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QUADRATIC STATISTICAL CHARACTERISTICS OF CYCLIC NOISE

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ABSTRACT
Time-frequency representation of cyclic noise is studied. For this purpose three quadratic statistical characteristics are considered. These are the complex and real instantaneous autospectra and gated autospectrum. Advantages and disadvantages of these characteristics are discussed. It is shown that the complex instantaneous autospectrum gives the best results. As a concrete example of the method, the complex instantaneous autospectrum of the gearbox vibration is computed.

1 - INTRODUCTION
Acoustic noises may be categorized as being either steady (stationary) or transient (nonstationary). The most important statistical characteristics of the stationary noise is an autospectral power density. This characteristics is a function of frequency only and hence is independent of time. In the case of the transient noise an autospectral energy density may be computed [1]. This density is also a function of frequency only and is independent of time again. However, this time independence may be a serious drawback here as the spectral content of the transient noise can vary with time. Therefore in the last years much attention has been paid to development of methods enabling time-frequency analysis of the transient noise [2].

Classification of noises as being either steady or transient certainly does not exhaust all possibilities. For example, a number of machinery work cyclically. Hence the noise they generate has a cyclic character, that is, the noise posses certain features repeating more or less regularly. From the statistical point of view the cyclic noise has an interesting property: one can decide whether to analyze the cyclic noise as a stationary or nonstationary signal. A traditional approach used to analyze the cyclic noise is based on assumption of its stationarity. The autospectral power density is then independent of time. However, should the cyclic noise be analyzed as a nonstationary signal, a possibility ensues to determine how the spectral composition of the noise varies with time and a much more detailed noise description may be obtained.

A number of time-frequency statistical characteristics of the cyclic noise can be designed. These are, e.g., a gated autospectrum, and real and complex instantaneous autospectra [3]. In the following section definitions and basic properties of these characteristics will be briefly reviewed.

2 - QUADRATIC STATISTICAL CHARACTERISTICS
The double autocorrelation function $R_{xx}(t_1, t_2)$ of a nonstationary noise $x(t)$ can be computed using the definition formula [1]:

$$R_{xx}(t_1, t_2) = E[x(t_1)\, x(t_2)] \qquad (1)$$

Here $E[\ ]$ denotes averaging operation. In the case of the cyclic noise the averaging can be performed over different cycles.

The complex instantaneous autospectrum $W_{xx}(f_1, t_2)$ is given by the inverse Fourier transform of $R_{xx}(t_1, t_2)$ [1]:

$$W_{xx}(f_1, t_2) = \int_{-\infty}^{\infty} R_{xx}(t_1, t_2)\, e^{j2\pi f_1 t_1} dt_1 \tag{2}$$

The statistical characteristics $W_{xx}(f_1, t_2)$ is a complex function of frequency and time and can take on negative values. Hence it is not a spectral density. However, it is free of artifacts and can be obtained with any desired frequency and time resolution.
A different quadratic statistical characteristics can be derived from (1) by introducing new variables $t = (t_1 + t_2)/2$, $\tau = (t_2 - t_1)$. After substituting these new variables into (1) one obtains [1]:

$$R_{xx}(\tau, t) = E\left[x(t - \tau/2)\, x(t + \tau/2)\right] \tag{3}$$

The real instantaneous autospectrum $W_{xx}(f, t)$ is then given by the direct Fourier transform of $R_{xx}(\tau, t)$ [1]:

$$W_{xx}(f, t) = \int_{-\infty}^{\infty} R_{xx}(\tau, t)\, e^{-j2\pi f \tau} d\tau \tag{4}$$

