following Fourier transform pair of the sound pressure in the two dimensions (x,y):

$$p\left(x,y,z\right) = \frac{1}{\left(2\pi\right)^2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} P\left(k_x,k_y,z\right)e^{-j\left(k_x x+k_y y\right)}dk_x dk_y \tag{2}$$

$$P\left(k_x,k_y,z\right) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} p\left(x,y,z\right)e^{j\left(k_x x+k_y y\right)}dxdy \tag{3}$$

where (k_x,k_y) are the spatial angular frequencies. This pair exists for any xy-plane with $z \geq 0$. If we insert the Fourier transform expression (2) for $p(\mathbf{r})$ into the wave equation (1) and take the Fourier transform, we obtain the following one-dimensional differential equation in z:

$$\left[\frac{\partial^2}{\partial z^2}+k_z^2\right]P\left(k_x,k_y,z\right)=0, \qquad k_z^2 \equiv k^2-k_x^2-k_y^2, \qquad z\geq 0 \tag{4}$$

When all sources of the sound field are in the half space $z< 0$, then the complete solution to (4) can be written as

$$P\left(k_x,k_y,z\right) = P\left(k_x,k_y,z_0\right)e^{-jk_z(z-z_0)}, \qquad z_0\geq 0, \qquad z\geq 0 \tag{5}$$

where k_z is a function of the spatial angular frequencies (k_x,k_y):

$$k_z \equiv \begin{cases} \sqrt{\left(\frac{\omega}{c}\right)^2-k_x^2-k_y^2} & \text{for } k_x^2+k_y^2 \leq \left(\frac{\omega}{c}\right)^2 \\ -j\sqrt{k_x^2+k_y^2-\left(\frac{\omega}{c}\right)^2} & \text{for } k_x^2+k_y^2 > \left(\frac{\omega}{c}\right)^2 \end{cases} \tag{6}$$

The circle in the spatial frequency plane defined by k_x^2+k_y^2= $\left(\frac{\omega}{\mathrm{c}}\right)^2$ is called the Radiation Circle. High spatial frequencies outside the radiation circle are seen from equation (5) and (6) to be exponentially attenuated in the direction away from the source.
Since the sound pressure field is measured in the plane $z = z_0$, the plane wave spectrum P can be obtained from equation (3) with z equal to z_0. Formulae (5) and (2) then allow the sound pressure field p for any $z \geq 0$ to be calculated.

2 - INTERPOLATION OF BAD MEASUREMENT POINTS

Since for the interpolation we are concerned only with the sound pressure in the measurement plane, $z = z_0$, we introduce a two-dimensional position vector $\mathbf{R} \equiv (x, y)$ in that plane. The pressure in the measurement plane can now be written as $p(\mathbf{R}, z_0, \omega)$. For convenience we shall omit the frequency parameter ω and write just $p(\mathbf{R})$.
We assume that the sound pressure $p(\mathbf{R}_n)$ has been measured at a set of N positions $\mathbf{R}_n \equiv (x_n, y_n)$ in the measurement plane. Without loss of generality we can restrict us to estimation of the sound pressure $p\left(\mathbf{0}\right) \equiv p\left(0,0\right)$ at the origin. We wish to estimate $p(\mathbf{0})$ as a linear combination of the measured sound pressure data $p(\mathbf{R}_n)$:

$$p\left(\mathbf{0}\right) \approx \tilde{p}\left(\mathbf{0}\right) \equiv \sum_{n=1}^{N} c_n \cdot p\left(\mathbf{R}_n\right) = \sum_{n=1}^{N}\left(a_n+jb_n\right)\cdot p\left(\mathbf{R}_n\right) \tag{7}$$

and we wish to use a set of complex coefficients $c_n = a_n + jb_n$ which minimizes the average estimation error for typical sound fields at a distance d from the sound sources. The characteristic that we shall make use of is the spatial frequency bandwidth limitation of the sound pressure distribution in the measurement plane after the propagation over the distance d from a parallel source plane. This bandwidth limitation follows from equation (5) and (6).
In the Fourier transform pair (2) and (3), the sound pressure in the measurement plane is expressed as an infinite sum of elementary waves $e^{-j\mathbf{K}\cdot\mathbf{R}}$, where $\mathbf{K} \equiv (k_x, k_y)$ is a "point" in the spatial frequency domain. Notice that these waves have the same "plane wave form" in the measurement plane, no matter if the spatial frequency $\mathbf{K}$ is inside the radiation circle or outside. Only the z-dependence is very different.
Based on the above considerations, we require the linear interpolation formula (7) to provide good interpolation estimates for all the elementary waves $e^{-j\mathbf{K}\cdot\mathbf{R}}$, but with a weight on the estimation error

