Thermoacoustic Refrigeration : Optimal Particle Velocity Vs Acoustic Pressure

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Introduction

Classical thermoacoustic refrigerators consist mainly of a standing wave resonator excited by a loudspeaker including a thermoacoustic core (a stack of plates). Following Ceperley's ideas [1], travelling waves can also be used in thermoacoustics. In 1990, Hofler [2] studies a standing wave thermoacoustic refrigerator and note that higher efficiency is obtained when there is a significant travelling wave component in the refrigerator. Raspet *and al.* [3] follow Hofler's conclusions by calculating the coefficient of performance as a function of the inverse of the standing wave ratio. Petculescu and Wilen [4] develop a resonant method which allows the ratio of travelling and standing wave components to be easily adjusted. Resulting devices of all these studies operate with an acoustic field from a more or less controlled superposition of standing and travelling waves. Thus, the goal of this paper is to calculate the optimal standing wave ratio for thermoacoustic refrigeration. To achieve this, the acoustic pressure p, the particle velocity u and their relative phase ϕ are uniform and independent one to the other in the stack region. Expressions for thermoacoustic given by Swift are rewritten with explicit pressure, velocity and relative phase. Optimal acoustic field is found to maximize the temperature gradient, the thermoacoustic heat flow and the coefficient of performance. A comparaison with a usual half-wavelength resonant refrigerator is done : more particularly, it is verified that, for high drive ratios, a standing wave refrigerator operates near the optimum in term of velocity. Some improvements can also be achieved by tuning the relative phase between pressure and velocity to its optimum.

Expressions of thermoacoutic quantities as a function of the acoustic pressure, the particle velocity and their relative phase

When assuming several classical hypotheses used in thermoacoustic theory (as quasi plane wave approximation, boundary layer approximation, and so on), thermoacoustic quantities can be written as presented in next subsections.

Thermoacoustic heat flux

The time-average heat flux per unit area $\overline{q_{th}}$ [?] is

$$\overline{q_{th}} = \frac{1}{2}\rho_0 C_p \Re(\tau u_z^*) - \frac{1}{2}T_0\alpha\Re(pu_z^*), \qquad (1)$$

where $\Re()$ standing for the real part and * denoting the complex conjugate. In the classical thermoacoustic linear theory, the second term of the right hand side of eq. (1) vanishes because p and u are out of phase in time (standing wave) (obviously, this hypothesis is not assumed in this paper). The total heat flux $\overline{Q_{th}}$ in the z-direction, given by

$$\overline{Q_{th}} = 2\int_0^{L_{sx}}\int_0^{y_0}\overline{q_{th}}dxdy, \qquad (2)$$

can then be written

$$\overline{Q_{th}} = \frac{\delta_h L_{sx}}{2(1-\frac{\delta_\nu}{y_0}+\frac{\delta_\nu^2}{2y_0^2})}\Big[\frac{T_0\alpha p|u|}{(1+\sigma)}([(-1+\sqrt{\sigma}-\frac{\delta_\nu}{y_0}\sqrt{\sigma})cos\phi$$
$$+(1+\sqrt{\sigma}-\frac{\delta_\nu}{y_0})sin\phi]) - \frac{(1-\sigma\sqrt{\sigma})}{(1-\sigma^2)}\frac{C_p\rho_0}{\omega}|u|^2\frac{\partial T_0}{\partial z}\Big], \qquad (3)$$

where the particle velocity is given by $u = |u|[cos\phi + isin\phi]$.

Temperature gradient

An estimation of the temperature gradient in steady state regime can be obtained when assuming that the heat flux $\overline{Q_{th}}$ in the gas is returned by diffusive conduction (throught the plates and throught the gas) [6]:

$$\frac{dT_0}{dz} = \frac{\delta_h T_0\alpha p|u|}{4(1+\sigma)(1-\frac{\delta_\nu}{y_0}+\frac{\delta_\nu^2}{2y_0^2})}$$
$$\frac{\left[(-1+\sqrt{\sigma}-\frac{\delta_\nu}{y_0}\sqrt{\sigma})\cos\phi+(1+\sqrt{\sigma}-\frac{\delta_\nu}{y_0})\sin\phi\right]}{[Ky_0+K_se_s]+\frac{\delta_h}{4}\frac{1-\sigma\sqrt{\sigma}}{(1-\sigma^2)(1-\frac{\delta_\nu}{y_0}+\frac{\delta_\nu^2}{2y_0^2})}\frac{C_p\rho_0}{\omega}|u|^2}, \qquad (4)$$

