A boundary element simulation tool for exterior acoustical problems

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Introduction

In the year 1989, the first author started with the development of boundary elements codes for the prediction of sound radiation from vibrating structures, especially arising from automotive industry like for example gearboxes and engine blocks. The first attempt was to combine the boundary element method (BEM) with an iterative solver resulting in a combination of the BEM with a multigrid solver. Since the generation of different grids seems to be a complicated process, the further development was stopped for some years. In 1994, the papers of Kleinmann and Roach (see Refs. given in [1]) initiate again the development of an iterative BE solver, which operates on the whole finite element mesh. The convergence of the iterative procedure is guarantied by premultiplying the system of equation by the adjoint operator. The iterative solver was implemented as an object oriented C++ code and applied to problems of acoustic scattering [1-3]. The present research tool is a further development of these scattered efforts.

The research tool

The C++ program "Acoustical Field (AFIELD)" is mathematically based on the surface Helmholtz integral equation (SHIE) [1]. For the numerical solution, the continuous equation (SHIE) must be discretizised and transformed into a system of linear equations, which can be solved by direct or iterative solvers.

As main solution method we use an iterative solver, which corresponds to the generalized minimum residual method on the normal equation (GMRESNE). More details can be found in [2, 3]. In addition, a direct solver – the Gaussian elimination - was implemented, in order to compare the performance and accuracy of the iterative and the direct solver. Also, as a common used high-frequency approximation the plane wave approximation [1] was implemented. Scattering as well as radiation problems can be treated.

In order to check the accuracy of the numerical implementation of the different BE solvers, we use the multipole test as described in [4]. For that reason, we compare the analytically known pressure (or related quantities like the normal velocity) of a multipole with the pressure, which is obtained by inserting the exact multipole pressure and multipole velocity into the SHIE. The resulting integral relative error is a measure for the quality of the approximation of the SHIE, i. e. for the accuracy of the numerical integration and the discretization of the structure.

In order to suppress instabilities at irregular frequencies, the Burton and Miller method (abbreviated: B&M method) was combined with the iterative and with the direct solver. The numerical implementation of the B&M method we have used, is described in [5].

Numerical calculations

The surface of the radiating structure should consist of triangular or rectangular surface elements (TRIAS or QUADS, respectively) in the usual NASTRAN of ANSYS format. The surface should be closed, and the normals should point into the exterior of the structure. Some options are provided for checking the geometry of the surface. For example, the direction of the normals can be checked and corrected if necessary. Elements are shown in dark color, if the corresponding normals are pointing into the interior of the structure.

First, a non-convex structure is investigated, which will be discretized by a small and a large number of surface elements. In [1], the scattering from such a structure was studied which consists of a sphere where the positive octant was cut out. The corresponding radius of the cateye is 1m. The structure is called "cateye", since it acts like a three-dimensional reflector. The mesh consists of 459 surface element and is shown in Fig. 1a).

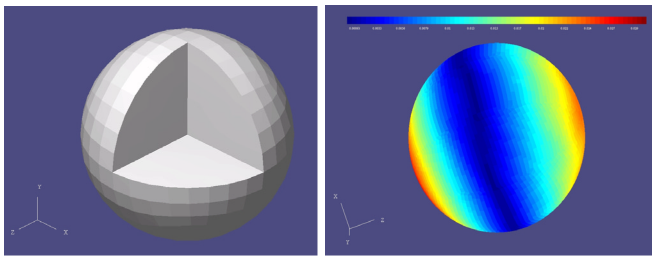

Figure 1: a) Left: FE mesh of the cateye; b) Right: pressure amplitude on a surrounding sphere.

We have used a small number of elements, in order to be able to perform many different test calculations very fast. We start with a multipole test and assume that a dipole is placed near the origin. The dipole should radiate with frequencies between 100 and 1000 Hz. The pressure error and the normal velocity error are shown for three selected frequencies in Table 1. These results show that the discretization of the surface is fine enough for the dipole velocity pattern only up to about 500 Hz. In fact, the length of one edge of most QUAD elements is about 0.2 m. Consequently, we have only about 3.4 elements per wavelength at 500 Hz.

Table 1

Frequency [Hz]	Pressure error in %	Normal velocity error in %
100	5.16	14.79
500	12.55	16.75
1000	79,32	82.98

In Fig 2a), a frequency sweep of the sound power level from 100 Hz to 1000 Hz in steps of 10 Hz is shown without any treatment of the irregular frequencies. The solid line represents the solution of the direct solver (Gaussian elimination). For the purpose of comparison, the exact solution (circles) is shown, too. It can be seen that some critical frequencies appear, leading to a remarkable deviation from the exact solution. If the same calculation is repeated by using the direct solver together with the B&M approach, a very good agreement between the numerical and the closed solution is obtained until 640 Hz. For higher frequencies small oscillations of the numerical solution around the analytical value can be observed, since the discretization is not fine enough in that frequency range.

