inter.noise 2000
The 29th International Congress and Exhibition on Noise Control Engineering
27-30 August 2000, Nice, FRANCE

I-INCE Classification: 2.4

THE EFFECTS OF ATMOSPHERIC TURBULENCE ON THE INTERFERENCE OF THE DIRECT AND GROUND REFLECTED WAVES

V. Ostashev*, S. Clifford*, R. Lataitis*, P. Blanc-Benon, D. Juve****

* NOAA/Environmental Technology Laboratory, 325 Broadway, R/ET4, CO, 80303-3328, Boulder, United States Of America

** Ecole Centrale de Lyon, 36, avenue Guy de Collongue, BP 163, 69131, Ecully Cedex, France

Tel.: 303-497-3712 / Fax: 303-497-6574 / Email: vostashev@etl.noaa.gov

Keywords:
SOUND, INTERFERENCE, TURBULENT, ATMOSPHERE

ABSTRACT
A rigorous theory of interference of direct and ground-reflected waves in a turbulent atmosphere was developed by Clifford and Lataitis (JASA, **73**, 1545-1550, 1983). The primary goal of the present paper is to generalize Clifford and Lataitis's theory by taking into account developments in atmospheric acoustics since 1983. In particular, we reformulate the geometry of the problem that allows us to use the parabolic equation method. Furthermore, we take into account that the direct and ground reflected waves are scattered by both temperature and wind velocity fluctuations, and use Kolmogorov, Gaussian, and von Karman spectra to model spectra of these fluctuations. Finally, we obtain a formula for the mean squared sound pressure and relate it to that of the coherence function of a spherical sound wave for line-of-sight propagation.

1 - INTRODUCTION
In many cases of sound propagation in the atmosphere, a source and receiver are located close the ground. If the horizontal distance L between the source and receiver is not too large, the effects of atmospheric refraction can be ignored and the sound field p at the receiver is a sum of the direct wave and that reflected from impedance ground. In the absence of atmospheric turbulence, the interference between these two waves could lead to very deep minima in the amplitude of the resulting sound field. However, temperature and wind velocity fluctuations always exist in the atmosphere. These fluctuations lead to partial or complete loss of coherence between the direct and ground reflected waves. As a result, the mean squared sound pressure $\left\langle |p|^2 \right\rangle$ at the interference minima can be increased by several dozen dB in comparison to that in a nonturbulent atmosphere. This phenomenon has been studied since the 1960s.
A first rigorous approach for calculating the interference of the direct and ground reflected waves in a turbulent atmosphere was developed by Clifford and Lataitis [1]. In this reference, it was assumed that atmospheric turbulence is caused by temperature fluctuations.
Furthermore, log-amplitude and phase fluctuations of the direct and ground reflected waves were calculated by using the Rytov method. Finally, an analytical formula for $\left\langle |p|^2 \right\rangle$ was derived. Using this formula, $\left\langle |p|^2 \right\rangle$ was calculated for a Gaussian spectrum of temperature fluctuations. These results have been widely used in subsequent research.
The main goal of the present paper is to generalize the theory developed by Clifford and Lataitis [1] in the following directions. First, we reformulate the geometry of the problem. A new geometry allows us to use the parabolic equation method, which has a wider range of applicability than the Rytov method. Secondly, we assume that atmospheric turbulence is caused by both temperature and wind velocity fluctuations since in most cases the effects of velocity fluctuations on $\left\langle |p|^2 \right\rangle$ are greater than the effects of temperature fluctuations (e.g. [2]). Finally, we derive analytical formulas for $\left\langle |p|^2 \right\rangle$ for Kolmogorov,

Gaussian, and von Karman spectra of temperature and wind velocity fluctuations. Using these formulas, the effects of atmospheric turbulence on $\left\langle |p|^2 \right\rangle$ are studied numerically.

