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BACKSCATTERING OF SOUND FROM A SCREEN USING A PARABOLIC EQUATION MODEL

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ABSTRACT
A new two-way parabolic equation including ground impedance for solving backscattering from a screen is described. Backscattering is handled by applying the acoustic continuity conditions at the vertical interface of a screen. An analytical model based on a rigorous geometrical theory of diffraction is formed for comparison. Results of an example involving homogeneous atmosphere and a thin screen show an excellent agreement. It is shown that backscattering causes a rise of sound level in the areas between a source and a screen. The parabolic equation model is potential for studying noise from barriers having various shapes.

1 - INTRODUCTION
In urban built-up areas, traffic noise often backscattered from surrounding barriers may cause extra environmental noise. Various models [1] have in recent years been developed concerning ground impedance, screens, winds and turbulence. However, backscattering is not included. Backscattering from screens involves the influences from ground and screens. As the ground effect is not the focus of the paper, a simple impedance model [2] is used, though more complex models [3] are applicable. The parabolic equation (PE) method has been applied to model forward outdoor sound propagation for years. Many existing PEs [4,5] handle the screen effect by setting the values of the points on the screen to zero (i.e. treat screens as rigid objects). This may not be accurate enough for absorbing screens. A new two-way PE including ground impedance for solving backscattering from absorbing screens is developed, which handles backscattering by applying the acoustic continuity conditions at the vertical interface of a screen. An analytical model based on the geometrical theory of diffraction [5,6] is created to verify the PE model using an example involving homogeneous atmosphere and a thin screen.

2 - THEORY
The PE model is derived in a cylindrical coordinate system with azimuthal symmetry. For a harmonic source with $e^{-i\omega t}$ removed, the acoustic pressure $p(r,h)$, where r is range and h is height, satisfies the Helmholtz equation [7]: $\rho\nabla\cdot\left(\rho^{-1}\nabla p\right)+\left(k_0\eta\right)^2 p=0$, where ρ is density and k_0 is a reference wavenumber. $\eta^2\approx\left(c_0/c\right)^2\left(1+i\alpha/27.29\right)$ defines a complex index of refraction (η), where c_0 is a reference velocity and α is the attenuation coefficient in dB/ λ (λ is the wavelength). A new variable u is used to replace p via $p^{+}{=}u^{+}H\ _0{}^{(1)}(k_0r)$ and $p^{-}{=}u^{-}H\ _0{}^{(2)}(k_0r)$, where $H_0{}^{(1)}$ and $H_0{}^{(2)}$ are the Hankel functions expressed in the far-field ($k\ _0r{>>}1$). Here, waves are split into two parts: one going out from the source (+) and the other coming back to the source (-). Assuming a weak range dependence, the propagation described by the Helmholtz equation can also be split into two parts: $\left(\partial/\partial r\right)u^{+}=ik_0\left(Q-1\right)u^{+}$ and $\left(\partial/\partial r\right)u^{-}=-ik_0\left(Q-1\right)u^{-}$. The outgoing wave equation (+) is the basis to obtain various parabolic approximations. The incoming wave equation (-) is unique only in two-way PEs. The operator Q is given by [7]:

$$Q=\sqrt{1+q}\quad,\quad q=\eta^2-1+\frac{1}{k_0^2}\rho\frac{\partial}{\partial h}\left(\frac{1}{\rho}\frac{\partial}{\partial h}\right)\tag{1}$$

In a discretized sound field with the grid sizes of Δr and Δh, the value at a position $(n\Delta r, m\Delta h)$ is denoted as u_m^n. An absorbing layer with a large α is normally set as the highest layer to prevent numerical reflections from the uppermost interface. The Crank-Nicolson [7] finite difference scheme is used to form a tridiagonal system of equations [7,8], which can be solved efficiently using a tridiagonal solver [9]. A column vector $\mathbf{u} = (u_1, u_2, \cdots, u_N)^T$ containing the values of all the points along a vertical line is marched sequentially from one range to the next range with a range step of Δr.
The ground effect is handled by applying a locally reacting boundary condition at h=0, i.e. $(\partial/\partial h)\, u + ik_0 u/Z^* = 0$. The normalized specific impedance [2], $Z^* = 1 + 9.08\,(f/\sigma)^{-0.75} + i11.9\,(f/\sigma)^{-0.73}$, is described by the effective flow resistivity σ (unit: kNsm^{-4}). The boundary condition above leads to $u_0 = u_1 Z^*/\,(Z^* - ik_0\Delta h)$, where u_0 and u_1 are the field values at h=0 and $h = \Delta h$, respectively.
The acoustic continuity conditions are applied at the vertical interface of a screen at r':

