Characterization of subwavelength elastic cylinders with the Decomposition of the Time Reversal Operator : theory and experiment.

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Introduction

The analysis of acoustic scattering is an important tool for object identification. It has applications among non-destructive evaluation, medical imaging or underwater acoustics. The D.O.R.T. method is a new approach to scattering analysis that was developed since 1994 and is applied to detection and focusing through non-homogeneous media[1].

This paper presents the application of the D.O.R.T. method to the analysis of scattering by elastic cylinders. We derive an expression for the array response matrix **K** in two dimensions for the case of a linear array in a fluid medium scattering acoustic energy from an elastic cylinder. The singular value decomposition (SVD) is applied to **K** to determine the number of eigenstate of the time reversal operator (TRO) generated by the cylinder. We use the approach found in recent analyses of time reversal for spheres [2]. We develop the theory for a discrete array at a single frequency ω.

Then experimental results for thin steel and nylon cylinders are shown. We show that the analysis of singular values and singular vectors of the transfer matrix **K** offers the possibility to characterize cylinders of sub-wavelength diameter.

Theory

First of all, an array of N transmit-receive transducers insonifying a scattering medium is considered as a linear, time-invariant system of N inputs and N outputs. It is characterized at each frequency ω by the array response matrix is $\mathbf{K}(\omega)$. The matrix $\mathbf{K}^*(\omega)\mathbf{K}(\omega)$ is called the Time Reversal Operator, TRO. Its eigenvectors can be interpreted as invariants of the time reversal process, and they also are the singular vectors of the array response matrix.

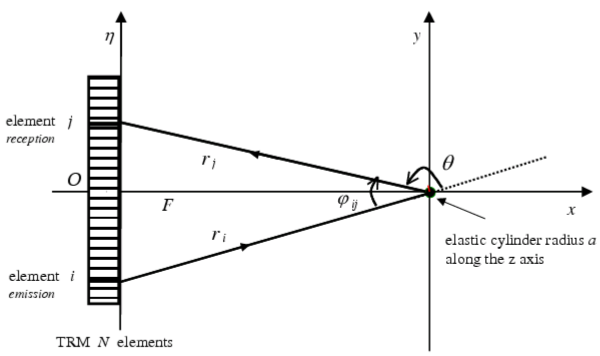

Figure 1: Geometry of the experiment.

In order to simplify the expression of **K**, we note T_{ii} the phase element $e^{ik_0 r_i}$ corresponding to the phase due to the propagation between the transducer number i and the scatterer. Thus, it is possible to write the array response matrix as :

$$\mathbf{K}(\omega) = p_0 \frac{2}{i\pi k_0} \mathbf{T}\mathbf{K}^{reduced}(\omega)\mathbf{T} \qquad (1)$$

Where **T** is a $N\times N$ unitary, diagonal matrix which contains the N phase elements $e^{ik_0 r_i}$. So it is equivalent to compute the SVD of **K** or of $\mathbf{K}^{\text{reduced}}$. For the case of a single elastic scattering cylinder, an element of the reduced array response matrix can be written as :

$$K_{ij}^{reduced}(\omega) = \frac{1}{\sqrt{r_i r_j}} \sum_{n=0}^{\infty} \varepsilon_n R_n (-1)^n \cos(n\varphi_{ij}) \qquad (2)$$

where R_n are the scattering coefficients given in Flax *et al.*[3] These are functions of the density ρ_l, the radius a, and transverse and longitudinal wave speeds (c_T and c_L) of the cylinder. Each term of the sum corresponds to a partial wave, or normal mode.

The Neumann coefficients are $\varepsilon_0 = 1$ and $\varepsilon_n = 2$ for $n \geq 1$. We have used the far field form of the field, replacing the Hankel functions with their asymptotic expressions for large arguments. The geometric parameters r_i, r_j and φ_{ij} are shown in Figure 1. We use equation (2) to compute the theoretical singular values and vectors. Comparaison with experimental values are shown in a second part.

Experimental results

Experiments have been carried out in a water tank on steel and nylon cylinders of diameters lying between 0.2 and 0.5 mm. The transducer array has 96 elements with central frequency 1.5 MHz and the array pitch is 0.5 mm. For each experiment, the distance F between the wire and the array is 50 mm.

As the cylinder diameters are less than half a wavelength, they have a low scattering power. In order to get a reasonable signal to noise ratio we used the Hadamard-Walsh basis to acquire the array response matrix. This emission basis is very convenient and increases the signal level by a factor of $\sqrt{N}$, N being the number of elements.

For comparison to experimental results, the simulation also takes into account the frequency response on transmit and receive, the directivity and the reception level of each transducer element.