2 - PARABOLIC EQUATION

Let the source and receiver be located at the heights h_s and h_r above the ground. We will use the Cartesian coordinate system $\vec{R} = (x, y, z)$ with the center at the source, z-axis directed upward, and z-axis in the direction from source to receiver. In this case, the plane $z{=}-h_s$ coincides with the surface of the ground. We assume that the mean wind velocity is zero and the mean temperature T_0 does not depend on z. Temperature and wind velocity fluctuations in the atmosphere are denoted as $T'\left(\vec{R}\right)$ and $\vec{\nu}\left(R'\right)$.

For this geometry, calculations of the statistical moments of a sound field are complicated by the presence of the surface of the ground. Therefore, we reformulate this geometry. In a new geometry, sound propagates in an unbounded turbulent medium which is symmetrical with respect to the plane $z{=}-h_s$. Furthermore, the sound field at the receiver is a sum of the sound field due to the source and that due to an image source of the strength Q located at the point $\vec{R}_i = (0, 0, -h_s)$. (The image source is symmetrical to the real source with respect to the plane $z{=}-h_s$.) Here, Q is the spherical-wave reflection coefficient given by [3]:

$$Q = \frac{2\beta\left[1 + i\sqrt{\pi}de^{-d^2}erfc\left(-id\right) - \beta + \left(h_s + h_r\right)/R_2\right]}{\beta + \left(h_s + h_r\right)/R_2}, \tag{1}$$

where the distance from the image source to the receiver is $R_2 = \sqrt{\left(h_s + h_r\right)^2 + L^2}$, the ground admittance is β, the complementary error function is *erfc*, and the numerical distance is

$$d = \left[\beta + \left(h_s + h_r\right)/R_2\right]\sqrt{ikR_2/2}$$

where k is the sound wavenumber.

We assume that fluctuations in temperature and wind velocity are relatively weak so that sound backscattering can be ignored. Then, for a new geometry, the sound pressure p at the receiver satisfies the following parabolic equation [2]:

$$\left[2ik\frac{\partial}{\partial x} + \left(\frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right) + 2k^2\left(1 + \varepsilon/2\right)\right]p = 0. \tag{2}$$

Here, $\varepsilon\left(\vec{R}\right) = -T'\left(\vec{R}\right)/T_0 - 2\nu_x\left(\vec{R}\right)/c_0$ is a linear combination of temperature and wind velocity fluctuations. In this formula, c_0 is the reference value of the sound speed and v_x is the x-component of $\vec{\nu}$. Note that the random field $\varepsilon\left(\vec{R}\right)$ is symmetrical with respect to the plane $x{=}-h_s$. In the plane $x = 0$, the sound field p satisfies the following initial condition:

$$p\left(\vec{r}\right) = \left(2i\pi/k\right)\left[\delta\left(\vec{r} - \vec{r}_1\right) + Q\delta\left(\vec{r} - \vec{r}_2\right)\right], \tag{3}$$

where $\vec{r} = (y, z)$, and δ is the delta function. Furthermore, $\vec{r}_1 = (0, 0)$ and $\vec{r}_2 = (0, -2h_s)$ are the coordinates of the source and image source in the plane $x = 0$.

It can be shown that Eqs. (2) and (3) are equivalent to initial equations describing sound propagation over the ground and boundary conditions for two cases: sound propagation over hard boundary in a turbulent atmosphere and sound propagation over impedance boundary in a nonturbulent atmosphere. Using physical reasoning, one would expect that Eqs. (2) and (3) approximately describe sound propagation over impedance boundary in a turbulent atmosphere if the turbulence is relatively weak.

A parabolic equation (2) with the initial condition (3) are the starting equations in our approach for calculating the mean squared sound pressure $\left\langle |p|^2 \right\rangle$. Equations (2) and (3) can also be used for calculating other statistical moments of a sound field propagating over impedance ground, for example, the coherence function. Note that parabolic equations have been found to be very convenient for studies of wave propagation in random media (e.g. [4]). Many classical methods for calculating the statistical moments of a sound or electromagnetic field are based on parabolic equations. Therefore, we expect Eqs. (2) and (3) to be very useful for studies of sound propagation over impedance ground in a turbulent atmosphere.

