inter.noise 2000
The 29th International Congress and Exhibition on Noise Control Engineering
27-30 August 2000, Nice, FRANCE

I-INCE Classification: 1.2

NOISE RADIATED BY THIN RECTANGULAR PLATES SUBJECTED TO INPLANE LOADS

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Keywords:
PLATE, NOISE, RADIATION, PARAMETRIC

ABSTRACT

The paper reveals recent developments of the noise radiated by thin rectangular plates parametrically excited. In the region of principal parametric resonance, starting from the temporal non-linear differential equation that describes the oscillatory movement and using the second order approximation of the asymptotic method was computed the amplitude as function of system parameters and geometric imperfections. In the anechoic chamber of Department of Mechanics were done the experiments concerning the non-linear dynamic response of such plates and the noise radiated in the region of principal parametric resonance. By recording and processing the two signals, were plotted three diagrams: spectral density of the two signals and their coherence. The three characteristic frequencies of the parametric vibration of the plate ($\Lambda/2$,Λ,$3\Lambda/2$), previously calculated, were put in evidence.

1 - NOMENCLATURE

A_1, A_2, B_1, B_2 = unknown functions in asymptotic expansion; C = viscous damping coefficient; D = flexural rigidity of plate; E = Young's modulus; M = coefficient of the non-linear term; $N_y(t)$ = external in-plane loading per unit width; N_{yo} = static in-plane loading per unit width; N_{yt} = amplitude of harmonic in-plane loading per unit width; N_{cr} = critical buckling load of the plate, defined as in [14] pp. 353; a = length of plate in x-direction; b = length of plate in y-direction; $f(x,y,t)$ = Airy's stress function; h = plate thickness; $s = \Lambda/2\Omega$ = frequency parameter; t=time; $w(x,y,t)$ = lateral midsurface displacement in z-direction; $w_{\ 0}(x,y)$ = initial geometric imperfection in z-direction; Δ = decrement of damping; $\Lambda(t)$ = instantaneous frequency of the external in-plane excitation, $\Lambda = d\ \theta/dt$; Ω_q = free vibration circular frequency of a rectangular plate loaded by a constant component of in-plane force; $\bar{\Omega}_q$ = free vibration circular frequency of a rectangular plate, with initial geometric imperfections, loaded by a constant component of in-plane force; ε = small positive parameter in asymptotic expansion, $0 < \varepsilon \ll 1$; $\theta\,(t)$ = total phase angle of harmonic excitation; μ = load parameter of the plate; ν = Poisson's ratio; ρ = mass density per unit volume of plate; τ = slowing time in asymptotic analysis; $\Psi_p\,(t)$ = phase angle of the parametric vibration; ω_q = free vibration frequency of unloaded rectangular plate; $\Delta\Delta$ = double iterated Laplace operator in R^2; $\dot{(\ )}$ = differentiation with respect to time; $(\ )_{,\ \xi}$ = partial differentiation with respect to ξ.

2 - INTRODUCTION

Extensive efforts and considerable amount of research has been concentrated on the prediction of the non-linear dynamic behavior of rectangular plates with small deviation from flatness called initial geometric imperfection. Excellent reviews on the subject can be found in articles written by Hui [2-8]. Studies of the effect of geometric imperfection on the small-amplitude vibration frequencies of simply supported rectangular plates have been done by Hui and Leissa [2], Ilanko and Dickinson [9] and Bugaru [1].
They found out that geometric imperfections of the order of the plate thickness may raised the vibration frequencies and may even cause the structures to exhibit soft-spring behavior [7]. The survey of the literature reveals that the work on the subject has been devoted to the investigation of various types of shapes, loadings, and boundary conditions [11-13].

The present work covers an existing gap in our understanding of the parametric resonance of continuous systems and presents a rational analysis of the influence of geometric imperfections upon the amplitude and the phase angle for the stationary non-linear dynamic response.

3 - CONCEPTUAL MODEL

The model under investigation is an imperfect rectangular plate simply supported along its edges and acted by periodic in-plane forces uniformly distributed along two opposite edges as shown in figure 1.

