Numerical modeling of infrasound propagation at very long distance

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Introduction

Compliance with the Comprehensive Nuclear-Test-Ban Treaty (CTBT) in the atmosphere will be monitored by a worldwide network of 60 stations in the frequency range 0.02 to 4Hz. This network is able to detect small explosions in the atmosphere as well as shock waves caused by supersonic aircraft or meteorites. Two wave-guides in the upper atmosphere, one between 10km and 30km and another at about 100km, allow infrasound waves to propagate over several thousands of kilometers.

The French National Data Centre (in the CEA) uses and develop numerical modeling tools to characterize and study these infrasonic events. To take into account the nonlinear phenomena at the source and during the propagation, we develop a numerical approach to solve the Euler equations. In this paper, this method is compared in the linear domain with two other numerical modeling approaches based on the ray tracing technique and the parabolic approach. In our test case, the source is on the ground and generates at a distance of 1m a 1Pa pressure pulse of about 10s centered at the frequency of 0.1Hz. The pressure waveforms are computed at 450km from the point source for different altitudes from 2km to 50km considering an atmosphere height of 200km and a constant density.

The Eulerian approach

In the numerical method, all variables (pressure, density, velocity) are defined at the center of the cells. Their variations are assumed to be linear. At each time step the equations are solved in two phases. The first one is a Lagrangian phase; the second one is a remapping phase which consists in a projection of variables on the initial grid. Globally the method is then an Eulerian-like method. Moreover we use a splitting method for each spatial coordinates. In plane geometry, the Lagrangian equations used in the first phase are:

$$\begin{aligned}
&M\frac{\partial u}{\partial t} = -\int_{\Omega}\frac{\partial p}{\partial x}d\tau \\
&M\frac{\partial v}{\partial t} = -\int_{\Omega}\frac{\partial p}{\partial z}d\tau + \int_{\Omega}\rho g d\tau \\
&M\frac{\partial E}{\partial t} = -\int_{\Omega}\left(\frac{\partial pu}{\partial x}+\frac{\partial pv}{\partial z}\right)d\tau + \int_{\Omega}\rho g v d\tau \\
&E = e + \frac{1}{2}(u^2+v^2),\ \ p = (\gamma-1)\rho e
\end{aligned} \quad (1)$$

M is the mass of the cell Ω, u and v are the velocity components, p the pressure, ρ the density, g the gravitational acceleration, E the specific total energy and e is the specific internal energy. In the beginning of each Lagrangian phase, we have to evaluate pressures and velocities at the interfaces between cells to solve set of equations (1). In that purpose, we solve a Riemann problem and then an antidiffusion procedure is used to determine the desired values of velocity and pressure at the interfaces. The second phase consists in projecting the mass, the momentum and the energy on the initial grid. We assume linear variation of quantities in each cell using of slope limiters. Finally, after a computation time of 40 hours (CPU time), in cylindrical geometry, we obtain the pressure waveforms presented in Figure 1.

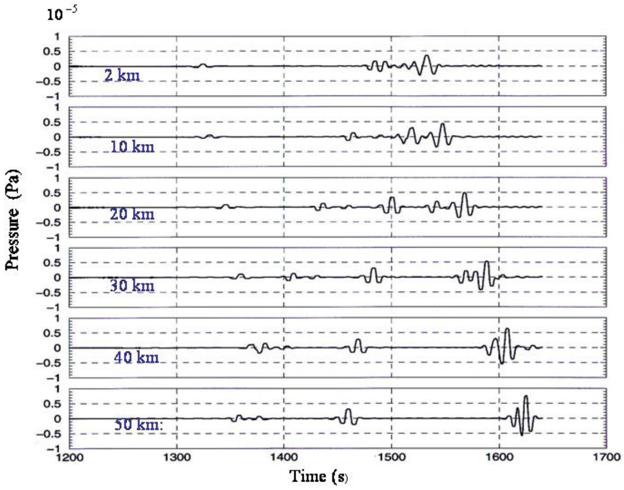

Figure 1: Waveforms computed by the Eulerian approach.