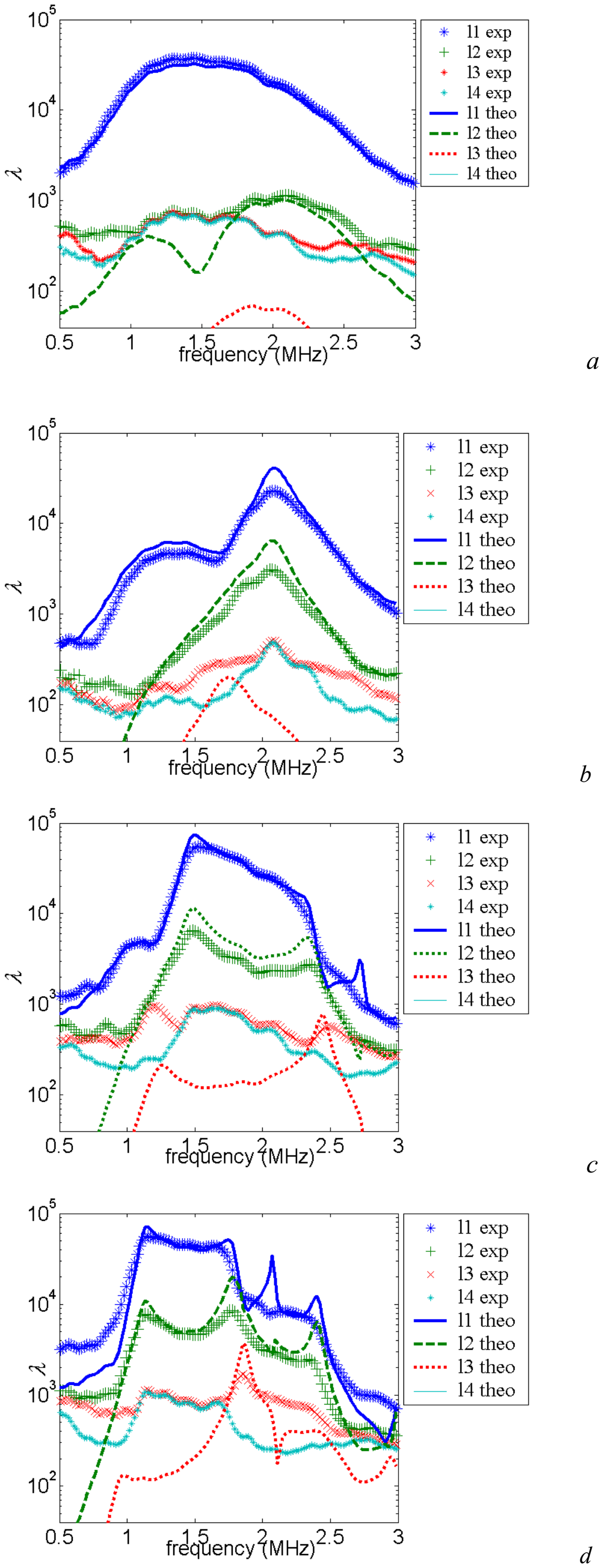

Figure 2 : singular values λ_n (log scale) simulated and experimental with frequency response. *a* **steel 0.32 mm**, *b* **nylon 0.25 mm**, *c* **nylon 0.35 mm** and *d* **nylon 0.46 mm** diameter.

Figure 2 presents the singular values for three different wires. Experimental and theoretical values are in good agreement. For the case of steel, the second singular value is visible between 1.6 and 2.5 MHz. For the case of nylon, the second singular values are visible between 1 and 2.5 MHz.

The third singular values are also above noise for narrow frequency window (around 1.1 and 2.5 MHz for the 0.35 mm nylon wire).

Figure 3 presents the three singular vectors for a single frequency. Experiment and theory are also in good agreement. The correction of experimental values (*) takes into account the reception level of each transducer. Thus the agreement is better with the corrected values (o).

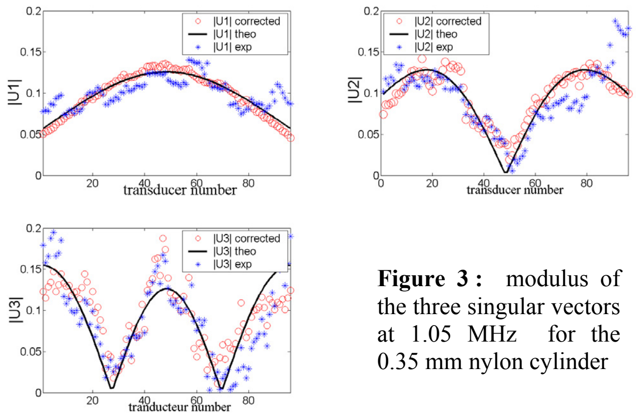

Figure 3 : modulus of the three singular vectors at 1.05 MHz for the 0.35 mm nylon cylinder

Conclusion

Initially, the DORT method was used assuming a one to one correspondence between point like scatterers and eigenmodes of the TRO. Here, we have shown that a subwavelength elastic cylinder is associated with at least three singular vectors and singular values. The singular vectors are linear combination of normal modes projected onto the array.

Experimental results show the multiple singular values for subwavelength scatterers. Three different scatterers are compared. Experimental results are in good agreement with theory when several partial waves where taken into account. Different behavior of nylon and steel are clearly shown. For the steel cylinders, the second eigenvalue was much smaller than the first and contributed little to the scattering. For the nylon cylinders, the second eigenvalue was significant but was generated by a combination of the monopole and quadrupole terms. The dipole term was negligible since the density constrast was small. These results show how the material properties of the cylinder affect the decomposition of the TRO. This opens a new approach to target characterization and the inverse problem based on analysis of the TRO.

References

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