The dynamic acoustic radiation force on cylinders: theory and simulations

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Abstract

The theory of the dynamic acoustic radiation force Y_d experienced by an infinite elastic cylinder of radius *a* is presented. Analytical solutions of the equations for the dynamic radiation force are derived. The equations relate the radiation force to the acoustic field and the cylinder's mechanical parameters. The results of numerical calculations are presented, showing how the frequency dependence of the dynamic radiation force function Y_d for a cylinder is affected by variations in its material parameters. The results are of interest when the dynamic radiation force is used to determine the dynamic properties of a cylinder target immersed in water.

Introduction

Acoustic radiation force is a phenomenon associated with the propagation of sound waves through a medium. It is caused by a transfer of momentum from the wave to the medium, arising either from absorption or reflection of the wave. This momentum transfer results in the application of a force in the direction of wave propagation. The magnitude of this force depends upon both the medium mechanical properties and the acoustic beam parameters. The duration of the force application is determined by the temporal profile of the acoustic wave. Generally speaking, radiation force is a mean steady force. Hasegawa *et al.*[1] have studied the acoustic radiation force experienced by a lossless cylinder immersed in water.

Applications involving time-dependent (harmonic or pulsed) [2] radiation force have been developed to measure the ultrasonic absorption coefficients in liquids by using amplitude-modulated sound waves, and to image the viscoelastic properties of tissues by using focused ultrasound pulses [3]. In these conditions, the radiation force is dynamic in the sense that it is generated by an amplitude-modulated ultrasound beam and has slow time variation.

In this work, we solve the fundamental problem of the dynamic radiation force experienced by a cylinder placed in an ideal fluid. The cylinder is excited by two plane waves of slightly different frequencies, thus, producing at their intersection an incident field modulated in amplitude (Figure 1). The main steps are developed and the principal analytical solutions are given.

Method

Usually, the calculation of acoustic radiation force on an object can be divided into two steps: solve the linear acoustic scattering problem, and determine the radiation-stress tensor in the propagating fluid medium. Here we extend the work previously done [1] to calculate the dynamic radiation force produced by interfering two sound

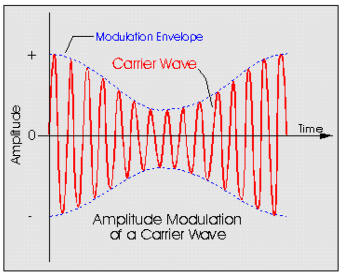

Figure 1: Representation of the incident amplitude-modulated sound field.

(or ultrasound) waves at slightly different frequencies (indicated by subscript 1 and 2 in the following equations). The velocity potential of the incoming waves can be written as:

$$\phi_i = \phi_{1,i} + \phi_{2,i} \qquad (1)$$

The scattered waves may be expressed as:

$$\phi_s = \phi_{1,s} + \phi_{2,s} \qquad (2)$$

The total velocity potential is given by:

$$\phi = \phi_i + \phi_s \qquad (3)$$

The radiation force is calculated by averaging the net force on the cylinder over time. In our typical applications, $|\omega_1 - \omega_2| << \omega_1 + \omega_2$ so that the radiation force has a slow time variation at the low-frequency $\Delta\omega$. To discriminate this slow variation from the total radiation force, we use the short-term time average of an arbitrary function $\chi(t)$ over the interval of *T* at time *t*, defined as: $\langle\chi(t)\rangle = \frac{1}{T}\int_{t-T/2}^{t+T/2}\chi(t)\,dt$ [3], where $\frac{2\pi}{\omega_1+\omega_2} << T << \frac{2\pi}{|\omega_1-\omega_2|}$.

The total radiation force <*F*> on the cylinder can be determined by integrating the total pressure field around the cylinder, and by using the formula given by Hasegawa *et al.* [1] we have:

$$\langle F\rangle = \langle F_r\rangle + \langle F_\theta\rangle + \langle F_{r,\theta}\rangle + \langle F_t\rangle \qquad (4)$$

where:

$$\langle F_r\rangle = \left\langle -\frac{1}{2}a\rho\int_0^{2\pi}\left(\frac{\partial\Psi}{\partial r}\right)^2_{r=a}\cos\theta\,d\theta\right\rangle$$

$$\langle F_\theta \rangle = \left\langle \frac{1}{2a}\rho \int_0^{2\pi} \left(\frac{\partial \Psi}{\partial \theta}\right)^2_{r=a} \cos\theta\, d\theta \right\rangle$$

$$\langle F_{r,\theta} \rangle = \left\langle \rho \int_0^{2\pi} \left(\frac{\partial \Psi}{\partial r}\right)_{r=a} \left(\frac{\partial \Psi}{\partial \theta}\right)_{r=a} \sin\theta\, d\theta \right\rangle$$

$$\langle F_t \rangle = \left\langle -\frac{1}{2c^2} a\rho \int_0^{2\pi} \left(\frac{\partial \Psi}{\partial t}\right)^2_{r=a} \cos\theta\, d\theta \right\rangle$$