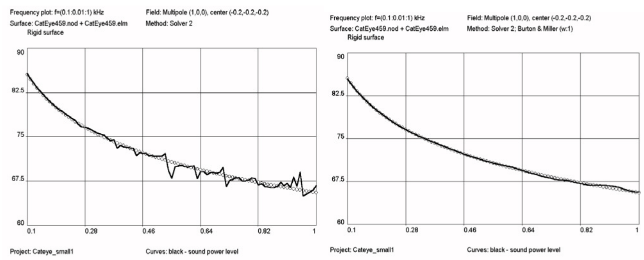

Figure 2: Frequency curves of the sound power level from 100 Hz to 1000 Hz in steps of 10 Hz; circles = analytical solution; a) Left: solid line = direct solver without treatment of critical frequencies, b) Right: solid line = direct solver combined with B&M method

In order to see the effect of the B&M method more clearly, we have investigated only the frequency range from 530 Hz to 560 Hz with a finer resolution, i. e. with a frequency step size of 0.3 Hz as shown in Fig. 3. Using again the B&M approach, no difference between the curves for the exact solution and for the numerical solutions (direct and iterative) can be discovered. In Fig. 3b, the results of the iterative GMRESNE solver are displayed. The direct solver gave the same results.

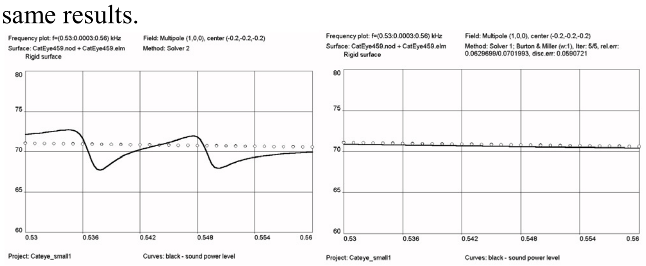

Figure 3: Frequency curves of the sound power level from 530 Hz to 560 Hz in steps of 0.3 Hz; circles = analytical solution; a) Left: solid line = direct solver, b) Right: solid line = iterative solver combined with B&M method.

An additional feature of AFIELD is that the pressure can be calculated and displayed on an arbitrary exterior auxiliary surface. In Fig. 1b), an outer sphere with radius 2m is chosen. Hence, the three-dimensional directivity pattern of the dipole can be clearly seen (iterative solver with 10 iterations and B&M method at 300 Hz).Second, a cylindrical structure with length to width ratio 3.73 and 13796 boundary elements is studied (see also [1]). For such a large FE structure, the iterative solver needs much less computer time than the direct Gaussian solver. After 5 iteration steps of the GMRESNE the residual error is below 1 %. Calculation time is about 30 minutes on a PC with a PIV 2.66 GHz processor (together with MS C++ Compiler) for a single frequency (100 Hz). The normal velocity of a monopole placed in the centroid is prescribed on the surface of the cylinder. The amplitude of the complex surface pressure is shown in Fig. 4a). The pressure field of the monopole projected onto the cylindrical surface can be clearly recognized. The last and largest example is the cateye with 57.470 boundary elements. About 100 minutes are necessary to compute one iteration for the radiating cateye-structure using the iterative solver on the same PC again. If the iterative solver is combined with the B&M method, the time for one iteration increases from 100 to 130 minutes. Hence, the computational cost is not much raised by the B&M method for large models. In Fig. 4b), another feature of AFIELD is presented: the absolute or relative multipole error can be depicted for every iteration step on the surface of the structure.

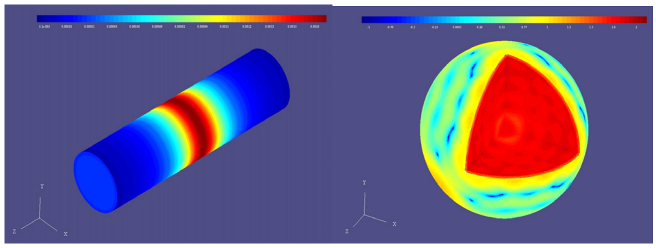

Figure 4: a) Left: Surface pressure on the cylinder at 100 Hz; 5th iteration. b) Right: relative error of the surface pressure for the 5th iteration at 1 kHz; B&M method.

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