3 - FORMULA FOR $\left\langle |p|^2 \right\rangle$

The solution of Eq. (2) can be sought in the following form

$$p = p_1 \exp\left(\chi_1 + i\phi_1\right) + p_2 \exp\left(\chi_2 + i\phi_2\right). \tag{4}$$

Here, p_1 and p_2 are the sound fields due to the source and image source in a nonturbulent atmosphere. Furthermore, χ_1 and ϕ_1 are the log-amplitude and phase fluctuations of a sound field emitted by a source in a turbulent atmosphere, while χ_2 and ϕ_2 are the log-amplitude and phase fluctuations of a sound field emitted by an image source.

We assume that χ_1, ϕ_1, χ_2 and ϕ_2 have Gaussian distributions. With the use of Eq. (4), we calculate the mean squared sound pressure:

$$\left\langle |p|^2 \right\rangle = \frac{1}{R_1^2} + \frac{|Q|^2}{R_2^2} + \frac{2\,|Q|\,C}{R_1 R_2} \cos\left[(R_2 - R_1)\,k + \Omega + \langle \chi_1 \phi_2 \rangle - \langle \chi_2 \phi_1 \rangle\right]. \tag{5}$$

Here, the brackets $\langle\rangle$ denote the ensemble average, Ω is the phase of Q so that $Q = |Q|\,e^{i\Omega}$, and C is given by

$$C = \exp\left[\langle \chi_1 \chi_2 \rangle + \langle \phi_1 \phi_2 \rangle - \frac{1}{2}\left(\langle \chi_1^2 \rangle + \langle \chi_2^2 \rangle + \langle \phi_1^2 \rangle + \langle \phi_2^2 \rangle\right)\right]. \tag{6}$$

Equation (5) describes interference between the sound waves emitted by a source and an image source. The factor C in Eq. (5) characterizes the coherence between these waves and hereafter will be called the "coherence" factor. In a nonturbulent atmosphere $C = 1$, while in a turbulent atmosphere C is less than 1.

Equations (5) and (6) contain second statistical moments of phase and log-amplitude fluctuations, e.g. $\left\langle |p|^2 \right\rangle$. In order to calculate these statistical moments, we need to know expressions for χ_1, ϕ_1, χ_2 and ϕ_2 in terms of the random field ε. These expressions are found by substituting Eq. (4) into Eq. (2) and using the first Rytov approximation. Then, the statistical moments of phase and log-amplitude fluctuations in Eqs. (5) and (6) are calculated. As a result, we obtain that in Eq. (5) the difference $\langle \chi_1 \phi_2 \rangle - \langle \chi_2 \phi_1 \rangle$ is less than $\pi/2$ and can be omitted. Furthermore, the following formula is obtained for the coherence factor C:

$$C(h) = \exp\left[-\frac{k^2 L}{4h} \int_0^h \left[b(0,0) - b(0,z)\right] dz\right]. \tag{7}$$

Here, $h = 2h_s h_r / (h_s + h_r)$ and b is the two-dimensional correlation function determined by

$$b(\vec{r}) = \int_{-\infty}^{\infty} B(x, \vec{r})\, dx = 2\pi \int d^2 K_\perp e^{i\vec{K}_\perp \cdot \vec{r}} \Phi\left(0, \vec{K}_\perp\right). \tag{8}$$

