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STATISTICAL MODELING TECHNIQUE FOR ESTIMATING PROBABILITY CHARACTERISTICS OF ACOUSTIC FATIGUE WEAR

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ABSTRACT
A parametric mathematical model in terms of the queuing theory is used to describe how the probabilities of being in given intervals of wear rate are varied in time domain. These intervals are determined via the percentage of either service life duration or critical crack length. It is assumed that interval-to-interval transitions meet the properties of Poisson's flows of events. Flow densities and, in some cases, characteristics of initial wear distribution are considered as free parameters of the model to be fitted to observed data. Applied as goodness-of-fit measures are either likelihood-type function developed on the base of time histories of experimental patterns or chi-square statistic that results from comparison of observed and expected service life histograms. Model parameters of interest are estimated with the aid of a numerical optimization procedure.

1 - INTRODUCTION
The problem of prediction of fatigue wear dynamics is very important in practical applications. Information on wear time history makes it possible to determine expected service life, establish points of structure examinations for detection of fatigue damages, etc. Knowledge about probabilities of given wear levels at different time points constitutes a convenient basis to substantiate corresponding decisions.
Presented here is a statistical modeling technique intended for oscillating structures suffered excitation in acoustic frequency range. In contrast to traditional ways of fatigue wear prediction, which were developed on the base of physical models, the approach in question relies on statistical data only. These data may be obtained from structure tests, field experience and other sources.
Two ways of model identification are under consideration. First of them is based on comparison of expected and observed histograms describing distributions of service lives. The second one ensures selection of model parameters yielding the point of extremum of some likelihood-type function.
Presented technique is useful for designers and specialists on exploitation who need a tool for service life prediction and motivation of decisions concerning the schedule of structure examinations.

2 - MODELS AND METHODS
The analyzed quantity is wear percentage. Available actual range of this quantity is divided into several intervals Each interval is considered as a separate state in which a wearing structure has some probability to find itself. For example, if one selects *10%* wear range for a typical system state, the state x_0 will correspond to the wear from *0%* to *10%*, the state x_1 $-$ from *10%* to *20%* and so on. The state x_{10} will correspond to *100%* wear. In due course transitions between the states are the case.
The queuing theory yields a convenient mathematical model that may be used to describe dynamics of these transitions. The model is represented by a graph (an example is presented in Figure 1) in which nodes (depicted as rectangles) correspond to the states, branches (depicted as arrows) correspond to transitions. The process of wear development may be imagined as a random walk along the graph from one state to another following the arrows. Time is supposed to be continuous. State-to-state transitions are instantaneous and take place at random time points. These transitions are effects of

environment. To describe them mathematically means to show how the environment have an influence on a structure. Initial distribution of state probabilities at the starting point of service life reflects initial wear distribution (parameters of different patterns of the same structure may differ).
It is assumed that state-to-state transitions (corresponding to each branch of the graph) meet the properties of Poisson's flows of events. It may be proved [1] that the number of events X in these flows falling into any interval of the length t adjoining to time point t is distributed according to the law of small numbers:

$$P_{t,\tau}(X=m)=\frac{(a(t,\tau))^m}{m!}e^{-a(t,\tau)}$$

where $P_{t,\tau}(X=m)$ is the probability of appearance of m events during the considered interval, $a(t,\tau)$ - mean number of events falling into an interval of the length τ adjoining to time point t. Only stationary flows (where $a(t,\tau)=\eta\tau$, η=*const*) will be taken up here. Parameter η is the density of a stationary flow. It is equal to mean number of events per unit time interval.
Two variants of modeling are of practical importance in studying wear development. They differ in sorts of initial measured data the modeling is based on.

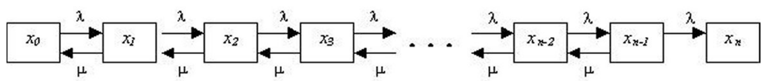

Figure 1: States x_i (i=0,1, ...,n-1): from $100i/n$ to 100 $(i+1)/n$ % of wear; state x_n: 100% of wear; $\lambda-$ wear density, $\mu-$ recovery density.

The first one employs histograms describing the distributions of service life duration for sufficiently large samples of structures. The system shown in Figure 1 is used to model wear development in this case. The state x_n corresponds to wear-out and has no exits. Flow densities are denoted as λ and μ. Parameter λ shows how the environment promotes the wear. Parameter μ represents the environment ability to recover the structure (this ability is usually not typical, however it is desirable to be assumed for completeness of categorization). Since states represent wear ranges in time percentage, flow densities must be invariable for different states.

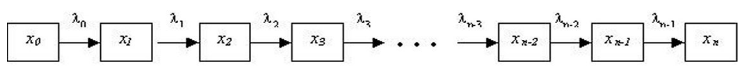

Figure 2: States x_i (i=0,1, ...,n-1): from $100i/n$ to 100 $(i+1)/n$ % of wear; state x_n: 100% of wear; λ_i $-$ wear flow density.

