Numerical simulation of aeroacoustic problems using Lattice Boltzmann Method

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Introduction

The Lattice Boltzmann Method (LBM) is an innovative numerical method based on kinetic theory to simulate various hydrodynamic systems. In its simplest form, LBM simulates the time-dependent motion of a perfect gas at low Mach number that is governed by the compressible Navier-Stokes equations. Then, this method is a reasonable candidate for the simulation of turbulence, flow-induced noise and sound propagation. At Renault, this method is now extensively used for aeroacoustic computations.

Theory of the Lattice Boltzmann Method

LBM is a discrete formulation of the Boltzmann kinetic theory. This theory describes the dynamical behavior of a gas with a continuum distribution function $f(\mathbf{x}, \mathbf{c}, t)$ which represents the number of particles whose positions and velocities are $\mathbf{x}$ and $\mathbf{c}$ at time t. Gas flows obey the Boltzmann equation [1]

$$\frac{\partial f}{\partial t} + c_i \frac{\partial f}{\partial x_i} = -\frac{f - f^{eq}}{\tau}$$

The right-hand side of this transport equation is the Bhatnagar-Gross-Krook (BGK) collision operator which determines physics of the flow. τ is the characteristic relaxation time of the distribution function f toward the local Maxwell-Boltzmann equilibrium function f^{eq}. The fluid density ρ, velocity $\mathbf{u}$ and internal energy e are defined via moments of the distribution function. Boltzmann's equation can be used to derive the fundamental macroscopic conservation laws (Navier-Stokes equations) using a Chapman-Enskog expansion [1]. It is possible to derive a simplified form of the Boltzmann equation for discrete velocities $\mathbf{c}_\alpha$, discrete time Δt and space $\mathbf{x}_k$ such as $\mathbf{x}_{k+1} = \mathbf{x}_k + \mathbf{c}_\alpha \Delta t$. The lattice Boltzmann equation is

$$g_\alpha(\mathbf{x}+\mathbf{c}_\alpha \Delta t, t+\Delta t) - g_\alpha(\mathbf{x}, t) = -\frac{\Delta t}{\tau_g}\left(g_\alpha(\mathbf{x}, t) - g_\alpha^{eq}(\mathbf{x}, t)\right)$$

where $\tau_g = \tau + \Delta t/2$ and g_α^{eq} is the equilibrium function associated with the particle velocity $\mathbf{c}_\alpha$

$$g_\alpha^{eq} = \frac{\rho h_\alpha}{2\pi\theta}\left\{1 + \frac{\mathbf{c}_\alpha \cdot \mathbf{u}}{\theta} + \frac{(\mathbf{c}_\alpha \cdot \mathbf{u})^2}{2\theta^2} - \frac{\mathbf{u}^2}{2\theta}\right\}$$

where $\theta = rT$ is the normalized temperature and h_α are weighting factors. In 2D, the Boltzmann equation can be solved using a nine velocity model shown in figure 1. The kinetic theory gives the link between the relaxation time and the viscosity of the fluid : $\nu = \tau\theta$. In the framework of lattice Boltzmann model, one obtains

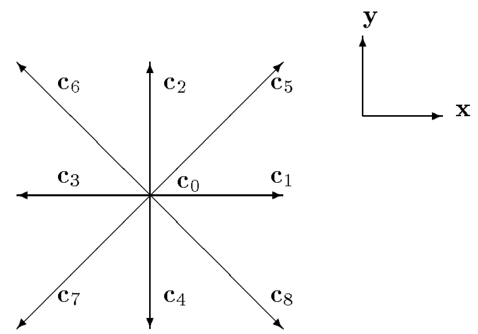

Figure 1: Discrete particle velocities of the two-dimensional D2Q9 lattice Boltzmann model.

$$\nu = \theta\left(\tau_g - \frac{1}{2}\right)\frac{\Delta x^2}{\Delta t}$$

This relation between the fluid viscosity and the relaxation time can be used to introduce a turbulence model based on the eddy viscosity approach[6]. The eddy viscosity is calculated independently of the Boltzmann scheme with a two-equation model ($k - \epsilon$, $k - \omega$...) or a sub-grid eddy viscosity model.

For more details about LBM, recent developments are highlighted in references[2, 5].

