Transient nonlinear processes in annular thermoacoustic engines

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Introduction

Thermoacoustic prime-movers employ for the transformation of the thermal energy into mechanical energy an interaction between an inhomogeneously heated stack of solid plates and resonant gas oscillations. In an annular thermoacoustic prime-mover, the stack is placed in a closed loop resonator, allowing the excitation of traveling acoustic waves (see Fig. 1).

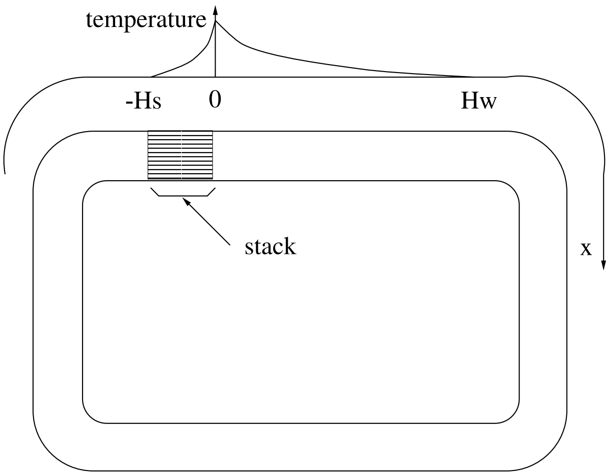

Figure 1: Schematic representation of the annular thermoacoustic prime-mover

In such a device, the saturation of the thermoacoustic instability is linked not only to classical non linear phenomena such as cascade process of higher harmonic generation and minor losses, but also to nonlinear processes influencing the temperature distribution in the inhomogeneously heated parts of the system such as acoustically enhanced thermal conductivity (equivalent to heat transport induced by gas oscillations) and the excitation of a unidirectional acoustic streaming. In order to understand the independant influence of each one of those nonlinear phenomena, the investigation of the transient regime may provide us useful informations, because the transient development of different nonlinear phenomena might proceed differently (with different time scales). Experimental observations of the transient regime has already been reported by ourselves. The obtained results show various regimes, demonstrating notably the possibility for the engine to turn on and off spontaneously and periodically, or the possibility for a double-threshold process during the amplification regime [1]. A simplified theoretical model is here presented, where the one-dimensional heat transfer equation is coupled to the acoustic problem by taking into account the forced convection due to acoustic streaming and the acoustically induced thermal conductivity. The obtained results notably provide a qualitative explanation for the double-threshold phenomenon.

The model

The device is divided into three parts, i.e. the stack ($-H_S \leq x \leq 0$), the inhomogeneously heated part of the resonator ($0 \leq x \leq H_W$), and the cold part at constant temperature T_C. The interval $-H_S \leq x \leq H_W$ is called the thermoacoustic core. It is made of three compounds (i.e. the stack, the tube walls and the fluid) with associated thermophysical properties (i.e. those of ceramics, stainless steel, and air at atmospheric pressure, respectively). It is assumed that the thermal coupling between those different elements is so strong that, at any position x along the thermoacoustic core, the temperatures $T_{air}(x)$, $T_{ceramics}(x)$ and $T_{stainless}(x)$ are equal. With this assumption, a simplified monodimensional heat transfer equation can be written in each part of the thermoacoustic core as follows:

$$\begin{aligned}\forall x \in [-H_S, 0], \partial_t T_-(x,t) + V_-(t)\partial_x T_-(x,t) = \\ (D_- + \delta D_-(t))\, \partial^2_{xx} T_-(x,t) + \frac{T_-(x,t)}{\tau_-},\end{aligned} \quad (1)$$

$$\begin{aligned}\forall x \in [0, H_W], \partial_t T_+(x,t) + V_+(t)\partial_x T_+(x,t) = \\ D_+ \partial^2_{xx} T_+(x,t) + \frac{T_+(x,t)}{\tau_+}.\end{aligned} \quad (2)$$

In Eqs. (1) and (2), the coefficients D_- and D_+ (with subscripts $-$ and $+$ standing for the stack region and the $[0, H_W]$ region respectively) are the global thermal diffusivities of the simplified monodimensional model. They are estimated from thermophysical properties and cross-sectional dimensions of each component (i.e. stack, tube walls, and fluid). The coefficients $\tau_\pm$ are phenomenological parameters which account for exchanges of the device with surroundings. In fact, the transient heat transfer equation is here coupled to the acoustic problem via the acoustically induced thermal diffusivity $\delta D_-(t)$ and the forced convection $V_\pm(t)\partial_x T_-(x,t)$ due to the excitation of a unidirectional acoustic streaming of velocity $V_\pm(t)$.