The parabolic approach

The parabolic approximation was introduced at the beginning of the forties in order to solve problems involved in the electromagnetic waves. Later, it was used in various fields such as underwater acoustics then was adapted for the atmospheric propagation by Gilbert [1]. A known initial field is propagated step by step in the frequency domain from the source toward the receiver by solving the Helmholtz equation in cylindrical coordinates (z,r):

$$\left(\frac{\partial^2}{\partial r^2}+\frac{1}{r}\frac{\partial}{\partial r}+\frac{\partial^2}{\partial z^2}+k(r,z)^2\right)P(r,z)=0 \quad (2)$$

where P is the acoustic pressure, k the wave number, z is the altitude, r the distance and c the sound speed. We assume that:

$$kr >> 1 \text{ and } P(r,z)=\frac{1}{\sqrt{r}}u(r,z)e^{jk_0 r} \quad (3)$$

Neglecting the backward waves, the solution of the equation (2) is:

$$u(r+\Delta r, z) = e^{j\Delta r\sqrt{Q}} u(r,z) \text{ (4) } where\ Q \approx \frac{\partial^2}{\partial z^2} + k^2 \text{ (4)}$$

After many developments described in the article of Gilbert [1], the field of pressure in a step can be written according to the field with the preceding step. We solve the pulse propagation problem via the frequency domain by Fourier synthesis in a post-data processing to obtain the signal waveforms in Figure 2.

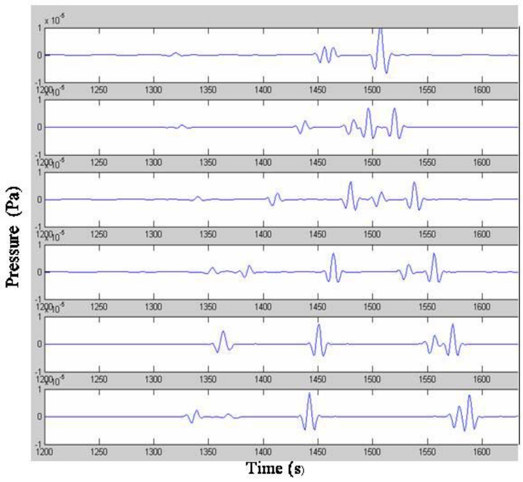

Figure 2: Waveforms computed by the parabolic approach for different altitudes. The computation time is small.

The ray tracing approach

From the early sixties, ray-based models have been used in underwater acoustics and in geophysics [2]. From the Helmhotz equation (2), after an inverse Fourier transform, the classical ray theory shows that the asymptotic pressure at a receiver point X may be written as a summation of contributions:

$$p(r,z,t) = \sum_{rays} p_r(r,z,t) \quad \text{where}$$
$$p_r(t) = A_r \cdot \Re[(-j)^{KMAH} S_a(t - t_r)] \qquad (7)$$

The summation is performed over each contribution p_r of rays which arrive at the receiver at the time t_r with the amplitude A_r. The wave form of a contribution is given by S_a, the analytic transform of the source time function S. The KMAH index (for Keller, Maslov, Arnold Hörmand), initially equal to zero, is the number of caustics crossed by a ray. Rays that arrive exactly at the receiver positions are obtained interpolating neighboring rays. Following this approach, we obtain the waveforms presented in Figure 3. The inherent problems of this technique are well known: waves that propagate in the shadow zone are not predicted, the amplitude near caustics are over estimated. But, ray tracing approach allows a straightforward physical analysis of wave propagation.

Discussions

Let consider the receiver at 2km from the ground in Figure 1 for example. The first arrival at about 1325s propagates in the shadow zone due to the decrease of the sound speed in the lower part of the atmosphere. Then, at about 1475s, we obtain phases that travel above 140km where the sound velocity reaches 500m/s. The last arrivals, at 1540s, are refracted at about 100km where the sound speed is close to 300m/s before to reach the receivers.

Conclusions

In this long range sound propagation test case, the Eulerian and parabolic approaches give very similar waveforms. In particular, their amplitudes, durations and arrival times are in good agreement. Results obtained by the ray tracing approach are better than expected especially on the ground receiver but the firsts arrivals are missing.

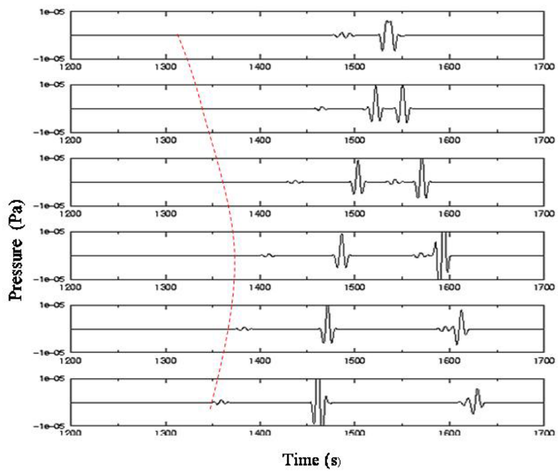

Figure 3: Waveforms computed by the ray tracing approach for different altitudes. The dashed lines indicate some missing arrivals. The computation time is small.

References

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