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ACTIVE CONTROL OF SOUND RADIATION FROM ENGINE MOUNTED PLATES WITH PIEZOELECTRIC ACTUATORS

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ABSTRACT
Active control of sound radiation from an engine mounted plate by a piezoelectric actuator is investigated. The optimal control voltage, which is applied on the piezoelectric actuator, is obtained by minimizing the sound radiation power. Case studies show that the highest reduction of the total power radiated from the plate could be 71 dB at resonant frequencies.

1 - INTRODUCTION
Active control of sound radiation from vibrating structures has become a very interesting research area. There exist two different active methods to attenuate the radiated sound. One, termed "active noise control" (ANC), is to use an appropriate pressure wave as the secondary source which optimally interferes with the primary pressure wave to effect sound cancellation in a local region. The other is to apply external forces to the vibrating structures directly to attenuate the radiated sound in a globe sense using a low number of control transducers by controlling the vibrations of the radiating structure. The method, also termed "active structural acoustic control" (ASAC), is suitable for controlling low to mid-frequency structural sound radiation [1].
Piezoelectric actuators show much potential in this field as the piezoelectric elements may be either attached to or imbedded directly into the structure. In this paper, active control of sound radiation from an engine mounted plate with a piezoelectric actuators is presented.

2 - PLATE VIBRATION
For consideration of sound radiation from an engine cabin, a simply model is created. The engine is mounted through four isolators on a flexible thin rectangular plate. For simplicity, the isolators only excite vertical point forces on the plates. The plate is supposed to be baffled to infinite, so the radiated pressure in the fields can be calculated with the Rayleigh's integral. Figure 1 shows the arrangement and coordinate system of the baffled, simply supported rectangular plate with the engine and the actuator.
To calculate the radiated sound field, a description is required of the plate vibration distribution. For the simply supported thin plate, the displacement distribution cab be given in modal form as

$$w\left(x,y,t\right)=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}W_{mn}\sin k_m x \sin k_n y e^{i\omega t} \tag{1}$$

where m and n are respectively the modal order in the x and y coordinates, ω is the circular frequency. The wavenumbers are given by $k_m = m\pi/L_x$, $k_n = n\pi/L_y$.
The plate modal amplitudes under the point force excitations are given by

$$W_{mn}=P_{mn}^{f}/\rho_1 h\left(\omega_{mn}^2-\omega^2\right) \tag{2}$$

where ω_{mn} are the natural frequencies, ρ_1 is the plate density and h is the plate thickness.
The modal forces, P_{mn}^{f}, due to the force excitations of isolators of the engine can be read as

$$\begin{array}{l}P_{mn}^{f} = 4F\{\sin(k_m x_{f1})\sin(k_n y_{f1}) + \sin(k_m x_{f1})\sin(k_n y_{f2}) \\ +\sin(k_m x_{f2})\sin(k_n y_{f1}) + \sin(k_m x_{f2})\sin(k_n y_{f2})\}/L_x L_y\end{array} \quad (3)$$

where F is the amplitude of force of the isolators, (x_{f1}, y_{f1}), (x_{f2}, y_{f1}), (x_{f1}, y_{f2}) and (x_{f2}, y_{f2}) are respectively the coordinates of the isolators, here the four point forces are supposed to be equal and in phase, the unequal and out of phase forces can be considered easily by changing the amplitude and sigh of each item in the equation (3).

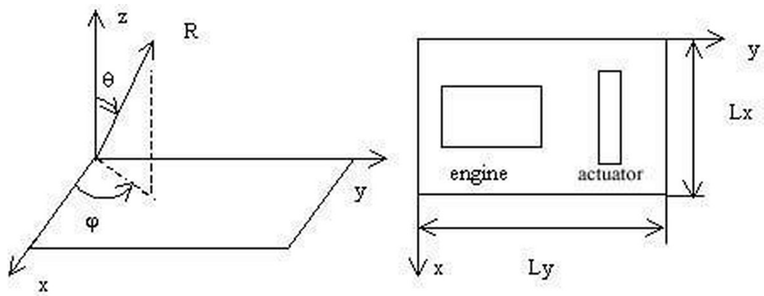

Figure 1: A rectangular baffled plate with the engine and the actuator.

