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A PROPOSAL ON A MEANS TO IDENTIFY THE NATURAL FREQUENCY OF RAILWAY AND HIGHWAY STRUCTURES

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ABSTRACT
A soundness estimation method using measured natural frequency has been applied to the pier of urban viaducts and bridges of railway and highway. The natural frequency is obtained from the damped vibration of a pier by percussion test using about 30 kilograms weight of ball. In the usual estimation method, the Fourier transform has been applied to identify the natural frequency of the pier. However, it is difficult to identify the natural frequency by only Fourier transform, when measured waveforms contain vibrations caused by the passages of a train, vehicles and so on. In this paper, a useful method based on the wavelet transform is developed to improve the precision of the identification of natural frequency.

1 - OUTLINE OF WAVELET TRANSFORM

The wavelet transform is the functional expansion with a special property. In the wavelet transform, there are continuous wavelet transform and discrete wavelet transform. The continuous wavelet transform is defined as follows.

$$\left(W_\psi f\right)(b,a)=\int_{-\infty}^{\infty}\frac{1}{\sqrt{|a|}}\cdot\psi\left(\frac{t-b}{a}\right)\cdot f(t)\,dt \tag{1}$$

in which $\Psi(t)$ the mother wavelet of a function $f(t)$, $1/a$ is a parameter of a frequency, and b is a parameter of a time. The discrete expression of equation (1) is given as follows.

$$\left(W_\psi f\right)(b,a)=\sum_n\frac{1}{\sqrt{|a|}}\cdot\psi\left(\frac{n\Delta t-b}{a}\right)\cdot f(n\Delta t)\,\Delta t \tag{2}$$

When the time-frequency spectrum is numerically obtained, these parameters have to be made as small as possible.

In the wavelet multi-resolution analysis, the function $f_j(t)$ is repeatedly decomposed in the following operation.

$$f_j(t)=g_{j-1}(t)+f_{j-1}(t) \tag{3}$$

in which j is an integer number which is called a level. The function f_j in level j is decomposed to functions f_{j-1} and $g_{\ j-1}$ in level $j-1$ below one order. Therefore, the function f_j in level j is expressed by the sum of the lower level functions g_{j-1} , g_{j-2},... as follows.

$$f_j(t)=\sum_{m=-\infty}^{j-1}g_m(t) \tag{4}$$

The function $g_j(t)$ is expanded in series of the wavelet function Ψ with coefficient $\alpha_{j,k}$ as following equation.

$$g_j(t) = \sum_{k=-\infty}^{\infty} \alpha_{j,k} \cdot \psi\left(2^j t - k\right) \tag{5}$$

in which k is an integer number and the wavelet function Ψ means the wave which exists locally in both domains of frequency and time.
As a level j becomes smaller, the function g_j gives the waveform in the lower frequency region. The level j defines the frequency band which is from ($2^{j-1}/\Delta t$) Hz to ($2^j/\Delta t$) Hz.

2 - MEASUREMENT OF VIBRATION

2.1 - Outline of measurement

In percussion test, the top of a pier is struck by an iron ball covered with rubber in the direction of perpendicular to the bridge axis, and horizontal particle velocities of the pier are measured on several points of the pier in a vertical line. The weight of the iron ball is about 30 kilograms, which is not so heavy considering handiness. The frequency spectrum of exciting force is nearly flat on the effect of rubber cover of iron ball. Therefore, the Fourier spectrum of vibrations of the pier by percussion test is similar to the frequency response function of the pier.
On the other hand, the amplitude of vibration by a stroke of iron ball is very small because it is not so heavy. Therefore, waveforms by multiple strokes are superposed by making the signal of a pick-up nearest to the struck point to a trigger. This operation improves the ratio of signal to noise.
The horizontal vibration of the girder is also measured to find its natural frequency in the direction of perpendicular to the bridge axis. The natural frequency of the girder must be removed from peak frequencies of the Fourier spectrum in the vibration of the pier.

2.2 - Object of investigation

The measurements are performed in the pier of railway bridge shown in figure 1. The pier is a three-dimensional rigid frame in which each column is connected by a beam at middle height of the pier. The type of foundation is a pile foundation. The pier supports a trussed bridge with about 64 meter span. A four-lane motorway runs under the trussed bridge and an crowded road with two-lane runs parallel to the railway truck. It is considered that the vibration of the pier by percussion test contains vibrations caused by the passage of vehicles in these roads because the traffic is heavy.