that depends on the attenuation of the waves over the distance d from the source surface. Explicitly, we require good estimates for the weighted elementary waves

$$p_{\mathbf{K}}(\mathbf{R}) \equiv W(\mathbf{K}) \cdot e^{-j\mathbf{KR}} = W(k_x, k_y) \cdot e^{-j(k_x x + k_y y)} \tag{8}$$

with

$$W(\mathbf{K}) \equiv \left\{ \begin{array}{ll} 1 & |\mathbf{K}| \leq k \\ e^{-d\sqrt{|\mathbf{K}|^2 - k^2}} & |\mathbf{K}| > k \end{array} \right\} \tag{9}$$

The weighting function (9) corresponds to an assumption about identical amplitude of all spatial frequency components of the sound pressure at the source surface, see equation (5).
The estimation error in (7) for each of the weighted elementary waves is

$$\varepsilon(\mathbf{c}, \mathbf{K}) \equiv p_{\mathbf{K}}(\mathbf{0}) - \sum_{n=1}^{N} c_n \cdot p_{\mathbf{K}}(\mathbf{R}_n) \tag{10}$$

and we define the average estimation error as

$$E(\mathbf{c}) \equiv \frac{1}{2\pi k^2} \int_0^\infty \int_0^{2\pi} |\varepsilon(\mathbf{c}, \mathbf{K})|^2 \, K d\psi dK \tag{11}$$

where (K, ψ) are the polar coordinates of the vector $\mathbf{K}$,

$$\mathbf{K} = (k_x, k_y) = (K\cos(\psi), K\sin(\psi)) \tag{12}$$

and $\mathbf{c} = \mathbf{a} + j\mathbf{b}$ is a vector containing the complex interpolation coefficients c_n, ref. equation (7). After removal of some terms, which can be shown to equal zero, the integral expression (11) for the average estimation error E reduces to

$$E(\mathbf{a} + j\mathbf{b}) = E_0 - 2 \cdot \sum_{n=1}^{N} a_n C(R_n) + \sum_{n=1}^{N} \sum_{m=1}^{N} [a_n a_m + b_n b_m] \, C(R_{nm}) \tag{13}$$

where $R_n \equiv |\mathbf{R}_n|$ are the distances from the estimation position to the measurement positions, $R_{nm} \equiv |\mathbf{R}_m - \mathbf{R}_n|$ are the distances between the measurement positions and

$$E_0 \equiv \frac{1}{k^2} \int_0^\infty W^2(K) \, K dK \tag{14}$$

$$C(R) \equiv \frac{1}{k^2} \int_0^\infty W^2(K) \, J_0(KR) \, K dK \tag{15}$$

Here, J_0 is the 0'th order Bessel function. We are looking for the coefficient vector $\mathbf{c} = \mathbf{a} + j\mathbf{b}$ that minimizes the average estimation error $E(\mathbf{c})$ in equation (13). The minimum of the quadratic function $E(\mathbf{c})$ is characterized by all partial derivatives being equal to zero:

$$\frac{\partial E}{\partial a_n} = 0 \qquad \text{and} \qquad \frac{\partial \text{ E}}{\partial b_n} = 0 \tag{16}$$

which can be shown to lead to the following equations for the interpolation coefficients:

$$\sum_{m=1}^{N} C(R_{nm}) \, a_m = C(R_n) \qquad n = 1, 2\ , ..., N \tag{17a}$$

$$b_n = 0 \qquad n = 1, 2, ..., N \tag{17b}$$

The minimum value $E_{\min}$ of the error $E(\mathbf{c})$ is easily shown to be

$$E_{\min} = E_0 - \sum C(R_n) \, a_n \tag{18}$$

In order to set up the system of linear equations in (17a), we need to calculate the integrals $C(R_{nm})$ and $C(R_n)$, the integral $C(R\ )$ being defined in equation (15). Here, the important part inside the radiation circle, i.e. for $|\mathbf{K}| \leq k$, can be integrated analytically, and we end up with

$$\begin{aligned} C(R) &= \frac{1}{k^2}\int_0^k J_0(KR)\,KdK + \frac{1}{k^2}\int_k^\infty J_0(KR)\,e^{-2d\sqrt{K^2-k^2}}KdK \\ &= \frac{J_1(kR)}{kR} + \frac{1}{(2kd)^2}\int_0^\infty \left(tJ_0\left(kR\sqrt{1+(t/2kd)^2}\right)\right)e^{-t}dt \end{aligned} \tag{19}$$