Then, the transient evolution of acoustic pressure for a given temperature distribution must be evaluated. This requires to solve the well-known classical linear differential equation of thermoacoustics in the thermoacoustic core, taking into account the specific boundary conditions linked to the annular geometry of the present device. Finally, it has been demonstrated [2] that the acoustic problem reduces to:

$$\partial_t p_{rms}(t) = \alpha\{T_\pm(x,t)\} p_{rms}(t), \quad (3)$$

where $p_{rms}(t)$ is the root-mean-square amplitude of the acoustic pressure. In Eq. (3), the thermoacoustic amplification coefficient α depends on the temperature distribution in the whole thermoacoustic core. The sign of α determines whether the acoustic wave is attenuated ($\alpha < 0$) or amplified ($\alpha > 0$) while $\alpha = 0$ corresponds to the threshold condition or to the stationnary regime. To model the influence of the acoustic field on the temperature field in Eqs. (1)-(2), simple expressions for $V_\pm$ and δD_- are introduced:

$$\delta D_-(t) = \Gamma_D p_{rms}^2(t). \tag{4}$$

$$\partial_t V_\pm(t) + \frac{V_\pm(t)}{\tau_V} = \frac{\Gamma_V}{\tau_V} p_{rms}^2(t). \tag{5}$$

So, the velocity of acoustic streaming and the acoustically induced diffusivity are simply assumed to depend on the square of acoustic pressure, with account of a time delay τ_V of streaming establishment. Finally, the transient regime is computed by solving Eqs. (1),(2) and (3) step by step (with appropriate boundary conditions) from the onset of the thermoacoustic instability to the saturation of the wave amplitude (the heat transfer equation being solved using a Cranck-Nicholson finite-difference method).

Results and discussion

For each numerical simulation, the input thermal power Q at position $x = 0$ (hot heat exchanger) is initially set just below its critical value corresponding to the onset of the thermoacoustic instability. A small ΔQ increment (at time $t = 0$) on Q is then sufficient for the acoustic wave to be amplified in the waveguide. In Fig. 2, the calculated transient regime of wave amplification/saturation is plotted for $\Delta Q/Q = 3\%$. It is remarkable in Fig. 2 that the double-threshold process is qualitatively reproduced by the model. During this transient operation, the initial exponential growth of oscillations (first threshold) is followed by a quasi-stabilization ($t >= 125s$), which before the final stabilization is followed by another exponential growth (second threshold, $t >= 150s$).

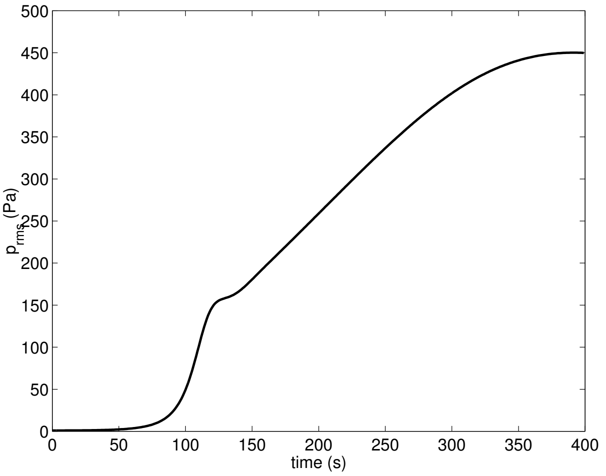

Figure 2: Calculated rms amplitude of acoustic pressure versus time in the transient regime. The model parameters are set to the following values: $\Delta Q/Q = 3\%$, $\Gamma_V = 8.10^{-7} ms^{-1}$, $\tau_V = 20s$, $\Gamma_D = 3.10^{-5} m^2 s^{-1}$.

In Fig. 3, the associated evolution of the temperature distribution (more precisely the difference between the time-dependent temperature distribution $T(x,t)$ along the thermoacoustic core and initial temperature distribution $T_0(x)$) is plotted at different moments during the transient regime. At time $t = 125s$, the effect of the acoustically enhanced thermal diffusivity results in the reduction of the temperature gradient along the stack with subsequent acoustic wave saturation. Then, from time $t = 150s$ to $t = 400s$ the second exponential growth of the wave amplitude occurs, while the mean temperature gradient is still decreasing. This second threshold is clearly due to the streaming induced change of the temperature distribution profile (see dotted curve in Fig. 3), which itself influences critically the wave amplification [3]. So, the observed double-threshold phenomenon is a consequence of the acoustically induced transient evolution of the temperature distribution, which proceeds with a different time scale than the characteristic time of wave amplification.

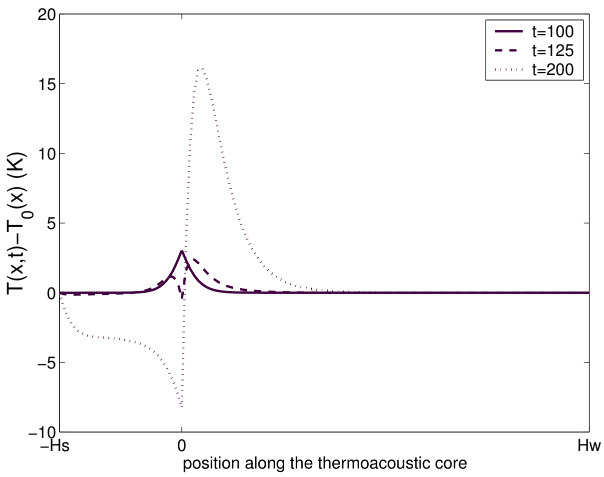

Figure 3: Calculated difference between temperature distribution $T(x,t)$ along the thermoacoustic core and initial temperature distribution $T_0(x)$ at different moments of the transient regime observed in Fig. 2

Acknowledgments

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