In this formula, $B\left(\vec{R}\right)$ is the correlation function of the random field ε, and $\Phi\left(K_x, \vec{K}_\perp\right)$ is the three-dimensional spectral density of this field, where K_x and $\vec{K}_\perp$ are the components of the wave vector $\vec{K} = \left(K_x, \vec{K}_\perp\right)$. These functions are described in detail in section 7.2.3 from [2]. It follows from that section that $B\left(\vec{R}\right) = B_T\left(\vec{R}\right)/T_0^2 + 4B_{xx}\left(\vec{R}\right)/c_0^2$, where B_T and B_{xx} are the correlation functions of the random fields T' and v_x. Furthermore, $\Phi\left(\vec{K}\right) = \Phi_T\left(\vec{K}\right)/T_0^2 + 4\Phi_{xx}\left(\vec{K}\right)/c_0^2$, where Φ_T and Φ_{xx} are the three-dimensional spectral densities of these random fields. Note that equations similar to Eqs. (5) and (7) were also derived in Ref. [5] using Feynman path-integral technique. In this reference, the case of electromagnetic wave propagation over perfectly reflecting boundary (Q=−1) in a turbulent atmosphere was considered. Interference of electromagnetic waves in a turbulent atmosphere has been studied since the 1960s [6].

4 - KOLMOGOROV, GAUSSIAN AND VON KARMAN SPECTRA

Equation (7) for the coherence factor is valid for anisotropic as well as for isotropic turbulence. For the latter case, this equation is simplified:

$$C(h) = \exp\left[-\pi^2 k^2 L \int_0^1 d\xi \int_0^\infty K\left[1 - J_0(hK\xi)\right]\left[\Phi_T(K)/T_0^2 + 4F(K)/c_0^2\right] dK\right]. \tag{9}$$

Here, $F(K)$ is the three-dimensional spectral density of wind velocity fluctuations (e.g. [2]), and J_0 is the Bessel function. Equation (9) is closely related to the coherence function $\Gamma(r)$ of a spherical sound wave in a turbulent atmosphere for line-of-sight sound propagation, given by Eq. (7.71) from [2]. Comparison between these equations reveals that

$$C\,(h) = \Gamma\,(h)\,/\Gamma_0\,(h)\,, \tag{10}$$

where $\Gamma_0(r)$ is the value of $\Gamma(r)$ in a nonturbulent atmosphere. In other words, the coherence factor C is equal to the normalized coherence function of a spherical sound wave.

Formulas for the coherence function $\Gamma(r)$ are presented in [2] for Kolmogorov, Gaussian, and von Karman spectra of temperature and wind velocity fluctuations. These formulas allow us to calculate the coherence factor C given by Eq. (10).

For the Kolmogorov spectrum, the spectral densities $\Phi_T\,(K)$ and $F(K)$ are given by Eqs. (6.38) from [2], and the coherence function $\Gamma(r)$ is given by Eq. (7.38) from [2]. With the use of Eq. (10), we have

$$C^K = \exp\left(-k^2 h^{5/3} L \frac{3DC_T^2}{8T_0^2} - k^2 h^{5/3} L \frac{11DC_\nu^2}{4c_0^2}\right). \tag{11}$$

Here, the superscript K stands for the Kolmogorov spectrum, C_T^2 and C_ν^2 are the structure parameters of temperature and wind velocity fluctuations, and the numerical coefficient $D \approx 0.364$. For the Gaussian spectrum, $\Phi_T\,(K)$ and $F(K)$ are given by Eqs. (6.40) and (6.43), and $\Gamma(r)$ is given by Eq. (7.107) from [2]. Using the latter formula and Eq. (10), one obtains

$$C^G = \exp\left\{-2\gamma_T^G L\left[1-\left(\sqrt{\pi}l/2h\right) erf\,(h/l)\right] - 2\gamma_\nu^G L\left[1-\left(\sqrt{\pi}l/4h\right) erf\,(h/l) - \frac{1}{2}e^{-h^2/l^2}\right]\right\}. \tag{12}$$

Here, the superscript G stands for the Gaussian spectrum, l is a scale of temperature and wind velocity fluctuations for the Gaussian spectrum, $erf\,(h/l)$ is the error function, $\gamma_T^G = \sqrt{\pi}k^2\sigma_T^2 l/\left(8T_0^2\right)$ and $\gamma_\nu^G = \sqrt{\pi}k^2\sigma_\nu^2 l/\left(2c_0^2\right)$ are the extinction coefficients of the mean sound field due to temperature and velocity fluctuations, respectively. Note that for the case when wind velocity fluctuations are zero Eq. (12), was obtained in [1].