and $\Psi = \mathrm{Re}[\phi]$. (ρ is the density of the fluid medium in which the sound speed is c).
After some tough and tedious arithmetic manipulations, Eq.(4) can be expressed by:

$$\langle F \rangle = 2a\rho \begin{pmatrix} \frac{1}{2}k_1^2 |A|^2 Y_1 + \frac{1}{2}\rho k_2^2 |A|^2 Y_2 \\ +\left(k_1 k_2 |A|^2\right) Y_3 \sin(\Delta\omega t) \\ +\left(k_1 k_2 |A|^2\right) Y_4 \cos(\Delta\omega t) \end{pmatrix} \tag{5}$$

where Y_i is a dimensionless factor called acoustic radiation force function, and k_1 and k_2 are the two incident wave numbers, respectively. The explicit form of the radiation force functions is given in reference [4].

Y_1 and Y_2 are the steady components of the radiation force that are related to the incident waves. The dynamic component of the radiation force is associated with Y_3 and Y_4 such as:

$$Y_d = \sqrt{Y_3^2 + Y_4^2} \tag{6}$$

Results

Eq. (6) was used to calculate the dynamic radiation force function for an elastic brass and bismuth cylinders immersed in water. The radiation force function curve is plotted versus the size parameters x_1 (i.e. k_1a) and x_2 (i.e. k_2a), and cover the range defined by the condition $|\omega_1 - \omega_2| << \omega_1 + \omega_2$ so that $\delta x = |x_1 - x_2| << x_1 + x_2$. Therefore, the range is chosen by $0.5 \le x_1 \le 10$ and $0 \le \delta x \le 0.1$ in increments of 0.01.
Figure 2 and Figure 3 show a 3-D plot of the radiation force function for the brass and bismuth cylinders, respectively.

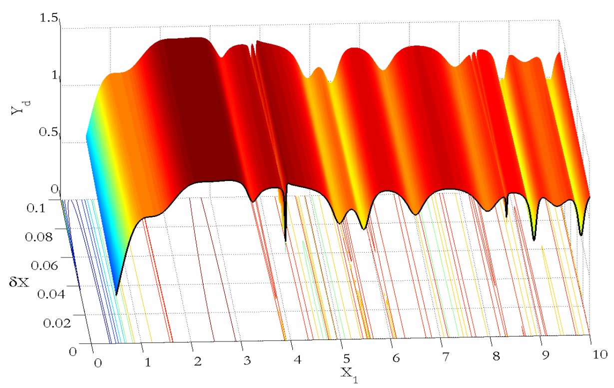

Figure 2: Dynamic radiation force function ($Y_d(x_1,x_2)$) curve for brass cylinder in water. The splitting phenomenon is also observed at the minimum ($x_1=4$, $\delta x=0.1$) of $Y_d(x_1,x_2)$.

The special case where the frequencies of the incident plane waves are equal (i.e. $\delta x = 0$) is also shown in the figure (curve in bold), and $Y_d\ (x_1,x_2)$ converges to the steady radiation force function Y_p given by Hasegawa *et al.* [1].

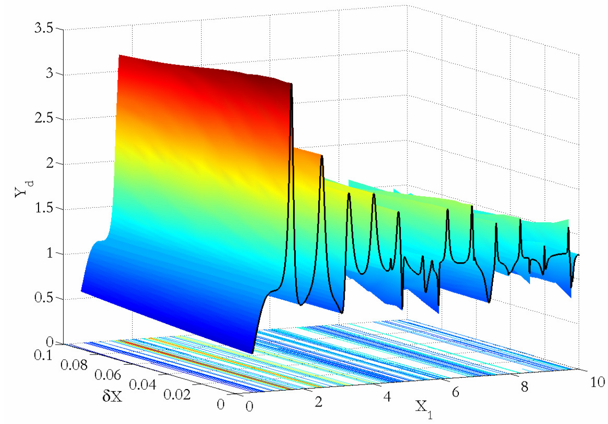

Figure 3: Dynamic radiation force function ($Y_d(x_1,x_2)$) curve for bismuth cylinder in water.

It is to be noticed that in the section of the traditional radiation force function where the $Y_p(x)$ curve is relatively flat, we see very little change in $Y_d(x_1,x_2)$, so that $Y_d(x_1,x_2)$ could be approximated to $Y_p(x)$. However, when δx increases, the value of $Y_d\ (x_1,x_2)$ starts to deviate from $Y_p\ (x)$, which is clearly shown in Figure 1 at the minimum $x_1=4$, $\delta x=0.1$ of the curve. Hence, $Y_p(x)$ is not valid and it is essential to use the expression of $Y_d(x_1,x_2)$ as given by Eq.(6).

Conclusion

The major achievement of this work is to calculate theoretically the dynamic components of radiation force experienced by a cylinder placed in an amplitude-modulated sound (or ultrasound) field (i.e. Eq.(6)). It is shown that radiation force is no longer static and has an oscillatory component.
These calculations could be useful to determine the dynamic properties of any cylinder target immersed in water.

References

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