The active control is achieved by using a rectangular piezoelectric actuator. The actuator consists of two piezoelectric layers located symmetrically on each side of the plate. The piezoelectric layers are driven anti-phase in the d_{31} mode for creating pure bending. The modal forces, P_{mn}^{a}, due to the piezoelectric actuator, were given by Fuller, etc. (1996) as

$$P_{mn}^{a} = 4C_0\varepsilon_{pe}\left(k_m^2 + k_n^2\right)\left(\cos(k_m x_{p1}) - \cos(k_m x_{p2})\right)\left(\cos(k_n y_{p1}) - \cos(k_n y_{p2})\right)/\left(mn\pi^2\right) \quad (4)$$

where (x_{p1}, y_{p1}), (x_{p2}, y_{p1}), (x_{p1}, y_{p2}) and (x_{p2}, y_{p2}) are respectively the coordinates of the actuator edges. The parameter C_0 is a constant that is function of the piezoelectric actuator/plate properties and geometries specified by

$$C_0 = -E\frac{1+\nu_a}{1-\nu}\frac{P}{1+\nu-(1+\nu_a)P}\frac{2}{3}\left(\frac{h}{2}\right)^2 \quad (5)$$

$$P = -\frac{E_a}{E}\frac{1-\nu^2}{1-\nu_a^2}\frac{3t_a(h/2)(h+t_a)}{2\left[(h/2)^3 + t_a^3\right] + 3(h/2)t_a^2} \quad (6)$$

where Poisson's ratios ν, ν_a and Young Modulus E, E_a are respectively for the plate and the actuator. $\varepsilon_{pe} = d_{31}V/t_a$ is the strain induced by an unconstrained piezoelectric layer of thickness t_a, when a voltage V is applied along its polarization direction, while d_{31} is the dielectric strain constant of the piezoelectric. For modal amplitude of the actuator ρ_1 and h in equation (2) are replaced by ρ_a and t_a, where ρ_a is the actuator density.

3 - SOUND RADIATION

The sound radiation caused by the plate vibration in the spherical coordinate (r,θ,ϕ) can be evaluated by the Rayleigh integral as

$$p(r,\theta,\phi) = K\sum_{m=1}^{\infty}\sum_{n=1}^{\infty} W_{mn} I_m I_n e^{i\omega t} \quad (7)$$

where

$$K = -\frac{\omega^2\rho_2 L_x L_y}{2\pi r}e^{-ikr},\ I_m = m\pi\frac{(-1)^m e^{-i\alpha} - 1}{\alpha^2 - (m\pi)^2},\ I_n = n\pi\frac{(-1)^n e^{-i\beta} - 1}{\beta^2 - (n\pi)^2} \quad (8)$$

where ρ_2 is the fluid density, $\alpha = kL_x\sin\theta\cos\phi$, $\beta = kL_y\sin\theta\sin\phi$, k is the wavenumber of acoustic medium.
The total pressure is due to the isolator forces and the actuator and can be read as

$$p_t(r,\theta,\phi) = p_f(r,\theta,\phi) + p_a(r,\theta,\phi) = C_f F + C_a V \quad (9)$$

4 - OPTIMAL CONTROL

The piezoelectric voltage V can be chosen appropriately to minimize the total radiated sound power into the far field. The cost function chosen to be minimized is the integral of the mean-squared sound pressure over a hemisphere of radius r=R in the far field. This cost function, which is the radiated sound power, can be written as

$$\Psi = \frac{1}{R^2}\int_s p_t^2\left(R,\theta,\phi\right)ds = \int_0^{2\pi}\int_0^{\pi/2} p_t^2\left(R,\theta,\phi\right)\sin\theta d\theta d\phi \tag{10}$$

or

$$\Psi = F^2 G_{ff} + FVG_{fv} + V^2 G_{vv} \tag{11}$$

where

$$G_{ff} = \int_0^{2\pi}\int_0^{\pi/2} C_f C_f^* \sin\theta d\theta d\phi \tag{12}$$

$$G_{fv} = \int_0^{2\pi}\int_0^{\pi/2} 2\mathrm{Re}\left(C_f C_a^*\right)\sin\theta d\theta d\phi \tag{13}$$

$$G_{vv} = \int_0^{2\pi}\int_0^{\pi/2} C_a C_a^* \sin\,\theta d\theta d\phi \tag{14}$$

where Re is the real part of a complex quantity.
To minimize the sound radiation power Ψ the control voltage V must satisfy