This statistical characteristics is, in a way, similar to the Wigner distribution used in time-frequency analysis of nonstationary signals [2]. Nevertheless it differs from the Wigner distribution in that it is a statistical characteristics obtained by averaging. The real instantaneous autospectrum can take on negative values and therefore it is not a spectral density, too. Similarly as the complex instantaneous autospectrum it can be obtained with any desired frequency and time resolution. Unfortunately, artifacts occurring in this characteristics deteriorate its interpretation significantly.
The last time-frequency statistical characteristics discussed here is a gated autospectrum. Denoting for a moment the time variable as t', then a time window of duration T_w centered at a time t is $w(t', t)$ and the gated noise is $x(t')\, w(t', t)$. The Fourier transform $X(f, t)$ of the gated noise equals

$$X(f, t) = \int_{-\infty}^{\infty} x(t')\, w(t', t)\, e^{-j2\pi f t'} dt' \tag{5}$$

and the gated autospectrum $P_{xx}(f, t)$ is given by the following formula

$$P_{xx}(f, t) = \frac{1}{T_w} E\left[|X(f, t)|^2\right] \tag{6}$$

The gated autospectrum is a cyclic variant of the well known short-time Fourier transform [2]. Unfortunately the window length T_w sets serious limits on frequency and time resolution of this characteristics. However, unlike the instantaneous autospectra the gated autospectrum is a spectral density.
To investigate the properties of the three statistical characteristics an artificial signal consisting of three sinusoids was analyzed. The frequencies of the sinusoids were 1 kHz, 3 kHz, and 20 kHz. The signal was sampled with a frequency f_s=128 kHz and the characteristics $W_{xx}(f_1, t_2)$, $W_{xx}(f, t)$ and $P_{xx}(f, t)$ were computed using formulas (1) $-$ (6). The time window length of the gated autospectrum was T_w=$T_p/2$, where T_p is the basic period of the signal. It should be remarked that frequency resolution Δf is different for each characteristics. It is 1 kHz for $W_{xx}(f_1, t_2)$, 500 Hz for $W_{xx}(f, t)$, and about 2 kHz for $P_{xx}(f, t)$. In the complex instantaneous autospectrum the three components could be clearly seen. In the real instantaneous autospectrum a much more complicated picture could be observed. Beside the three components a number of artifacts occurred here and these artifacts deteriorated the clarity of the characteristics significantly. Finally, the components with low frequencies were not resolved in the gated autospectrum and the amplitude of the third component was much lower than expected. Time dependence of this component was also suppressed. Comparison of the three characteristics revealed that the complex instantaneous autospectrum performs best. Hence the following discussion will concentrate on this characteristics only.

3 - GEARBOX VIBRATION ANALYSIS

In this section an example of a real signal analysis will be given. The signal is vibration acceleration measured on a passenger car gearbox. The gearbox had three shafts (input, main, output). Vibration acceleration had been recorded at an input shaft velocity of 3000 rpm, under load of 20 Nm, and at 3$^{\mathrm{rd}}$ gear. The data were recorded with a HP signal analyzer, the sampling frequency was f_s=32768 Hz. Concurrently with the vibration signal, tachopulses derived from input shaft rotation have also been recorded.

A basic assumption for computation of the double autocorrelation function $R_{xx}(t_1, t_2)$ is a cyclostationarity of the analyzed signal. This means that in each cycle the signal must be sampled in the same number of points and these points always correspond to the same phase in the respective cycle. However, in a real signal duration of different cycles and position of samples will vary from cycle to cycle. Hence the recorded signal had to be cyclostationarized first. This was done by signal resampling [4]. The starting point of each cycle was defined by a corresponding tachopulse.

Velocity ratios between different shafts are given by numbers of teeth on the respective gears. In the case considered here these ratios were n_1/n_2=30/38 and n_3/n_4=18/75. Denoting the periods associated with the three shafts as T_{p1}, T_{p2}, and T_{p3}, then the basic period of the gearbox T_p is given by the relation T_p=95T_{p1}=75T_{p2}=18T_{p3}. The average period of the input shaft as determined from the tachopulses was T_{p1}=19.85 ms, hence T_p=1.866 s. After the measured signal was resampled the number of samples in T_p was 65536, the new time sampling interval was Δt=28.8 μs and the new frequency sampling interval was Δf=0.53 Hz.

A complete instantaneous autospectrum would be represented by a square matrix of 65636 ×65536 elements. Evidently the computation of the whole characteristics $W_{xx}(f_1, t_2)$ would be rather time demanding. Fortunately it is usually not necessary to know the complete instantaneous autospectrum $W_{xx}(f_1, t_2)$ and it often suffices to know only selected slices. An example of such a slice corresponding to t_2=1s is shown in Fig. 1. As can be seen, the magnitude of the complex instantaneous autospectrum has the same composition as the standard autospectral power density, i.e. it consists of a great number of discrete spectral components overlaid over continuous spectrum. However, now the autospectrum can be obtained for different instants.