For the von Karman spectrum, $\Phi_T\,(K)$ and $F(K)$ are given by Eqs. (6.44) and (6.45), and Γ is given by Eq. (7.114) from [2]. For this spectrum, the coherence factor C is given by

$$C^{\nu K} = \exp\left\{-\frac{2L}{K_0 h}\int_0^{K_0 h} d\xi\left[\gamma_T^{\nu K}\left[1-\frac{2^{\frac{1}{6}}\xi^{\frac{1}{6}}}{6}K_{\frac{5}{6}}\,(\xi)\right]\right] + \gamma_\nu^{\nu K}\left[1-\frac{2^{\frac{1}{6}}\xi^{\frac{1}{6}}}{6}\left(K_{\frac{5}{6}}\,(\xi)-\frac{\xi}{2}K_{\frac{1}{6}}\,(\xi)\right)\right]\right\}.$$

Here, the superscript vK stands for the von Karman spectrum, K_0 is inversely proportional to the integral scale length of temperature and wind velocity fluctuations, K_α is the modified Bessel function, and $\gamma_T^{\nu K} = 10\pi^2 Ak^2 K_0^{-5/3} C_T^2/\left(10T_0^2\right)$ and $\gamma_\nu^{\nu K} = 6\pi^2 Ak^2 K_0^{-5/3} C_\nu^2/\left(5c_0^2\right)$ are the extinction coefficients of the mean sound field due to temperature and wind velocity fluctuations, where the numerical coefficient $A = 0.033$.

Equations presented in this section are used to study numerically the effects of temperature and wind velocity fluctuations on the interference of the direct and ground reflected waves in a turbulent atmosphere. Numerical examples will be given during the presentation. Furthermore, in a companion paper [7], theoretical results presented above are used to explain experimental data obtained in a laboratory experiment which models sound propagation through a turbulent atmosphere.

ACKNOWLEDGEMENTS

V. Ostashev's research was supported by an NRC-ETL Research Associateship and by the U.S. Army Research Office under contract number DAAG55-98-1-0463 (administered by W. Bach).

REFERENCES

1. **S. F. Clifford and R. J. Lataitis**, Turbulence effects on acoustic wave propagation over a smooth surface, *J. Acoust. Soc. Am*, Vol. 73, pp. 1545-1550, 1983

2. **V. E. Ostashev**, *Acoustics in Moving Inhomogeneous Media*, E&FN SPON, London, 1997

3. **K. Attenborough, S. I. Hayek, and J. M. Lawther**, Propagation of sound above a porous half-space, *J. Acoust. Soc. Am.*, Vol. 68, pp. 1493-1501, 1980

4. **S. M. Rytov, Yu. A. Kravtzov, and V. I. Tatarskii**, *Principles of statistical radio physics. Part 4. Wave propagation through random media*, Springer, Berlin, 1989

5. **A. V. Kukushkin and I. M. Fuks**, Reflection formulas for a wave field in a random nonuniform medium above an ideally reflecting interface, *Radiophysics and Quantum Electronics*, Vol. 31, pp. 1035-1042, 1980

6. **F. G. Bass, S. Ya. Braude, E. A. Kaner, and A. V. Men**, Fluctuations of electromagnetic waves in the troposphere in the presence of separation boundaries, *Soviet Physics Uspekhi*, Vol. 73, pp. 89-119, 1961

7. **Ph. Blanc-Benon, J. Wasier, D. Juvé, and V.E. Ostashev**, Experimental studies of sound propagation through thermal turbulence near a boundary, In *Proceedings of InterNoise 2000 (Nice, France)*, 2000