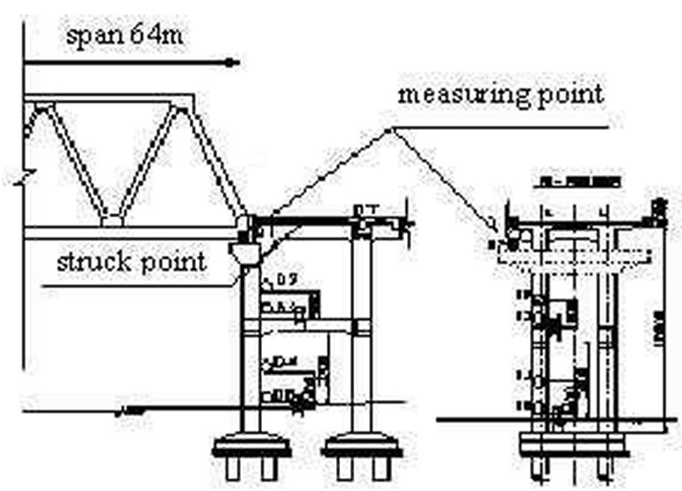


Figure 1: Test pier.

3 - IDENTIFICATION BY FOURIER TRANSFORM

In the percussion test, the peak frequency in which phase angle is equal to 180 degrees in the Fourier spectrum of measured waveform corresponds to the natural frequency of the structure. The waveform of particle velocity and its amplitude and phase spectra at the crown of the pier are shown in figure 2. It is seen that there are six peaks at 2.0Hz, 2.7Hz, 3.3Hz, 5.2Hz, 6.0Hz and 7.3Hz, except the peak at 0.3Hz in which the phase angle is away from 180 degrees. From the Fourier spectrum, it is difficult to judge which of the six peak frequencies is the natural frequency of the pier.

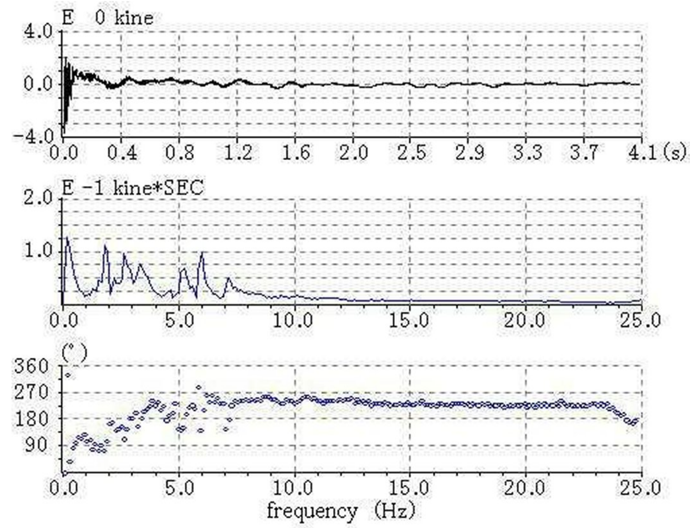


Figure 2: Measured waveform and Fourier spectrum at crown of pier.

Next, the waveform and its Fourier spectrum at the center of the trussed bridge are shown in figure 3. It is inferred that the prominent peak at 2.0Hz is the natural frequency of the trussed bridge, because the phase angle is equal to 180 degrees. However, the small peak at 3.0Hz or 7.3Hz has the possibility of the natural frequency of the trussed bridge, because the phase angle at each peak changes through 180 degrees. The above indicates that the Fourier spectrum does not always give sufficient information on the identification of the natural frequency of structures.

