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PREDICTION OF GEARBOX NOISE VARIABILITY FROM TOLERANCES ON PROFILE AND HELIX ANGLE

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ABSTRACT
We consider a gearbox fitted out with a single helical gear and a housing with only one elastic face which radiates noise. For this gearbox manufactured in large number, a high noise level variability can result from variability of geometry faults induced by tolerances on each gear design parameters. In this paper, this noise variability is induced by only tolerances on two gear design parameters: profile and helix angle. A full factorial experiments and a variance analysis are used to rank the influence of each geometrical parameter and to analyze their interaction on the total radiated sound power. Then, the mean value and of the standard deviation for noise level are estimated using a modified Taguchi's method.

1 - INTRODUCTION

Gears generally constitute the best technological choice to transmit a high rotation speed under a high engine torque, because of high efficiency and minimum disturbance of input-output law (transmission error). This transmission error is the main source of vibratory and acoustic nuisances [1] which designers of geared transmissions wish to reduce.

Besides, for identical geared transmissions manufactured in large number, the problem to consider is the high variability of the transmission error responsible for acoustic nuisances dispersion. The sources of this dispersion result mainly from geometry faults authorized by designers who introduce necessary tolerances on the nominal dimensions of gears. The set of tolerances values on the different geometrical gear characteristics defines a "quality class" which strongly governs the manufacturing cost, so that, reducing all the tolerances in order to reduce acoustic nuisances and their dispersion is not economically conceivable.

So, theoretical, numerical or experimental searches owing to reduce acoustic nuisances and their dispersion must be based on the selection of some influential nominal characteristics and then, on the intervention on tolerances associated to these characteristics. This paper presents numerical simulations dealing with the vibratory and acoustic dispersion of a typical transmission fitted out with a single helical gear, two shafts, four bearings and a steel housing.

We consider only the profile error and helix angle error, two quality classes for each one and one rotation speed. The profile error and the helix angle error are characterized respectively by a distance between the real profile and the theoretical profile of teeth and by a distance between the real helix angle and the theoretical helix angle.

2 - DESCRIPTION OF THE GEARED TRANSMISSION

The geared transmission retained is fitted out with a single helical gear. Each shaft is mounted between two tapered roller bearings. The steel housing (650x420x150 mm) is of the right-angled parallelepiped form made up of five 40 mm thick faces (frame) and one 6 mm thin face (450x300 mm) fixed to the frame (Figure 1). The thin face holds up two bearings supporting respectively the input and the output shafts. This geometry was chosen so that the thin elastic face is the main source of acoustic radiation of the geared transmission for low frequencies. The main characteristics of the gear provided in Table 1 are similar to those of a gear used in car gearboxes.

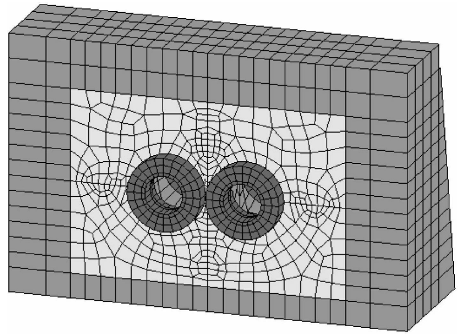

Figure 1: Housing finite element modeling.

	Pinion	Driven wheel
Number of teeth	17	71
Base radii (mm)	23.397	97.718
Normal module (mm)	2.676	2.676
Base helix angle	24°	
Transverse pressure angle	18.877°	
Facewidth (mm)	20	
Center distance (mm)	128	

Table 1: Main geometrical characteristics of the helical gear.

3 - METHOD OF COMPUTATION OF THE RADIATED SOUND POWER

The computation of the sound power radiated by the housing for several values of the profile and helix angle errors is done in three steps:

- computation of the housing vibratory excitation source (static transmission error),
- computation of the vibratory response of the housing and,
- computation of the sound power radiated by the housing.

3.1 - Computation of the static transmission error under load

The method used is described in detail in [2]. It allows to compute static transmission error of the helical gear from its geometrical characteristics, from geometry faults and from input torque. Computation of the static transmission error requires estimation of the loaded teeth deflections, from a 3D finite element modeling of each toothed wheel, estimation of the hertzian deformations and solving of the matrix equation which governs static equilibrium of the gear pair, for a set of successive positions of the pinion. The meshing stiffness is then defined by linearizing the static transmission error under load around the static equilibrium position. For a gear without geometry faults and for a low rotation speed, the time evolutions of the static transmission error and the meshing stiffness are periodic functions, fundamental frequency of which is equal to the meshing frequency f_{mesh}.