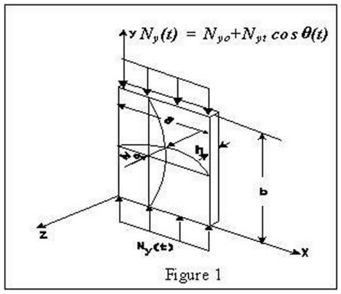


Figure 1.

It is assumed that the plate is of uniform thickness, "stress free", elastic, homogeneous and isotropic and also the plate thickness and the resulting displacements are small compared with the wavelength of lateral vibration in order to be able to use thin plate theory. Consequently, since thin plate theory is used in the analysis, the loading frequencies over which lateral vibrations occur are considerably below the natural frequencies of longitudinal vibrations and in-plane inertia forces can be neglected.

4 - BASIC EQUATIONS

The plate theory used in this analysis may be considered as the dynamic analogue of the Von Karman large-deflection theory and is derived in terms of Airy's stress function, the lateral displacement and the initial geometric imperfection. The differential equations governing the non-linear flexural vibrations of the plate are:

$$\begin{aligned} \Delta\Delta f &= E\left[\left((w+w_0)_{,xy}\right)^2 - (w_{0,xy})^2 - (w+w_0)_{,xx}(w+w_0)_{,yy} + w_{0,xx}w_{0,yy}\right] \\ \Delta\Delta w &= h/D\left[f_{,yy}(w+w_0)_{,xx} - 2f_{,xy}(w+w_0)_{,xy} + f_{,xx}(w+w_0)_{,yy} - \rho\, w_{,tt}\right] \end{aligned} \quad (1)$$

where $D = Eh^3/12\left(1-\nu^2\right)$.
The boundary stress conditions (in-plane movable edges) are expressed as:

$$\begin{array}{ll} f_{,YY} = 0 \text{ and } f_{,XY} = 0 & \text{along } x = 0, a \\ f_{,XX} = -N_Y(t) \text{ and } f_{,XY} = 0 & \text{along } y = 0, b \end{array} \quad (2)$$

The boundary supporting conditions are expressed as:

$$\begin{array}{ll} w = w_{,xx} + vw_{,yy} = 0 & \text{along } x = 0, a \\ w = w_{,yy} + vw_{,XX} = 0 & \text{along } y = 0, b \end{array} \quad (3)$$

The problem consists in determining the functions f and w, for a given function w_0, which satisfy the governing equations (1) together with the boundary conditions (2) and (3).

5 - METHOD OF SOLUTION

Applying the Kantorovich's method to the governing equations (3) as in [1], introducing linear damping and taking one term in the expansion for the lateral displacement, the system is reduced to the following differential equation of motion:

$$\begin{gathered} \ddot{w} + 2C\dot{w} + \overline{\Omega}^2\ \left[1 - 2\mu\left(\Omega/\overline{\Omega}\right)^2 cos\left[\theta(t)\right]\right] w - 2\mu cos\left[\theta(t)\right]\Omega^2\ (w_0+d) + \\ +Mw^3 + 3Mw^2\left(w_0 + \text{d}\ \right) = 0 \end{gathered} \quad (4)$$

where d is the amplitude of the static deformation of the plate and

$$\mu_q = N_{yt}/\left[2\left(N_{cr} - N_{yo}\right)\right] \quad (5)$$

This is a second-order non-linear differential equation with periodic coefficients, which may be considered as an extension of the standard Mathieu-Hill's equation.