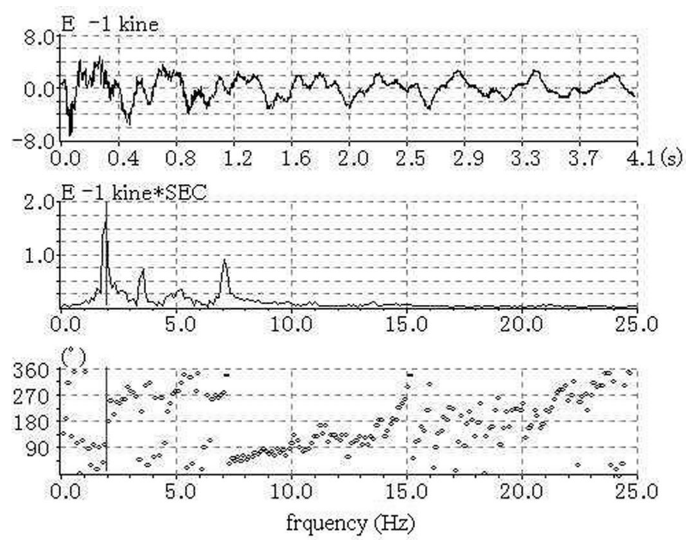


Figure 3: Measured waveform and Fourier spectrum at center of trussed.

4 - IDENTIFICATION BY WAVELET SPECTRUM

The wavelet function on the wavelet analysis contains two parameters in a frequency and time. The wavelet spectrum is used to examine the variation of frequency characteristics in a time domain. Therefore, it is probable that the natural frequency is identified from the change of dominant frequency in the time domain. Figure 4 shows the contour map of the wavelet spectrum for the measured waveform obtained at the crown of the pier. In the contour map, the abscissa and ordinate are time and frequency respectively. As the color becomes white, the spectrum value increases and the highest contour indicates a dominant frequency. A dominant frequency is found at about 3Hz and in the region of 5 to 6Hz. However, it is considered that the latter is not the natural frequency, because the peak contour of this component appears repeatedly. A dominant frequency is also found at about 2Hz. This component continues until about 4 seconds and the attenuation of it is very slow. From these fact and figure 3, it is seen that the vibration of the trussed bridge brings about the vibration of about 2Hz. On the other hand, the vibration of about 3Hz appears only in the initial stage and this component does not form a clear contour after this stage. Therefore, it is estimated that the first natural frequency of the structure is about 3.0Hz.

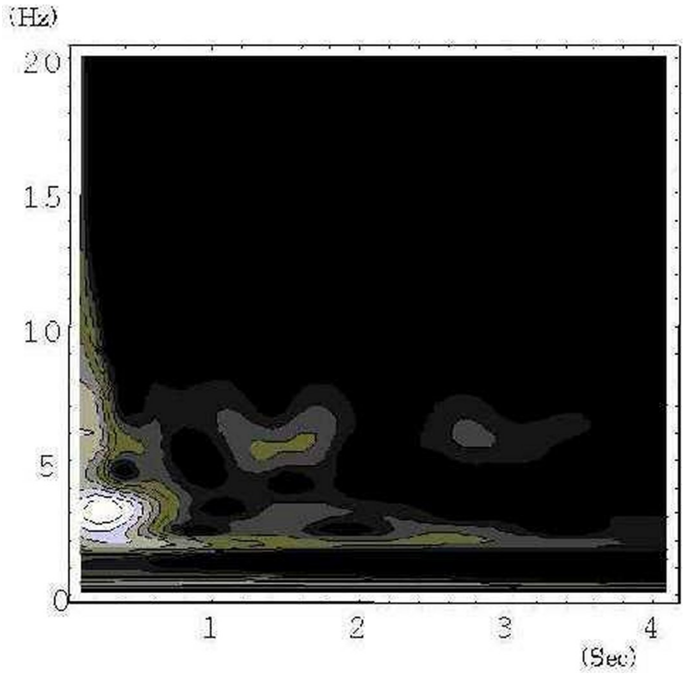


Figure 4: Wavelet spectrum.

