Focusing of sonic boom at caustics induced by flight manoeuvres

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Introduction

Sonic boom is an aero-acoustical phenomenon associated to supersonic speeds. Aircraft manoeuvres, especially acceleration (Fig1) can cause focusing, and thus a large amplification of sonic boom on special surfaces called caustics and classified by the theory of catastrophes [1], depending on their geometries. The two simplest caustics are the fold (Figure 1a) and the cusp (Figure 1b) caustic.



Figure 1: Caustics induced by flight manoeuvres: a) fold caustic and b) cusp caustic.

The nonlinear theory of geometrical acoustics used for describing sonic boom [2] fails in the vicinity of a caustic, where it predicts an infinite pressure. In 1965, Guiraud [3] suggested that linear diffraction and local nonlinear effects are the main physical mechanisms in the vicinity of a fold caustic and showed that the pressure field there is described by the nonlinear Tricomi equation. This equation was numerically solved only recently with a fully validated algorithm [4,5]. With similar assumptions, it is possible to establish that the pressure field around a cusp caustic is governed by the Khokhlov-Zabolotskaya (KZ) equation [6,7] with suitable boundary conditions. A fully validated algorithm has been developed to simulate numerically the focusing of shock waves at a cusp caustic [8].

Even if the theory was established 40 years ago, no quantitative comparison between theoretical or numerical results and experimental ones has been performed since. only qualitative agreements have been obtained either by flight tests [9] or laboratory experiments [10]. Flight tests have the advantage to measure sonic boom focusing directly but a lot of parameters are to be controlled for a quantitative comparison (aircraft shape and trajectory, meteorology, atmospherical turbulence, ground impedance, etc...) Laboratory experiments allow to discard these problems but up to now none has been done with exact scaling with sonic boom. In this paper, we propose a new experiment to study the focusing of shock waves on caustics, which scales the focusing of sonic boom. First of all, the experimental set-up is described, in particular, the experimental method to synthesize caustics is presented. Then, the experimental

results for the fold and cusp caustics are compared to the numerical ones predicted by the theory.

Experimental synthesis of caustics

There are three similarity parameters controlling the focusing of shock waves, associated with diffraction, nonlinearity and absorption[11]. In our experiments, sonic boom focusing is scaled at 1:100 000 with ultrasonic shock waves in water, this choice ensuring constant similarity parameters for both cases. Experiments are made in a water tank. The frequency of the waves is 1MHz (wavelength of 1.5mm). The waves are emitted by an array of 256 transducers. Each transducer is rectangular $(11 \times 5 \text{ mm})$, so is the array $(191 \times 95 \text{ mm})$. Each transducer is powered individually by a broadband amplifier controlled by a PC. So the amplitude, phase and shape of each signal emitted by a transducer are controlled. The pressure field is measured with a PVDF bilaminar membrane hydrophone. The signals received by the hydrophone are first acquired by a digital oscilloscope and then stored in the PC. The hydrophone is set on a three-axes-motor system also controlled by the PC.



Figure 2: Experimental set-up to synthesize: a) fold caustic and b) cusp caustic.

To simulate the focusing experimentally with ultrasonic shock waves, two stages are required. The first one consists in synthesizing the fold or cusp caustic in linear regime (monochromatic waves) to know the wavefront to be emitted by the array of transducers. This is achieved by the inverse filtering method [12]. This powerful tool to synthesize wavefield in linear regime is based on the knowledge of the propagation operator between the array and a set of control points. For both caustics, we choose to use a control segment. The location and geometry of this segment depend on the caustic as illustrated Figure 2 (for the fold caustic, the control segment is 6cm, 1m away from the array of transducers - for the cusp caustic the control segment is 5.4cm length, 37.5cm away from the array). Once measured, the propagation operator is numerically inversed. This new operator is used to calculate the signal to be emitted by the array of transducers to synthesize the Airy or Pearcey functions associated to either a fold or a cusp caustic. The second stage consists in emitting the same wavefront as calculated in linear regime, but now with a higher amplitude. Nonlinear effects arise and shock waves are created during the propagation. Not all details are presented here (see [8,11]), only the comparisons with numerical simulations.

Comparisons between numerical and experimental results

Figure 3a presents the measured pressure field (time versus distance from the caustic) in colour scale (blue to red) around the fold caustic. We can clearly see the wavefront folding. Figure 3b shows comparisons between numerical simulations and the experimental data at 5 different positions along the control segment. The nonlinear simulations (blue dotted lines) are very similar to the experimental measurements. There is only one small difference: the second shock is a little bit underestimated by the numerical simulation. This may be due to the difference between the experiment, where the field is not invariant along the caustic, contrary to the theory. But these results show that it is now possible to simulate the focusing of shock waves at a fold caustic numerically, with a very good precision.





Figure 4a presents the measured pressure field at three different distances from the cusp caustic (z=-10cm, z=0 and z=10cm) in nonlinear regime. Before the cusp, the wavefront is converging, at the tip, the beam is very narrow and after the tip of the cusp, the structure of the wavefield is complex (swallow tail). Figure 4b shows the temporal profile in x=0 for the three distances. Experimental measurements (blue curves) and nonlinear simulations (red curves) are compared. Again we can note the excellent agreement between the shapes of all three curves and especially the position of the shock. The amplitude is normalised for the two rows of curves by the amplitude at z=-10cm. So the amplitude is very well recovered by the simulation code which reproduces the amplification with a very good precision : there is only a small difference of about 5%.



Figure 4: Comparisons between measurements and nonlinear numerical simulations for the cusp caustic.

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