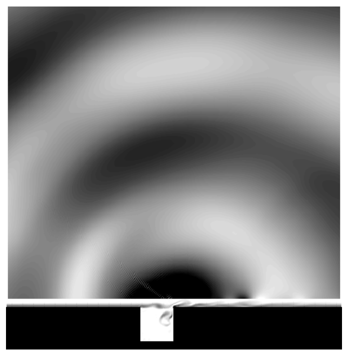

Figure 2: Example of the pressure and vorticity fields produced by an unsteady cavity flow. $L/D = 1$, $M = 0.25$.

Noise radiated by a cavity flow

A lattice Boltzmann code based on the two-dimensional nine-velocity model has been developed for basic aeroacoustic studies[3]. Cavity flow oscillations at low Mach number have been investigated. The phase loop mechanism associated with self-sustained oscillations and the generation of noise by the impingement of vortices upon the downstream corner of the cavity are well recovered. A snapshot of the vorticity over the cavity and the radiated pressure is shown in fig. 2. LBM is a low-dissipative scheme, it is therefore possible to compute simultaneously the aerodynamic and acoustic fluctuations of the flow.

Wall pressure fluctuations on a car side-glass

For industrial simulations, the commercial code PowerFLOW based on LBM is used at Renault. For high Reynolds number simulations, a modified $k-\epsilon$ turbulence model is incorporated[6]. Full scale 3D unsteady simulations are performed on real vehicle shape as shown in fig. 3. The objective of these simulations is to calculate the flow-induced structural loading to be coupled with the car panels. The main sources of sound for a passenger are the side windows. The most energetic flow pattern on the front side-glass is the A-pillar vortex. Figure 4 shows an example of the calculated and measured pressure fluctuations on the front side-glass. The agreement is good up to 6000 Hz for this point taken in the separated region of the A-pillar vortex. This cut-off frequency where numerical spectrum curve separates from the experimental curve is lower for re-attached flow regions and boundary layer flow regions.

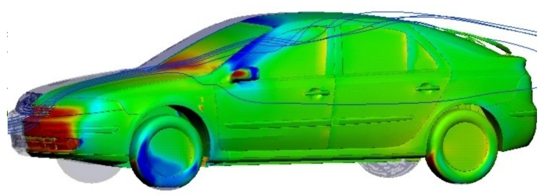

Figure 3: Simulation of the turbulent flow around a vehicle. The car is colored by the mean static pressure. The streamlines show the A-pillar vortex.

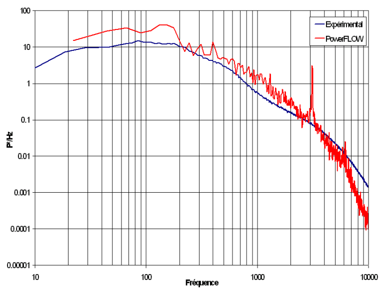

Figure 4: Power spectral density of the wall pressure fluctuations for a point situated on the front side-glass, in the separated region of the A-pillar vortex. In blue : experiments; in red : LBM simulations.

Sunroof buffeting

The compressible nature of LBM is great advantage for the simulation of flow oscillation over resonant cavities. The strong coupling between the vortex shedding and the acoustic resonance (the Helmholtz resonance in case of sunroof buffeting problem) can be taken into account directly in the computations[4]. The strong acoustic coupling induces a frequency lock-on of the oscillation around the resonance frequency of the cavity and a large increase of the fluctuation level. Numerical and experimental studies are shown in figure 5 for the sunroof of the Espace IV. In the simulations, the amplitude of the buffeting is over-estimated but the LBM approach can be used for parametric studies of wind deflectors.

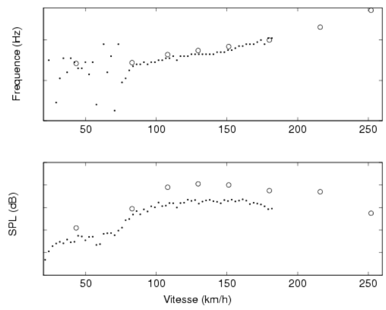

Figure 5: Frequency and amplitude of the pressure oscillation as a function of vehicle velocity. · measurements; ∘ LBM computations.

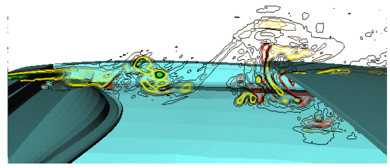

Figure 6: Snapshot of the vorticity over the sunroof opening.

Conclusion

At this time, there are few developments of LBM for aeroacoustic computations but the method already gives very encouraging results. However, typical CAA problems such as the turbulence modeling or the non-reflective boundary conditions have to be improved.

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