3.2 - Computation of the vibratory response of the housing

A finite element modeling of all transmission components i.e. gear, shafts, bearings and housing is required. The modeling is described in detail in [3]. From this modeling, a modal analysis of the complete transmission can be done, considering the mean value of meshing stiffness. Assuming there is no loss of contact between sets of loaded tooth pair and assuming that the dynamic mesh load remains weak enough in front of static load induced by the input torque, the mean value of the meshing stiffness is not affected by the dynamic response of the transmission. Vibratory response of the modeled geared transmission is then governed by:

$$\mathbf{M}\ddot{\mathbf{X}}\left(t\right)+\mathbf{C}\dot{\mathbf{X}}\left(t\right)+\mathbf{K}\mathbf{X}\left(t\right)+k\left(t\right)\mathbf{D}\mathbf{X}\left(t\right)=\mathbf{F}\left(t\right)+\mathbf{E}\left(t\right) \tag{1}$$

Elastic coupling between toothed wheels is introduced by $k(t)$ which represents the periodic meshing stiffness variation. Matrix $\boldsymbol{D}$ is derived from geometric characteristics of the gear. Finally, $\boldsymbol{F}(t)$ and

$\boldsymbol{E}(t)$ are equivalent force vectors associated to possible external and internal excitations. In order to solve the modal equation deduced from equation (1), we used a spectral and iterative method [4] which decreases computation times. This method is based on a spectral description of the meshing stiffness fluctuation and of the external force vector induced by the static transmission error.

3.3 - Computation of the radiated sound power

The acoustic problem can be solved using a classical formulation, that is to say the integral direct baffled method coupled with the boundary element method for the description of vibrating surface. The fluid surrounding the studied gearbox is assumed to be air so that we can neglect the fluid-structure interactions. The use of this integral formulation only needs to know the normal velocity of the vibrating face and leads to the Rayleigh integral.
The numerical solving of this integral can be explained in three steps:

- The first one is the discretization of the vibrating face with boundary elements. For this step, we keep the same mesh as the one used to compute the vibratory response (Figure 1). The nodal velocity values are those computed in the section 3.2.
- The second step consists on the numerical integration of the discretized Rayleigh integral. The retained method is a simple gaussian quadrature method.
- The last step is the solving of the linear matrix system obtained with the two first steps.

At this time, the available results allow to compute the radiated sound power for an harmonic excitation with frequency ω:

$$\Pi_{ac}(\omega)\,\frac{1}{2}\sum_{m}\int_{S_m}\int Re\left(P_m(\omega)\,\dot{U}_m^*(\omega)\right)dS_m \tag{2}$$

where m represents the element of surface S_m.

4 - CALCULATION OF THE RADIATED NOISE VARIABILITY: STATISTICAL TOOLS

Two issues are addressed here: we want to perform an analysis of the influence of each geometric faults and their interaction and secondly, we want to predict the first two moments of the noise distribution. The two retained methods perform a factorial experiments.

4.1 - Influence analysis method

The full factorial experiments method or the fractional factorial experiments method allows to take into account interactions between parameters with reasonable response computation number. This method requires the building of a linear matrix regression model of the response associated with an appropriate orthogonal array. Responses are computed for each event of the orthogonal array and the model coefficients are calculated [5]. Then, an analysis of variance using the matrix regression model coefficients is implemented in order to estimate variances inferred by every geometry fault. The more influent a fault is, the higher inferred variance is. In order to reveal interaction, we used the graphic technique proposed in [6].
The factorial experiments which has been retained is the orthogonal array called $L_9(3^4)$ by Taguchi [7]. For each of the 3 levels, the corresponding value of the geometry fault to be introduced in the orthogonal array is not subjected to any mathematical criterion. On the other hand, the method to calculate dispersion induced by the two faults imposes a particular value attributed to every level. This method is described below.