$$\mathbf{u}^i H_0^{(1)}(k_0 r') + \mathbf{u}^r H_0^{(2)}(k_0 r') = \mathbf{u}^t H_0^{(1)}(k_0 r') \quad , \quad \frac{1}{\rho_A}\frac{\partial}{\partial r}\left[\mathbf{u}^i H_0^{(1)} + \mathbf{u}^r H_0^{(2)}\right]_{r'} = \frac{1}{\rho_B}\frac{\partial}{\partial r}\left[\mathbf{u}^t H_0^{(1)}\right]_{r'} \tag{2}$$

where $\mathbf{u}^{\mathrm{i}}$, $\mathbf{u}^{\mathrm{r}}$ and $\mathbf{u}^{\mathrm{t}}$ are the column vectors of the incident, reflected and transmitted fields, respectively. The media on the left side and right side of the vertical interface are denoted by A and B. Making use of the outgoing and incoming equations, the unknown $\mathbf{u}^{\mathrm{r}}$ is eliminated by combining the equations in Eq. (2), and the unknown $\mathbf{u}^{\mathrm{t}}$ can be solved from

$$[Q_A/\rho_A + Q_B/\rho_B - (i/2k_0 r')\,(1/\rho_A - 1/\rho_B)]\,\mathbf{u}^t = [2Q_A/\rho_A]\,\mathbf{u}^i \tag{3}$$

An analytical model based on the geometrical theory of diffraction is formed to verify the proposed PE model for solving backscattering from a rigid screen placed on a flat, impedance ground. Obviously, backscattering involves diffraction and reflection, i.e. p^{scat}=p^{diff}+p^{ref}. The p^{diff} is due to the four rays, see figure 1(a)-1(d), whose pressures are given by Eqs. (A1), (A4), (A7) and (A8) in Ref. [5]. While, $p^{\mathrm{ref}} = \exp(ikr_1)/r_1 + \zeta\exp(ikr_2)/r_2 + \zeta\exp(ikr_3)/r_3$, see figure 1(e)-1(g), where r_1, r_2 and r_3 are the lengths of three paths and ζ is the reflection coefficient of spherical waves given as Q in Eq. (A5) in Ref. [5]. The reflection coefficient on the screen surface is assumed to be one. Unlike those diffracting rays, any of the three reflecting rays can be invalid if the corresponding path cannot be constructed according to geometrical acoustics.

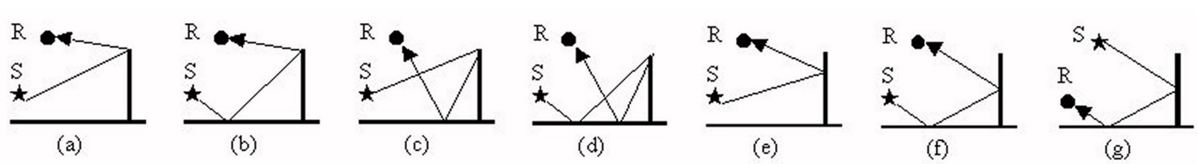

Figure 1: The four ray paths of diffraction from the screen top, (a)-(d), and the three optional ray paths of reflection from the screen surface, from the source S to the receiver R.

3 - RESULTS

Results are given as Excess Pressure Level (EPL) in dB defined as EPL=20log$_{10}|p/p_0|$, where p_0 is the free-field pressure. An example involves a point source (1 m high), a flat ground (σ=300 kNsm^{-4}) and an acoustically hard screen (10 m high and 10 m from the source) described by c=2300 m/s, ρ=2000 kg/m^3 and α=0.5 dB/ λ. Backscattering obtained by (a) the PE model and (b) the analytical model are compared in figures 2 (f=100 Hz) and 3 (f=400Hz). The results showing the reflection from the screen surface and the diffraction from the screen top are in a very good agreement.
The levels along a vertical line at r=9.9 m in figure 3 are given in figure 4. An excellent agreement is seen below 14 m, while above 14 m the difference may be due to the PE angular limitation. Figure 5 shows the PE calculations (f=400Hz) of the forward and the total fields at h=1.5 m. The total field shows a higher level due to the strong interference between backscattering and the forward propagating field. The interference is shown in figure 6. It is noted that the level of the total field rises to about 5dB at r=10 m. This indicates that the reflection coefficient near the screen surface is 0.89 which is close to one.

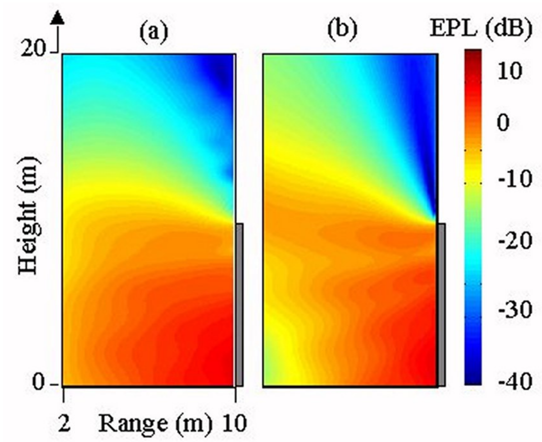

Figure 2: Backscattering by (a) PE and (b) the analytical model. Frequency is 100 Hz.

