Attenuation of acoustical waves in duct with flow

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Introduction

A study of the attenuation of an acoustical wave in a rigid duct with shear flow is presented. The flow is considered as quasi-laminar. Two methods for solving these equations are presented: First, a perturbation expansion of the visco-thermal propagation equations is given. A second method,using the Chebyshev polynomials is also presented and compared to the perturbation expansion.

List of symbol

- $u_0(y) + \tilde{u} = c_0(M + u)$,$\tilde{v} = c_0 v$ are the axial and transverse velocities
- $p_0 + \tilde{p} = p_0(1 + p)$ is the pressure
- $\rho_0 + \tilde{\rho} = \rho_0(1 + \rho)$ is the density
- $T_0 + \tilde{T} = T_0(1 + \tau)$ is the temperature
- u, v, p, τ, ρ; dimensionless acoustic variables

Basic equations

The linear equations governing the propagation of waves in a mean shear flow, when the diffusion coefficients are assumed to be constants, are:

$$\rho_0(\frac{\partial \tilde{u}}{\partial t}+u_0\frac{\partial \tilde{u}}{\partial x}+\frac{du_0}{dy}\tilde{v}) = -\frac{\partial \tilde{p}}{\partial x}+\mu_0\Delta\tilde{u}+(\lambda_0+\frac{\mu_0}{3})\frac{\partial}{\partial x}(\frac{\partial \tilde{u}}{\partial x}+\frac{\partial \tilde{v}}{\partial y})$$

$$\rho_0(\frac{\partial \tilde{v}}{\partial t}+u_0\frac{\partial \tilde{v}}{\partial x}) = -\frac{\partial \tilde{p}}{\partial y}+\mu_0\Delta\tilde{v}+(\lambda_0+\frac{\mu_0}{3})\frac{\partial}{\partial y}(\frac{\partial \tilde{u}}{\partial x}+\frac{\partial \tilde{v}}{\partial y})$$

$$\frac{\partial \tilde{\rho}}{\partial t}+u_0\frac{\partial \tilde{\rho}}{\partial x}+\rho_0(\frac{\partial \tilde{u}}{\partial x}+\frac{\partial \tilde{v}}{\partial y}) = 0 \qquad (1)$$

$$\rho_0 C_p(\frac{\partial \tilde{T}}{\partial t}+u_0\frac{\partial \tilde{T}}{\partial x}) = \frac{\partial \tilde{p}}{\partial t}+u_0\frac{\partial \tilde{p}}{\partial x}+\kappa\Delta\tilde{T}+2\mu_0\frac{du_0}{dy}(\frac{\partial \tilde{u}}{\partial y}+\frac{\partial \tilde{v}}{\partial x})$$

$$p = R\rho_0\tilde{T} + RT_0\tilde{\rho}$$

Perturbation expansion

The dimensionless equations are developed in terms of $\Omega = \omega H/c_0$ and M (the mean Mach number) and the dimensionless wavenumber is sought under the form:

$$K = K_{00}+\Omega K_{10}+MK_{01}+\Omega MK_{11}+\Omega^2K_{20}+M^2K_{02}+\cdots$$

First order K_{00}

At the first order, we obtain:

$$\frac{1}{\gamma}p'_{00} = 0$$

$$i\tau_{00} - i\frac{\gamma-1}{\gamma}p_{00} - \frac{1}{\sigma^2 s^2}\tau_{00}" = 0$$

$$iu_{00} - \frac{1}{\gamma}iK_{00}p_{00} - \frac{1}{s^2}u_{00}" = 0$$

$$v'_{10} - iK_{00}u_{00} + ip_{00} - i\tau_{00} = 0$$

where $s = \delta_{ac}/H$, δ_{ac} is the thickness of the acoustical boundary layer and σ is the Prandtl number. This set of equations can be formally written:

$$L_{K_{00}}\begin{pmatrix} p_{00} \\ \tau_{00} \\ u_{00} \\ v_{10} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

