On the Acoustics of Lateral Open Cavities

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Introduction

A major part of the sound radiating from vehicles, such as cars and trains is generated by the engine or the auxiliaries, transmitted into the underneath cavity, and finally propagated to the far field. The underneath cavity can be seen as a shallow room with laterally open sides. For computations of the transmission and propagation, some different methods can be applied, for instance FEM, BEM and SEA. The common drawback of these methods, however, is their complexity and inefficiency for engineering practice. At the design stage, a method is required with high efficiency and simplicity, which, above all, reveals the underlying physics.

Although a rectangular model of the underneath car cavity can be easily constructed, an even simpler geometry is chosen for the mathematical model. This is to circumvent the complex eigen-value problem, implied by the rectangular geometry. Herein, a simple combination of a tube model and a cylinder model is proposed to describe sound field of the underneath car cavity.

Cylindrical Cavity Model

Consider the configuration shown in Figure 1, consisting of a semi-infinite circular cylinder of radius *R*, elevated perpendicularly above an infinite plane. The end of the cylinder, at a distance *L* above the plane, is capped with a flat, acoustically hard plate. Similarly, the infinite plane as well as the cylinder wall is assumed to be acoustically hard.

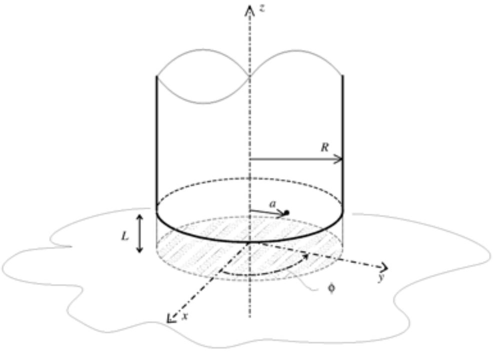

Figure 1: Cylindrical cavity model.

The fields inside and outside the cavity are governed by the wave equation

$$(\nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2})p = 0 \tag{1}$$

where the Laplace operator in cylindrical coordinates reads

$$\nabla^2 = \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{1}{r^2}\frac{\partial^2}{\partial \phi^2} + \frac{\partial^2}{\partial z^2}. \tag{2}$$

For harmonic processes, the general solution is given by [1]

$$\begin{aligned} p(r,\phi,z,\omega) = \sum_{n=-\infty}^{\infty} e^{in\phi}\frac{1}{2\pi}\int_{-\infty}^{\infty}\Big[A_n\left(k_z,\omega\right)H_n^{(1)}(k_r r) \\ +B_n\left(k_z,\omega\right)H_n^{(2)}(k_r r)\Big]e^{ik_z z}dk_z \end{aligned} \tag{3}$$

The coefficients A_n and B_n are determined from the boundary conditions. In a case where the top and bottom boundaries of the cavity can be taken acoustically hard, however, the radial velocity v_r can be written in the form

$$\begin{aligned} v_r(r,\phi,z,\omega) = \sum_{n=-\infty}^{\infty} e^{in\phi}\sum_{m=0}^{\infty}\Big[A_n H_n^{'(1)}(k_r r) \\ +B_n(k_m,\omega)H_n^{'(2)}(k_r r)\Big]\cos(k_m z) \end{aligned} \tag{4}$$

where $k_m = \frac{m\pi}{L}$, $m = 0,1,2\ldots$ is the modal wavenumber component in the *z*-direction, such that the radial component becomes $k_r = \sqrt{k^2 - k_m^2}$. Outside the cavity, the field consists of outwards propagating waves only. The velocity field at the cylindrical interface between the inner and outer domains is described by the series

$$v_r(R) = \begin{cases} \sum_\nu \sum_\mu C_\nu \sin(k_\mu |z|)e^{i\nu\phi} & ; \ |z| \le L \\ 0 & ; \ |z| > L \end{cases} \tag{5}$$

where $k_\mu = \mu\pi/L$, $\mu = 0,1,2,...$ At the cylindrical boundary, continuity in velocity and pressure demands

$$v_i(R,\phi,z) = v_o(R,\phi,z) \ ; \ |z| \le L, \tag{6}$$

$$p_i\left(R,\phi,z\right) = p_o\left(R,\phi,z\right); |z| \le L. \tag{7}$$

With these boundary conditions and a third set by the interior source, the acoustic transfer impedance

$$Z(r,\phi,z|a,0,L) = \frac{p(r,\phi,z)}{q_a} \tag{8}$$

can be calculated [2].