The second variant employs time histories of damage accumulation in per cent of some critical value of non-temporal nature (such as critical crack length, for example) for a given set of patterns. Number of these histories may be sufficiently small. At least, one exemplar is necessary. The system shown in Figure 2 is used to model corresponding wear development. Parameters λ_k have the same meaning as in the first variant.
For the first variant, the following set of ordinary differential equations [1] may be drawn to describe the time history of state probabilities:

$$\frac{dp_0(t)}{dt}=-\lambda p_0(t)+\mu p_1(t)$$
$$\frac{dp_k(t)}{dt}=-(\lambda+\mu)p_k(t)+\lambda p_{k-1}(t)+\mu p_{k+1}(t),\quad (k=1,2,\ldots,n-1)$$
$$\frac{dp_n(t)}{dt}=\lambda p_{n-1}(t)$$

where $p_k(t)$ is the probability to be within the state x_k at the time point t.
For the second variant the set of equations is simpler:

$$\frac{dp_0(t)}{dt}=-\lambda_0 p_0(t)$$
$$\frac{dp_k(t)}{dt}=-\lambda_k p_k(t)+\lambda_{k-1}p_{k-1}(t),\quad (k=1,2,\ldots,n-1)$$
$$\frac{dp_n(t)}{dt}=\lambda_{n-1}p_{n-1}(t)$$

In this case flow densities may differ for various indices k: $\lambda_i \neq \lambda_j$ $(i \neq j)$.
To integrate these equations, one has to assign initial conditions. The normalization condition

$$\sum_{k=0}^{n} p_k(t) = 1$$

is valid at any time point.
As for the first variant of modeling, it may be supposed in some situations that starting structure wear differences are described by a truncated normal distribution with non-negative mean and some standard deviation σ_{ini} that is used to characterize initial structure wear due to manufacturing defects. The distribution is truncated (values within the range from 0% to 100% are acceptable only) and standardized so as the total probability would be equal to unity.
In the above mentioned terms, estimation wear parameters is brought to the calculation of flow densities λ and μ, standard deviation σ_{ini} and mean of the normal distribution. Taken as estimations of these independent parameters are the values ensuring the best fit of observed data and expected frequencies of falling into a given state at the specified time points. Expected state probabilities are calculated by means of numerical integration of one of the presented sets of differential equations. Each of the considered modeling variants needed its own way to compare observed and expected data, viz.: chi-square measure was used for the first variant and function of maximum likelihood type $-$ for the second one.
Expected frequency to fall at the k-th state equals to $p_k N$ where $p_k -$ probability of being in this state, N $-$ number of cases. Corresponding observed frequencies F_k result from the data describing distributions of service lives in practice. Under some conditions, the following statistic is distributed asymptotically according to a chi-square distribution:

$$\sum_{k=0}^{n} (F_k - p_k N)^2 / p_k N$$

One should regard this sum as a goodness-of-fit measure for the first modeling variant in the sense that its large values correspond to bad fit and its small values correspond to good fit. The number of degrees of freedom (that is equal to n-l, where l is the total number of independent parameters) serves as a standard by which one can judge whether such measure is large or small.
Another criterion is necessary for the second modeling variant since there is no sufficient sample for comparison. In case of one pattern, calculated are the values of λ_k parameters that ensure maximum of the likelihood-type function

$$ML = \prod_{j=1}^{J} P_{t_j,z}^{z}$$

where $P_{t_j,z}$ is probability to fall into the interval corresponding to z% of wear at the time moment t_j in which the measurement took place, j=1,. . .,J $-$ numbers of measurement points. One can see that z-th power of the probability in the expression for the ML function is used as weight to make the states of greater wear more important than the states of lesser wear.
In case of several patterns, a sum of all ML functions corresponding to each pattern is regarded as the total ML function:

$$ML_{total} = \sum_{i=1}^{I} ML_i$$

The second criterion is less convenient than the first one as it does not give the opportunity to get goodness-of-fit measure for modeling. Only parameters of the best model may be here estimated.
The employed procedure of computing parameters to be estimated consists of two stages. On the preparatory stage, some numerical integration scheme for the aforementioned differential equations is coded to calculate all p_k using the Microsoft® Excel spreadsheet. The probability functions are computed with some specified time step h from initial zero time point to the given specified upper time bound. Runge-Kutta methods [2] (or their equivalents) proved to be sufficient to get acceptable accuracy of solution.
It is of vital importance that Excel supports dynamic links between cell contents. If one locates free parameters and time step h in the separate cells to which the cells containing formulas for calculation of $p_k(t)$ and initial state probabilities are referred, all the solution will be automatically modified when values of free parameters are changed.

On the final stage, a numerical optimization procedure to get required values of free parameters is run. Obtained values of free parameters are considered as wear characteristics, which have become apparent during observations.
If parameters of the optimization procedure in use are tuned for running one of the quasi-Newton algorithm variants [4], finding strict local minima, if any, was guaranteed within the specified accuracy range. As the procedure finds a point in which the gradient equals to zero, such point is unique in some neighborhood of the solution that has been found (up to the corresponding numerical method error) [3].

3 - CONCLUSION

Even a single history of pattern wear may be used as a base for estimations of probabilities if the maximum likelihood−type approach is in use. When a sample of patterns is sufficiently representative, one can choose the best fitted model comparing the observed data and results of estimations on the base of different sets of model components to figure out which components are important and which ones may be dropped. With the aid of statistical criteria, it may be also evaluated whether the model fit is acceptable or not.

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