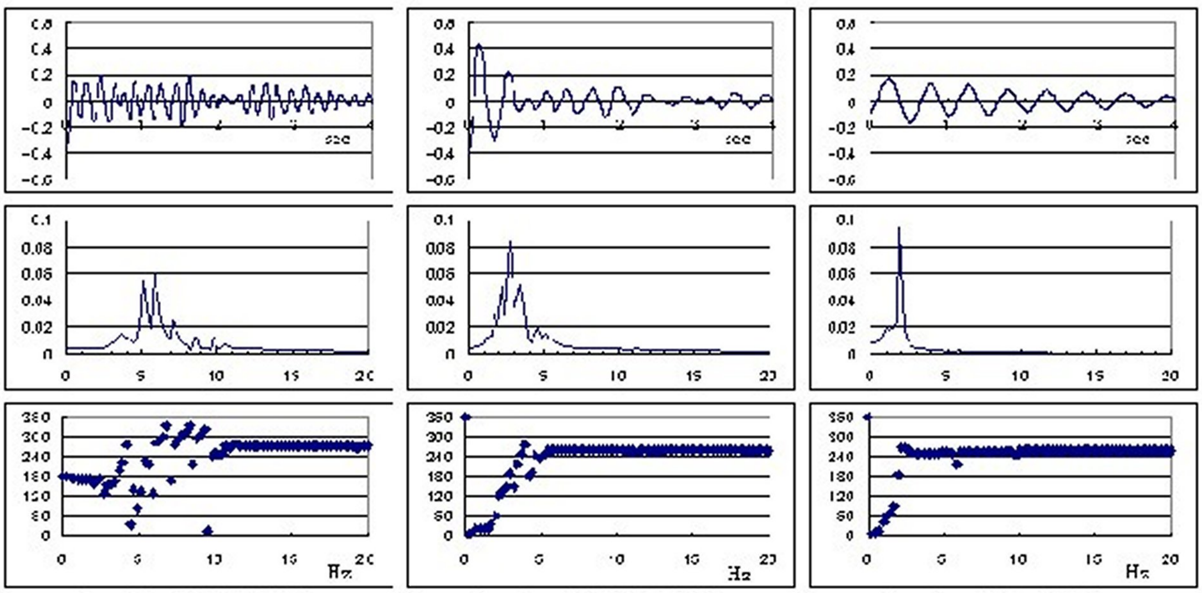
5 - IDENTIFICATION BY WAVELET MULTI-RESOLUTION ANALYSIS

In order to increase the accuracy of identification, the wavelet multi-resolution analysis is applied to the measured waveform. First, the measured waveform is decomposed into waveforms belong to several frequency bands by using the wavelet multi-resolution analysis. Next, the Fourier transform is performed for the decomposed waveform in which it is expected to contain the natural frequency.

Figure 5 shows the decomposed waveform and the amplitude and phase spectra for it in each level of j=-6, -7 and -8. In level j=-6 in the frequency band of 3.9 to 7.8Hz, there are two peaks at about 5Hz and 6Hz. However, the decomposed waveform shows no attenuation and the phase spectrum is not clear in this frequency band. Therefore, it is considered that this decomposed waveform is generated by some external sources and does not contain the natural frequency component. In level j=-7, the frequency band is from 2.0Hz to 3.7Hz. There are three peaks at 2.2Hz, 2.7Hz and 3.3Hz in this band. The phase angle is not equal to 180 degrees in the peaks at 2.2Hz and 3.3Hz, while the phase angle at 2.7Hz coincides with 180 degrees. Therefore, it is estimated that 2.7Hz is the natural frequency of the pier considering the magnitude of the peak. In level j=-8 in the frequency band of 1.0 to 2.0Hz, there is an isolated peak at 2.0Hz and the phase angle also equals 180 degrees. The decomposed waveform attenuates slowly. It is estimated that the vibration of the pier at 2.0Hz is caused by the vibration of the trussed bridge. From the above-mentioned, it is decided that the natural frequency of the pier is 2.7Hz.

6 - CONCLUSION

In this paper, the wavelet transform is applied to identify accurately the natural frequency from measured waveforms by percussion test. The wavelet spectrum gives the useful information on the variation of frequency characteristics in the time domain. Therefore, it is probable that the natural frequency is picked out from measured waveforms that contain vibrations caused by the passages of a train, vehicles and so on. On the other hand, the measured waveform is decomposed into waveforms in several frequency bands by applying the wavelet multi-resolution analysis. From the above-mentioned, it is probable that the accuracy of identification of the natural frequency increases by drawing the wavelet spectrum of the waveform decomposed by the wavelet multiple resolution analysis.



Level j=-6 (3.9 - 7.8 Hz) Level j=-7 (2.0 - 3.9 Hz) Level j=-8 (1.0 - 2.0 Hz)

Figure 5: Results of wavelet multi-resolution analysis and Fourier spectra.