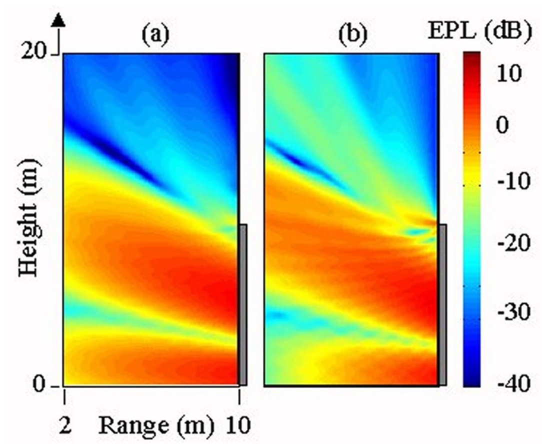

Figure 3: Backscattering by (a) PE and (b) the analytical model. Frequency is 400 Hz.

4 - CONCLUSIONS

A new PE model for solving backscattering from a screen on an impedance ground has been developed. In an example involving homogeneous atmosphere and a thin screen, results are compared with a rigorous analytical model and an excellent agreement is observed. It is found that the reflection from the screen surface dominates the backscattering and the diffraction from the screen top is relatively weak. Discrepancies seen at large heights are likely due to the PE angular limitation. However, sound field at such large heights is often out of practical importance because only the noise propagating near ground surfaces is normally concerned. Of great interest is the interference between the forward propagation and backscattering, which changes the field patterns and increases sound levels. Thus, backscattering is not negligible in the areas between a source and a screen. Further test cases show that results are not sensitive to the change of the screen materials. This indicates that materials may not be crucial factors for designing sound barriers because barriers made by different materials, such as soil, wood, glasses and metals, all may behave similar to rigid objects. The PE may be useful to study noise problems related to backscattering from barriers having various shapes.

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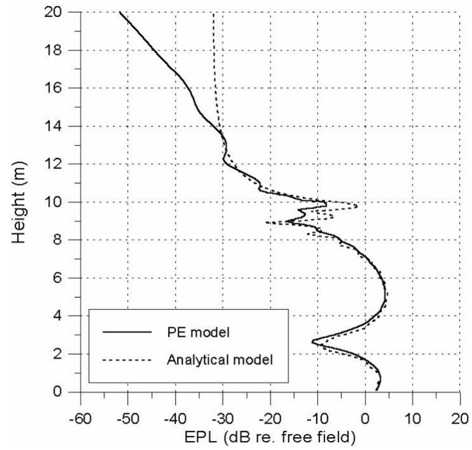

Figure 4: Backscattering levels at r=9.9 m.

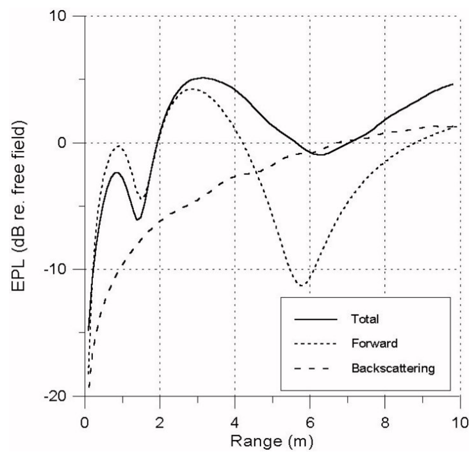

Figure 5: The field levels at h=1.5 m by PE.

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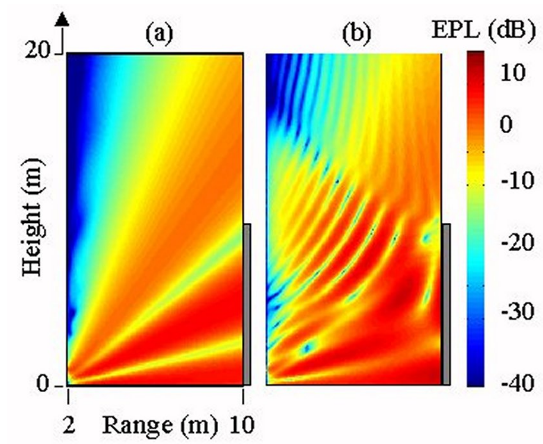

Figure 6: (a) The forward and (b) the total fields by PE. Frequency is 400 Hz.