4.2 - Moments estimation method

This method is proposed by D'Errico [8]. It allows to characterize statistical distribution of a function with several random variables having known distribution, from the calculation of its moments. It is based on the use of a full factorial experiments, here the $L_9(3^{\,4})$. Sound powers (responses Y_k) are computed in N=3^n combinations of geometry faults of the full factorial experiments. The first two moments of the distribution of the radiated sound power are then given by:

$$m_1=\sum_{k=1}^{N}W_kY_k,\qquad m_2=\sum_{k=1}^{N}W_k\left(Y_k-m_1\right)^2 \tag{3}$$

W_k is the product of the weights of every combination of levels.

The value v of a level depends on the statistical distribution of factors. For products manufactured in large numbers, one assumes that the distributions of the geometry fault values are normal distributions. According to the D'Errico's method, the level values v are: level 1, $\nu = \mu_i - \sigma_i \cdot \sqrt{3}$, weight 1/6, level 2, $\nu = \mu_i$, weight 4/6 and level 3, $\nu = \mu_i + \sigma_i \cdot \sqrt{3}$, weight 1/6 ($\mu_i$ represents mean value of every factor and σ_i represents its standard deviation).
Standard deviation of every geometry fault is unknown but the tolerance range Δ in which every fault can evolve is known. Δ depends on the quality class and on the kind of geometry fault. For a normal distribution of geometry fault we take $\Delta=3\sigma$.
For the geared transmission studied, the mean value of every fault is zero. The corresponding value of the total sound power is taken as the nominal reference (attempt 1).

5 - RESULTS

All gear tolerances correspond to AFNOR French standard NF E 23-006. The first numerical simulations correspond to a quality class 8 for the profile error and the helix angle error (P8-H8 configuration). The tolerance ranges associated to these two geometry faults are equal respectively to ± 18 μm and ± 20 μm. This quality class is often used in industrial applications (gearbox, machine-tool, ...). For the whole presented results, the rotation speed of the input shaft of the geared transmission is equal to 1000 rpm, the engine torque is equal to 60 Nm and the mean value of the meshing stiffness is equal to 350 N/μm. Figure 2 displays the 9 spectra of the sound power level radiated by the housing in the increasing order of the attempts. The first ray is relative to the meshing frequency f_{mesh}. Dispersion of the sound power level can be observed for each of the 5 rays. For the last ray, it reaches more than 25 dB. The level of the first harmonic of the spectrum (f=2 f_{mesh}) is the highest for all attempts. The reference attempt without geometry faults (attempt 1) is not always the less noisy. Nevertheless, it is the lowest one for the dominant ray (f=2 f_{mesh}).

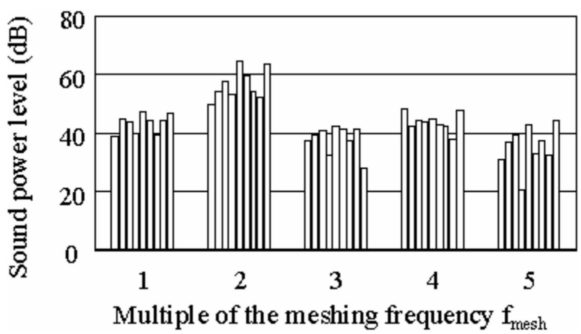

Figure 2: Spectra of sound power level.

The housing radiation efficiency is displayed in Figure 3 . In our case, the critical frequency of the housing is close to 1900 Hz.

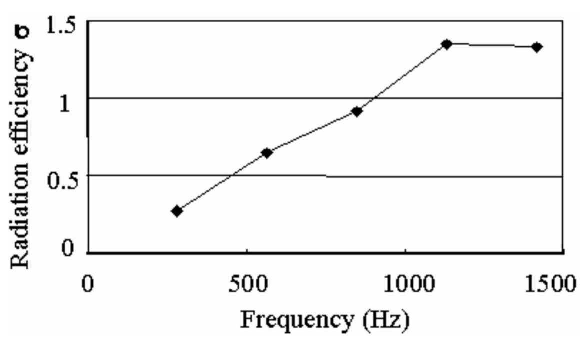

Figure 3: Housing radiation efficiency.

Figure 4 displays the total sound power level for each attempt of the factorial experiments, calculated from the spectrum of sound power. Dispersion is larger than 10 dB. Attempt 1 (reference) is the less noisy but attempt 8 is noiseless too. The noise level is low because rotation speed of the geared transmission is relatively weak (1000 rpm) and it does not allow to excite some particular modes leading to high vibratory and acoustic levels in a resonant manner [3].