6 - SOLUTION OF THE TEMPORAL EQUATION OF MOTION

Mathematical techniques for solving such problems are limited and approximate methods are generally used. The method of asymptotic expansion in powers of a small parameter ε, developed by Mitropolskii [10], is a most effective tool for studying non-linear vibrating systems with slowly varying parameters. Assuming that the viscous damping and the non-linearity are small and the instantaneous frequency of excitation and the load parameter vary slow with the time the equation (5) can be written, by denoting W=w and $\Theta = \theta$, in the following asymptotic form:

$$\ddot{W} + \overline{\Omega}^2 W = \varepsilon \left[2\mu cos\left[\Theta\left(T\right)\right] \; \Omega^2 \; \left(W + W_0 + d\right) - 2C\dot{W} - MW^3 - 3MW^2\left(W_0 + d\right)\right. \tag{6}$$

where $\tau = \varepsilon t$ is the "slowing" time. For the second order of approximation in ε, we seek a solution for the equation (6) in the following form:

$$W = W_p\left(\tau\right)\cos\left[\left(1/2\right)\Theta + \psi_p\right] + \varepsilon u\left(\tau, W_p, \Theta, \left(1/2\right)\Theta + \psi_p\right) \tag{7}$$

where W_p, ψ_p are functions of time defined by the system of differential equations:

$$\begin{aligned} dW_p/dt &= \varepsilon A_1\left(\tau, W_p, \psi_p\right) + \varepsilon^2 A_2\left(\tau, W_p, \psi_p\right) \\ d\psi_p/dt &= \overline{\Omega} - \left(1/2\right)\Lambda + \varepsilon B_1\left(\tau, W_p, \psi_p\right) + \varepsilon^2 B_2\left(\tau, W_p, \psi_p\right) \end{aligned} \tag{8}$$

and $d\Theta\left(t\right)/dt = \Lambda\left(t\right)$. Functions u, A_1, $A_{\ 2}$, B_1, B_2 are selected in such a way that the W, given by (7), will represent a solution of the equation (6), after replacing W_p and ψ_p by the functions defined in the system (8).

Following the general scheme of constructing asymptotic solutions and performing numerous transformations and manipulations, we can finally arrive at a system of equations describing the non-stationary response of the discretized system. By integrating this system of equations, amplitude W_p and phase angle ψ_p can be obtained as functions of time. The solution W of the equation (6) is

$$\begin{aligned} W\left(t\right) = {} & W_p\cos\left(\left(1/2\right)\Theta + \psi_p\right) - \left[\left(\mu\Omega^2\right)/\left(\Lambda\left(\Lambda + 2\overline{\Omega}\right)\right)\right] W_p\mathrm{co\ s}\left(\left(3/2\right)\Theta + \psi_p\right) + \\ & + \left[M/\left(32\overline{\Omega}^2\right)\right] W_p^3\cos\left(\left(3/2\right)\Theta + 3\psi_p\right) - \left[2\mu\Omega^2/\left(\Lambda^2 - \overline{\Omega}^2\right)\right]\left(W_o + d\right)\cos\Theta - \\ & - \left[3M/\left(2\overline{\Omega}^2\right)\right]\left(W_o + d\right)W_p^2 + \left[M/\left(2\overline{\Omega}^2\right)\right]\left(W_o + d\right)W_p^2\cos\left(\Theta + 2\psi_p\right) \end{aligned} \tag{9}$$

The solution (9) was computed for the region of principal parametric resonance. The parametric resonance occurs when the excitation frequency is approximately equal to twice the natural frequency and can be written as:

$$\Lambda \cong 2\overline{\Omega} \tag{10}$$

Analyzing relation (9), the paper reveals, for the first time, new terms not yet mentioned by the researchers in the field.

7 - EXPERIMENTAL DATA

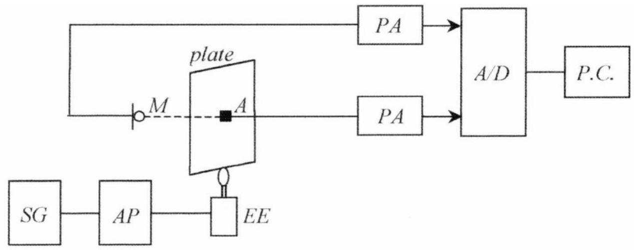
The experiment was realized in the anechoic chamber of the Mechanical Department in University Politehnica Bucharest. The plate (a=500 mm, b=1000 mm, h=0,8 mm) was clamped on three sides and excited on the free lower side.