$$\partial\Psi/\partial V = 0 \tag{15}$$

Therefore, the control voltage V must be

$$V = -G_{fv}/\left(2G_{vv}\right)F \tag{16}$$

and the minimum radiation power will be

$$\Psi = \left\{G_{ff} - G_{fv}^2/\left(4G_{vv}\right)\right\}F^2 \tag{17}$$

5 - CASE STUDIES

Case studies are presented for the active control of sound radiation from the engine mounted plate with the piezoelectric actuator. The plate was assumed to be steel with material properties given as: ρ_1 = 7850 kg/m^3, v= 0.3, h = 56 mm and E=2.1e+11 N/m^3. The dimensions of the plate were: L_x = 10 m, L_y = 10 m. Resonant frequencies for modes (m, n) were calculated and are given in Table 1. For frequency response analysis of the plate, the damping of plate is introduced by E= $\mathrm{E}\left(1+\mathrm{i}\eta\right)$, where $\eta = 0.001$. The four isolators were located at x_{f1} = 2 m, x_{f2} = 6 m, y_{f1} = 2 m and y_{f2} = 4 m. The piezoelectric actuator is PZT with the properties given as: ρ_a = 7650 kg/m^3, ν_a= 0.3, $d_{31} = -166\mathrm{e}-12$ m/V, t_a = 2 mm and E_a=6.3e + 10 N/m^3. The layer was patched on the plate at x_{p1} = 7 m, x_{p2} = 9 m, y_{p1} = 2 m and y_{p2} = 4 m. The acoustic medium is air with sound speed c = 343 m/s and ρ_2 = 1.21 kg/m^3. Figure 2 shows the radiated power in the frequency range of 0 and 300 (rad/sec) without and with the actuator. The calculated frequency increment is 0.4 (rad/sec). The power has four peaks at frequencies of 32.4, 58.0, 101.2 and 161.2 (rad/sec). These resonant frequencies are due to the mode (1, 1), (1, 2), (1, 3) and (1, 4). The powers are respectively 154.7 dB, 116.7 dB, 119.5 dB and 135.6 dB at these frequencies. After the optimal voltages are applied to the actuator, the powers are respectively 83.3 dB, 98.4 dB, 87.5 dB and 99.3 dB at these frequencies. The reductions are respectively 71.4 dB, 18.3 dB, 32 dB and 36.3 dB at these peaks. It can be seen from the Fig. 2 that the control effects are also obtained at lower non-resonant frequencies.

n / m	1	2	3	4	5	6
1	32.404	103.88	223.02	389.81	604.25	866.35
2	58.138	129.61	248.75	415.54	629.98	892.08
3	101.02	172.5	291.64	458.43	672.87	934.97
4	161.07	232.55	351.68	518.47	732.92	995.02
5	238.27	309.75	428.88	595.67	810.12	1072.2
6	332.62	404.1	523.24	690.03	904.47	1166.5

Table 1: Plate resonant frequencies, ω_{mn} (rad/sec).

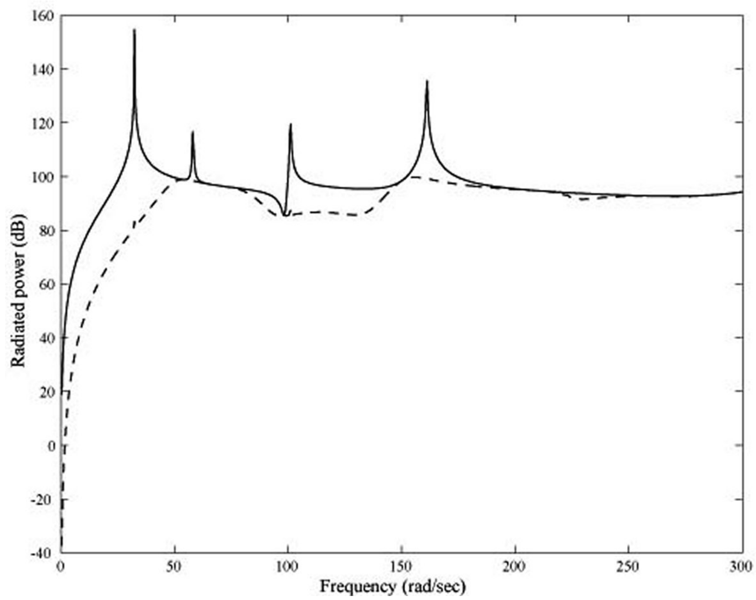

Figure 2: Total radiated power from the plate; __, without the actuator; - - -, with the actuator.

6 - CONCLUSIONS

Active control of sound radiation from an engine mounted plate by a piezoelectric actuators is investigated. The optimal control voltage is obtained by minimizing the sound radiation power. The high reduction of the total power radiated from the plate can be achieved at resonant frequencies. The benefit of reduction of radiation power can be also obtained at lower non-resonant frequencies.

REFERENCES

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