The solution of this problem gives the Zwikker and Kosten solution with $k_h^2 = -i\sigma^2 s^2$ and $k_v^2 = -is^2$:

$$p_{00} = 1$$

$$\tau_{00} = \frac{\gamma-1}{\gamma}(1-\frac{\cos(k_h y)}{\cos(k_h)}) = \frac{\gamma-1}{\gamma}f_h$$

$$u_{00} = \frac{K_{00}}{\gamma}(1-\frac{\cos(k_v y)}{\cos(k_v)}) = \frac{K_{00}}{\gamma}f_v$$

$$v_{10} = -i(\frac{K_{00}^2-1}{\gamma}y - \frac{K_{00}^2}{\gamma}\frac{\sin(k_v y)}{k_v\cos(k_v)} - \frac{\gamma-1}{\gamma}\frac{\sin(k_h y)}{k_h\cos(k_h)})$$

and the wavenumber is given by

$$K_{00}^2 = \frac{1+(\gamma-1)\frac{\tan k_h}{k_h}}{1-\frac{\tan k_v}{k_v}} \qquad (2)$$

The adjoint solution is given by:

$$\mathbf{\Phi} = \begin{pmatrix} \phi_1 = -v_{10} \\ \phi_2 = f_h \\ \phi_3 = K_{00}f_v \\ \phi_4 = 1 \end{pmatrix}$$

Second order K_{10} (term in Ω)

The set of equations giving K_{10} can be written:

$$L_{K_{00}}\begin{pmatrix} p_{10} \\ \tau_{10} \\ u_{10} \\ v_{20} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ \frac{1}{\gamma}iK_{10}p_{00} \\ iK_{10}u_{00} \end{pmatrix}$$

By use of the Fredholm theorem, this set of equations has a solution only if

$$K_{10}\int(\frac{1}{\gamma}p_{00}\phi_3+u_{00}\phi_4)\mathrm{d}y=0 \qquad (3)$$

so that

$$K_{10}=0$$

Second order K_{01} (term in M)

The set of equations giving K_{01} can be written:

$$L_{K_{00}}\begin{pmatrix}p_{01}\\ \tau_{01}\\ u_{01}\\ v_{11}\end{pmatrix}= \qquad (4)$$

$$\begin{pmatrix}0\\ imK_{00}\tau_{00}-iK_{00}\frac{\gamma-1}{\gamma}mp_{00}+2\frac{\gamma-1}{s^2}m'u'_{00}\\ \frac{1}{\gamma}iK_{01}p_{00}-v_{10}m'+imK_{00}u_{00}\\ iK_{01}u_{00}+iK_{00}mp_{00}-iK_{00}m\tau_{00}\end{pmatrix}$$

where m is the transverse dependence of the mean velocity ($u_0(y)=Mm(y)$).

By use of the Fredholm theorem, this set of equations has a solution only if

$$\int(\phi_2(imK_{00}\tau_{00}-iK_{00}\frac{\gamma-1}{\gamma}mp_{00}+2\frac{\gamma-1}{s^2}m'u'_{00})$$
$$+\phi_3(\frac{1}{\gamma}iK_{01}p_{00}-v_{10}m'+imK_{00}u_{00})$$
$$+\phi_4(iK_{01}u_{00}+iK_{00}mp_{00}-iK_{00}m\tau_{00}))\mathrm{d}y$$

or

$$2(\int f_v\mathrm{d}y)K_{01}=-\int(m((\gamma-1)(f_h-2)f_h \qquad (5)$$
$$+K_{00}^2f_v^2+\gamma)+\frac{1}{i}m'(-\gamma f_v v_{10}+2\frac{\gamma-1}{s^2}f_hf'_v))\mathrm{d}y$$

The value of K_{00} and K_{01} as a function of s is given in Figures 1 and 2 for two values of the mean flow profile $m=(2n_M+1)(1-y^{2n_M})/(2n_M)$.