The accelerometer was mounted in the center of the plate. On the opposite side of the plate, at 1,5 m of its center was placed a microphone. The block-schema of the assembly is presented in figure 2.

There made tests by exciting plate with the frequencies: 80 Hz, 160 Hz, 240 Hz. For every excitation frequency Λ, there were put in evidence one group ($\Lambda/2$, Λ, $3\Lambda/2$) of resonant frequencies of parametric vibration of the plate.

It was calculated the main parametric frequency of this plate, by the formula presented in [1]. This frequency is Λ=160 Hz.

In the figure 3 there is shown the plotted diagram of the coherence between the two registered signals, that is the power spectral density of the vibrations and the power spectral density of the noise radiated by the plate at the above mentioned frequency which corresponds to the principal parametric resonance at 80 Hz.



Legend: M-microphone; A-accelerometer; PA-preamplifier; A/D-analog-digital convertor; P.C.-computer; SG-signal generator; AP-power amplifier; EE-electrodynamic excitator.

Figure 2.

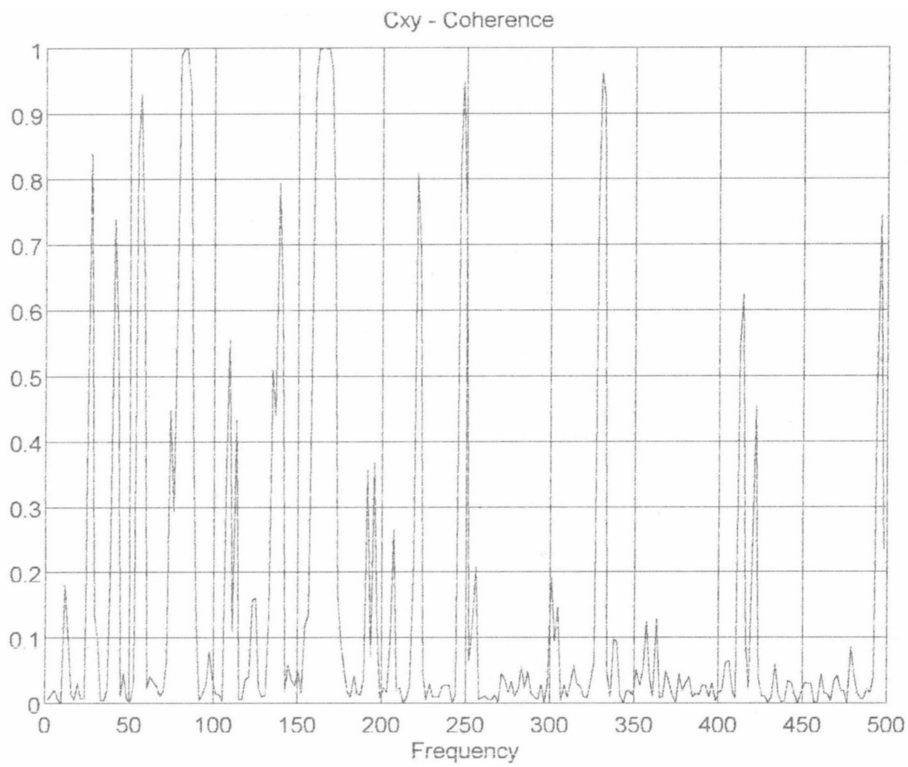


Figure 3.

In the diagrams it is put in evidence the resonant components: f_1=83 Hz; f_2=166 Hz; f_3=249 Hz. These values are in good correspondence with the calculated group of resonant frequencies of parametric vibration of the plate.

8 - CONCLUSIONS

The parametric vibrations, there are found in numerous technical situations, are produced in large surfaces of steel's sheets due to the geometrical imperfections in manufacturing or in edge mounting. The measuring and processing of the radiated noise in terms of acoustical power signal may be an efficient method of control in industry. Therefore the paper reveals the possibility of controlling the manufacturing imperfections of the plates by means of acoustical measurements in the area of principal parametric resonance.

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