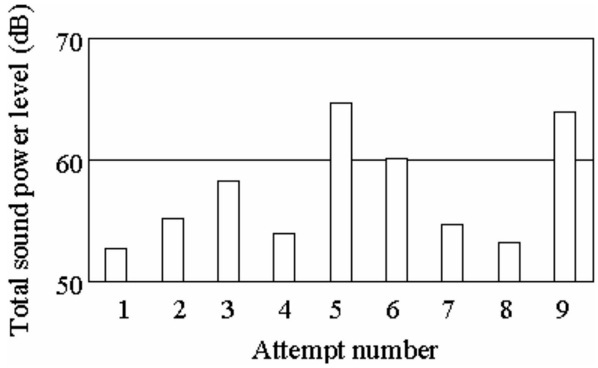

Figure 4: Total sound power level.

A variance analysis is done in order to analyze the influence of each geometry fault. Variances inferred by profile error and helix angle error on the total sound power are respectively equal to 8 10^{-13} and 11 10^{-13} Watts^2. The helix angle error dominates. So, reduction of the dispersion may certainly be obtained by a decrease of the quality class affected to helix angle, the quality class relative to profile error remaining unchanged.
Let's consider the problem of interaction between the profile error and the helix angle error. The results obtained from the graphic technique are displayed in Figure 5. Every point corresponds to the value of the total sound power radiated for one attempt of the factorial experiments. The evolutions of the total sound power are quite different when we change the level of helix angle error. This result demonstrates clearly the importance of interaction between the two geometry faults.

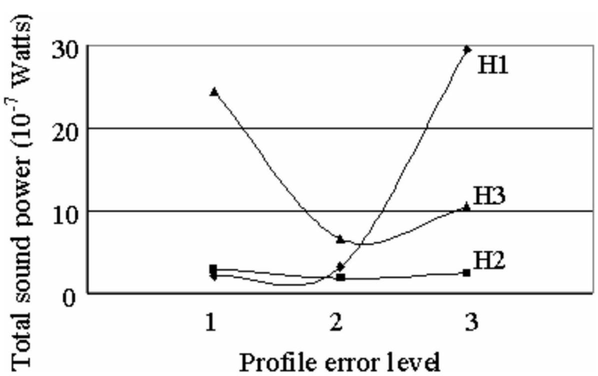

Figure 5: Representation of interaction between profile error and helix angle error.

The estimation of the acoustic dispersion due to the two geometry faults is done with the method described in section 4.2. Obtained results are presented in Table 2 for the whole tested configuration (P8-H8, P8-H7, P7-H8). For a quality class 7, tolerances ranges are equal to ±13 μm for profile error and ±12 μm for helix angle error. Average deviation is defined by the difference between mean value and reference value. The optimal configuration is clearly P8-H7, P7-H8 configuration is lightly better than P8-H8 configuration so that the results of variance analysis are confirmed.

	P8-H8	P8-H7	P7-H8
$\overline{\Pi_{actot}}$ (dB) (Watts)	56.4 4.35 10^{-7}	55 3.17 10^{-7}	55.5 3.57 10^{-7}
Average deviation (dB)	3.8	2.4	2.9
Standard deviation (Watts)	5.85 10^{-7}	2.67 10^{-7}	3.51 10^{-7}

Table 2: Statistical characteristics of total sound power distribution.

The standard deviation of the total sound power could be higher than its mean value. The noise statistical distribution is not a normal distribution so that such a result is not surprising. Most of manufactured gearboxes would radiate noise level close to the reference value. In order to analyze the meaning of this standard deviation, one can use Tchebycheff's inequality: for P8-H8, P8-H7 and P7-H8 configurations we fix a maximum probability threshold equal to 4 % so that a total sound power level higher than respectively 65.3 dB, 62.2 dB and 63.2 dB has a probability smaller than 4 %.

6 - CONCLUSION

Among all the geometry faults of the gear being able to contribute to the variability of the radiated noise, the analysis was focused on two faults: profile error P and helix angle error H.
Obtained results lead to the following main conclusions:

- For the P8-H8 configuration, the variability can reaches more than 25 dB for some rays of the noise spectrum,
- Interaction between the profile error and the helix angle error has been demonstrated.
- The comparison between P8-H8, P8-H7 and P7-H8 configurations demonstrates that the helix angle error has more influence on the variability of the radiated noise than the profile error.
- The reduction of the range of variation associated with helix angle error is the most effective to reduce appreciably the variability of radiated noise.

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