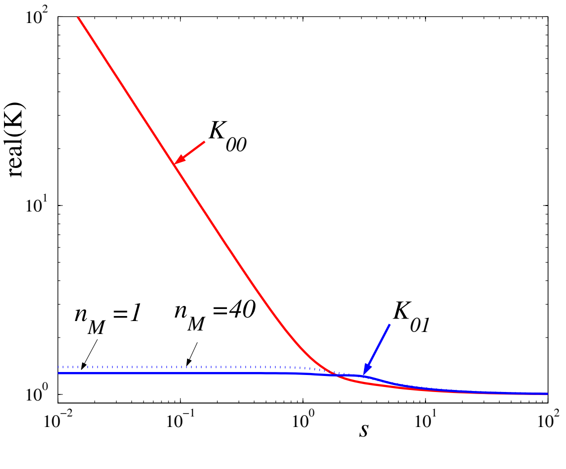

Figure 1: Real part of K_{00} and K_{01} as a function of s.

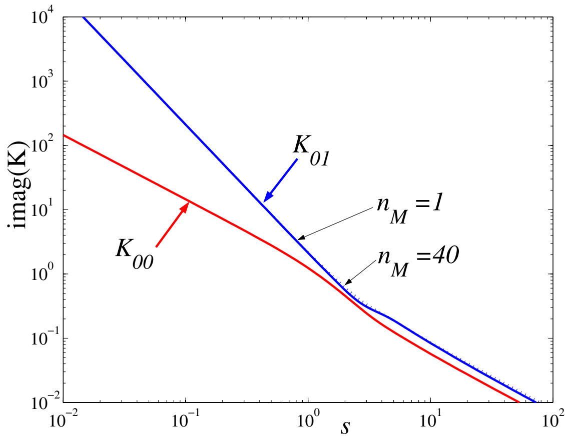

Figure 2: Imaginary part of K_{00} and K_{01} as a function of s.

Use of Chebyshev polynomials

The equations (1) can be rearranged to eliminate the variables p and ρ. The remaining variables u, v, τ have to vanish at the wall. The vector $\mathbf{U}$ represents the value of u at the N Chebyshev points taken along the transverse direction. The equations (1) can be put under the form

$$K\begin{pmatrix}\mathbf{U}\\ \mathbf{V}\\ \mathbf{T}\\ K\mathbf{U}\\ K\mathbf{V}\\ K\mathbf{T}\\ K^2\mathbf{U}\\ K^2\mathbf{V}\\ K^3\mathbf{V}\end{pmatrix}=\mathbf{M}\begin{pmatrix}\mathbf{U}\\ \mathbf{V}\\ \mathbf{T}\\ K\mathbf{U}\\ K\mathbf{V}\\ K\mathbf{T}\\ K^2\mathbf{U}\\ K^2\mathbf{V}\\ K^3\mathbf{V}\end{pmatrix}$$

The modes can be found by computing the eigenvalues of the $9N\times 9N$ matrix $\mathbf{M}$. The values find by this method for the quasi-plane wave are compared to the value find by the perturbation expansion in Figure 3.

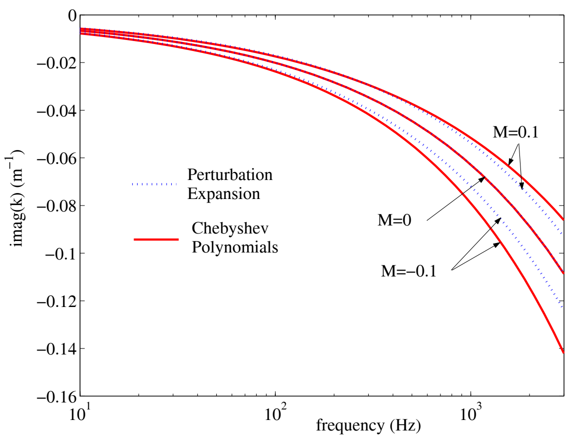

Figure 3: Imaginary part of the wavenumber k as a function of the frequency f for a channel of height 15 mm at 20 °C ($15 < s < 265$ and $0.0014 < \Omega < 0.41$).