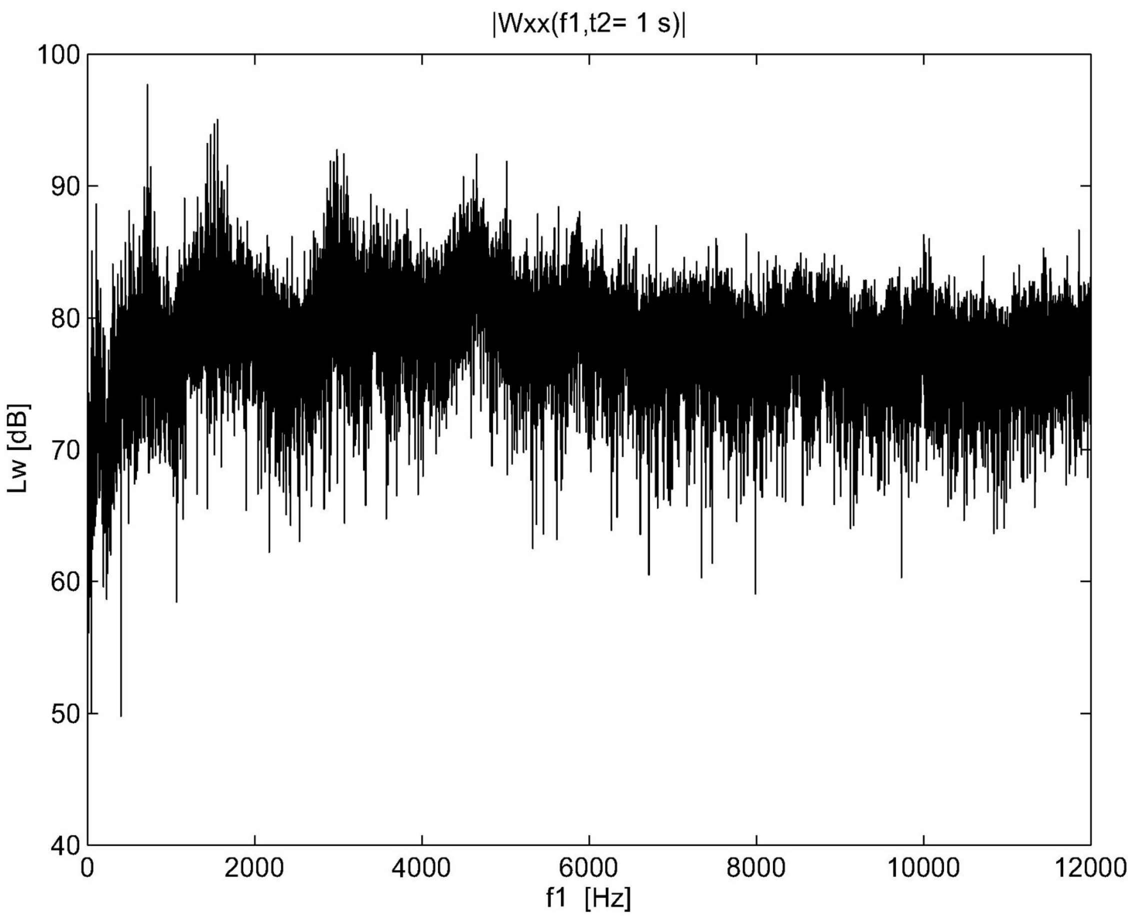

Figure 1: A slice of the complex instantaneous autospectrum of the gearbox vibration at t_2=1 s; frequency resolution Δf=0.53 Hz.

4 - CONCLUSION

Three quadratic statistical characteristics have been discussed. These are the complex and real instantaneous autospectra and gated autospectrum. It has been shown that the complex instantaneous autospectrum gives the best results. It can be obtained with an excellent frequency and time resolution and is free of artifacts. Along the frequency axis the characteristics shows spectral details similar to those obtained by a standard spectral analysis. However, now the spectral components are time dependent.

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