K and K_s being the diffusive thermal conductivity of the gas and the plates respectively. Assuming zero viscosity, this temperature gradient becomes

$$\frac{dT_0}{dz} = \frac{-\frac{\delta_h}{4}T_0\alpha p|u|[cos\phi - sin\phi]}{[K_se_s+Ky_0]+\frac{\delta_h}{4}\frac{\rho_0 T_0}{\omega}|u|^2}. \qquad (5)$$

Coefficient of performance

The coefficient of performance, that is the ratio of the heat flow from the cold thermal reservoir and the work flow can be written, assuming zero viscosity,

$$COP = COP_c\Gamma\frac{\left[(cos\phi - sin\phi)+\Gamma\right]}{\left[\Gamma[cos\phi+sin\phi]-1\right]}, \qquad (6)$$

where COP_c is the Carnot's coefficient of performance.

Optimal acoustic field

The heat flux, the temperature gradient and the COP are written as functions of the independent variables pressure p, velocity v and relative phase ϕ. Sets of values of these independent variables can be found which optimise each of thermoacoustic quantity. An example is given below concerning the temperature gradient.

Particle velocity optimizing temperature gradient

It is clear from the expressions (4) or (5) that it exists a value of the particle velocity which optimises the temperature gradient. This value is given by

$$|u|_{opt} = \sqrt{\frac{4\omega(Ky_o + K_s e_s)(1-\sigma^2)(1-\frac{\delta_\nu}{y_0}+\frac{\delta_\nu^2}{2y_0^2})}{\delta_h \rho_0 C_p (1-\sigma\sqrt{\sigma})}}, \quad (7)$$

or, for zero viscosity,

$$|u|_{opt} = \sqrt{\frac{4\omega(Ky_o + K_s e_s)}{\delta_h \rho_0 C_p}}. \quad (8)$$

It is interesting to note that this optimal value of the velocity depends only on the geometry and physical parameters of the stack.

Relative phase optimizing temperature gradient

Equation (5) shows that optimal phase for the temperature gradient (with inviscid fluid) is $\phi_{opt} = -\frac{\pi}{4}$ or $\phi_{opt} = \frac{3\pi}{4}$, corresponding to the cases where heat flux associated with standing wave and heat flux associated with travelling wave combine their effects. When viscosity is taken into account, the expression of ϕ_{opt} becomes

$$\phi_{opt} = arctan\left[-\frac{1+\sqrt{\sigma}-\frac{\delta_\nu}{y_0}}{1-\sqrt{\sigma}+\frac{\delta_\nu}{y_0}\sqrt{\sigma}}\right] + n\pi. \quad (9)$$

The optimal phase, which depends on geometry of the stack and physical parameters of the working gas, is close but different to the phase of a pure standing wave.

conclusion

As a conclusion, results has been obtained which show that an usual half-wavelength standing wave refrigerator operates near the optimum in terms of velocity when a high drive ratio is reached. In this configuration, it is demonstrated that the best position for the stack in the resonator is closed to the one usually recommended in the literature, when resonator is tuned at a very high acoustic pressure level. Moreover, some improvements would be achieved by tuning the relative phase between pressure and velocity to its optimum.

References

[1] P. H. Ceprley. A pistonless stirling engine - the travelling wave heat engine. *J. Acoust. Soc. Am.* **66** (1979) 1508–1513.

[2] T. J. Hofler, Performance of a short parallel-plate thermoacoustic stack with arbitrary plate spacing. *J. Acoust. Soc. Am.* **88** (1990) 594.

[3] R. Raspet, J. Brewster, and H. E. Bass. A new approximation method for thermoacoustic calculations. *J. Acoust. Soc. Am.* **103(5)** (1998) 2395–2402.

[4] G. Petculescu and L. A. Wilen, Alternative-geometry travelling wave thermoacoustic devices. *First International Workschop on Thermoacoustics, Hertogenbosh, April 22-25* (2001).

[5] G. Swift, Thermoacoustic engines. *J. Acoust. Soc. Am.* **84(4)** (1988) 1145–1180.

[6] J. Wheatley, T. Hofler, G. W. Swift and A. Migiori. An intrinsically irreversible thermoacoustic heat engine. *J. Acoust. Soc. Am.* **74(1)** (1983) 153–170.