Because of the exponential factor in the last integral, the integration interval can be truncated, and the integration can be performed by a numerical quadrature formula, such as a Gauss-Legendre formula.
By minimizing the sum-of-squares interpolation error for all the elementary (plane) waves, we have implicitly assumed no correlation between these waves. Such a correlation would be present, if for example the sources were assumed to be always within the area covered by the array. Slightly better interpolation could be obtained, if such a restriction could be made and exploited, but then the method would also break down if the assumption were not fulfilled.

3 - COMPUTER SIMULATIONS

The present section will describe a set of simulated measurements with a 6x6 microphone array with 5 cm element spacing. The sound source is a monopole in a source plane at some distance from the array plane. One of the measured microphone signals is then considered non-applicable, and instead the pressure at that point is estimated (interpolated) from the remaining 35 microphone signals. The interpolated pressure is then compared with the true pressure.
In order to gain overview, some averaging of the pressure estimation error is performed, both over the source positions and over the calculation position in the array. As one should expect, the errors turned out to be largest close to the borders of the array and smallest at the center. It was chosen to calculate one average error over the central 4x4=16 calculation positions (*Center*) and one over the remaining 20 calculation positions along the borders (*Border*). In both cases, the relative averaged error has been calculated from the following formula

$$\bar{E} \equiv 10 \cdot \log\left(\frac{\sum \|p-\tilde{p}\|_2^2}{\sum \|p\|_2^2}\right) \tag{20}$$

where the summation is over the relevant calculation positions and also over the monopole source positions used in the averaging. The monopole source positions covered a 20x20 grid of the same size as the microphone array and in a parallel plane.
Figure 2 shows the relative averaged error in the frequency interval from 50 Hz to 3000 Hz for the case where the monopole source positions are in a plane 7.5 cm from the array plane ($z_0 = 7.5$ cm). However, for the interpolation procedure a 50% larger source distance was specified ($d = 1.5z_0$).

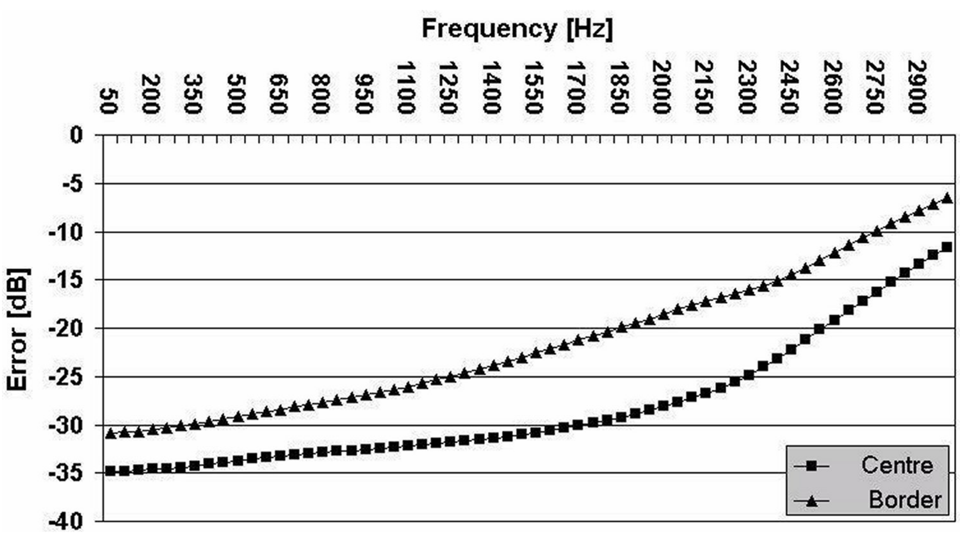

Figure 2: Relative average interpolation error for monopoles at 7.5 cm distance.

4 - MEASUREMENTS

The above interpolation procedure has been implemented in the Non-stationary STSF software package Type 7712 from Brüel & Kjær and applied in connection with a series of engine measurements, [1], where it was discovered after the measurement that one of the microphones had an error. These measurements were taken with an array having 12 rows and 10 columns of microphones. The bad microphone was in row 12 and column 9, see Figure 3, which is a screen copy of some of the graphically oriented window for selection of interpolation positions. The output time signal from the faulty microphone showed rather strong impulses, perhaps due to electrical noise from the engine.