Experimental Validation

Two experiments are undertaken in an anechoic chamber. In the first, the very simple rectangular plate, shown in Figure 2, is used to simulate the car floor, which together with a set of plates realizing the road surface, establish the underneath car cavity.

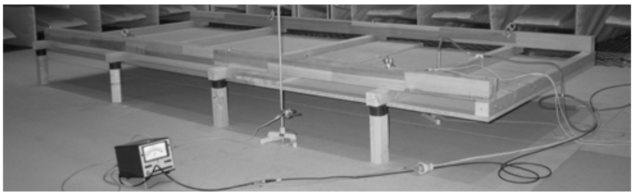

Figure 2: Rectangular experimental set-up.

The second is performed with a real car, shown in Figure 3. In both experiments, the transfer impedance between a source position at the interface of the engine compartment to the cavity and receiver points around the cavity are measured.

Figure 3: Receiver positions at the left long side of an underneath car cavity.

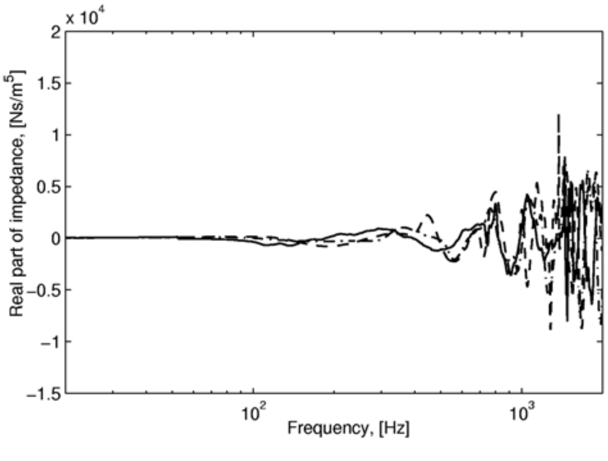

Figure 4: Comparison of calculated and measured real part of transfer impedance. Measurement results (——), calculated based on equal volume (— —) and equal distance (— · —).

In all cases the acoustic transfer impedance is calculated, using a numerical routine coded in MatLab®. Figure 4 shows the comparison of measured and calculated results for a receiver position at the boundary of the cavity. In the calculation, two different cylindrical models are used:

- The equal volume model, which has the same cavity volume as the rectangular experimental rig and
- the equal source-receiver distance model, in which the distance between the source and calculation point is set to that from the source to microphone in the experiment.

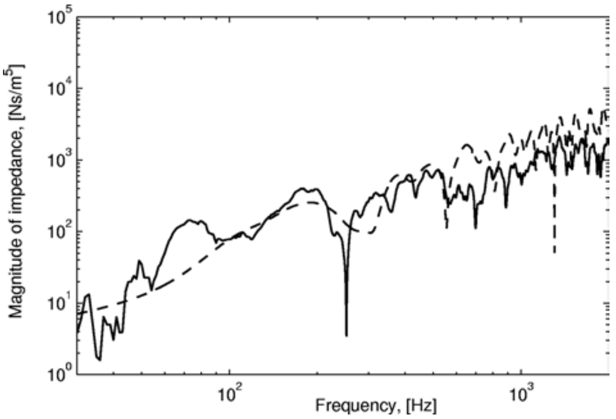

Figure 5: Comparison of calculated and measured magnitudes of transfer impedance for the rear side position in Figure 3. Measured (——) and calculated results(— —).

By comparing the measured results with those from the calculation, it is seen that the calculated results based on the equal volume model essentially captures the physics for all of the frequency range considered. In the upper range, however, the equal source-receiver distance criterion slightly improves the predictions. This can be explained by an improved matching of the impedances for the inner and outer domains such that the direct paths predominates.

From the results presented in Figure 5 it is seen that the cylindrical modelling is also able to capture the dominant features of the more irregular cavity underneath the car.

Concluding Remarks

The comparison of theoretical an experimental results demonstrate that a simple cylindrical model can be efficiently employed in simulating the sound field in the cavity underneath the car. This modelling captures the salient physics. It is shown that the actual geometry of a laterally open cavity is of subordinate importance for the sound transmission via the open sides. The modelling can be employed for further work on sound propagation from vehicles with a view towards simplified methods for low noise design.

Acknowledgements

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References

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