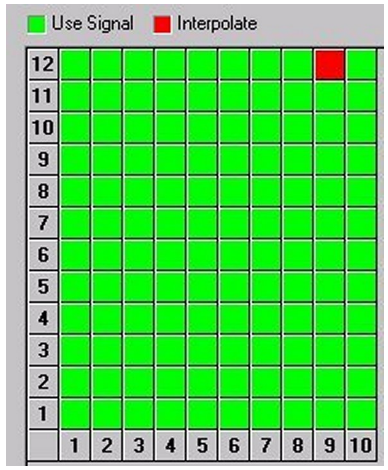

Figure 3: Selection of interpolation points in 12x10 array.

Figure 4 shows the (Envelope) Active Intensity at the engine surface for the 1/3-octave band at 1000 Hz − before and after the replacement of the bad microphone signal with interpolated data. The two plots represent the same instant in time, where a strong noise impulse was measured with the bad microphone. When the bad microphone signal is used (left plot in Figure 4), then the sound field at the engine surface needs a very special form in order to focus onto only a single position in the array plane − the position of the bad microphone.

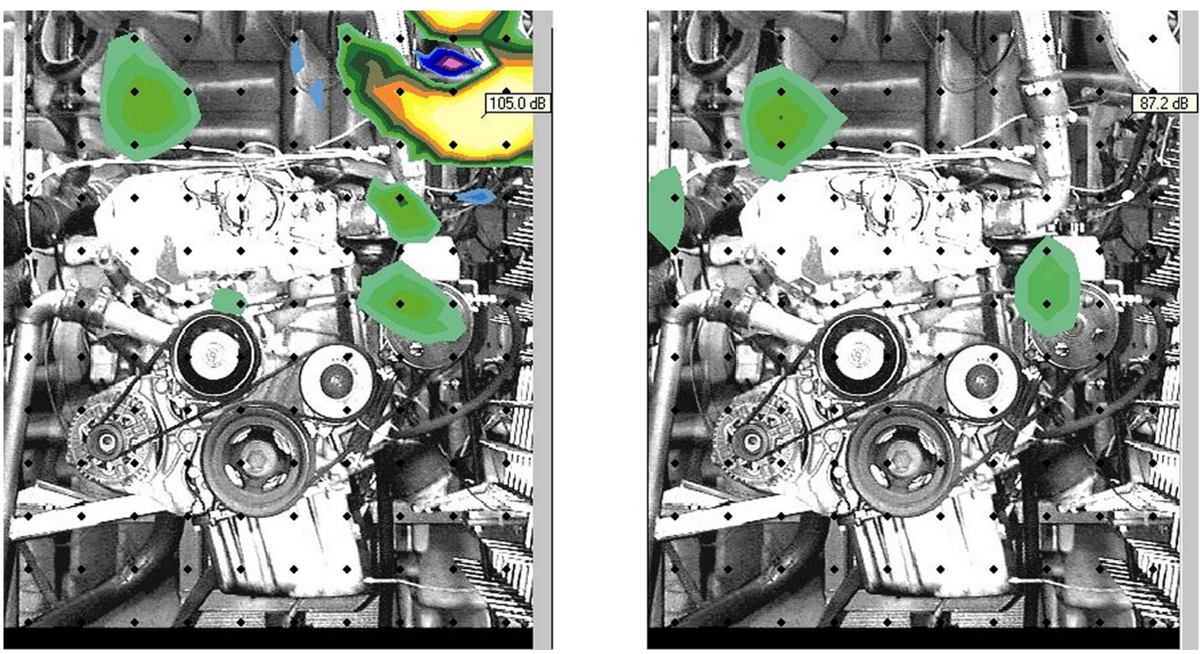

Figure 4: Active Intensity at the engine surface calculated without (left) and with (right) the use of interpolation for the bad microphone signal; the dots indicate microphone positions.

5 - CONCLUSION

The planar array interpolation method presented above has proven to be efficient and accurate for interpolation of bad or non-existing measurement points in a planar grid. Applications include situations, where a bad signal is discovered only after a measurement has been completed and situations, where some positions in a regular rectangular grid cannot be measured due to obstacles or hazardous measurement conditions, such as high temperatures or moving parts. A future application of the interpolation method could be interpolation of sparse microphone grid data into a full rectangular grid for Near-field Acoustical Holography applications.

REFERENCES

1. **Hald J.**, Use of Non-stationary STSF for analysis of transient engine noise radiation, In *Proceedings Inter-Noise*, 1999