Energy residuals for localization of structural areas inducing hypersensitive behaviour

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Introduction

This paper describes a method allowing one to localize structural areas inducing hypersensitive vibrating behavior: with a very low calculation cost, the use of energy residual in FEM post processing can be very efficient to detect the zones in which small changes can induce large dispersions on responses. The principle of the method is presented here, including a comparison between some energy residuals, in term of efficiency and localization properties.

Localization procedure

The localization procedure has been described in details in references [1] and [2], to which readers are invited to reefer for more details. It is based on concepts used in model updating [3], and its basics are quite simple: the nominal structure (characterized in equations with the "0" subscript) is considered, its eigen characteristics are evaluated using any available method, solving modal equation 1. "i" indicates the mode number, K and M are the stiffness and mass matrices.

$$\left(K_0 - \omega_0^{i2} M_0\right) U_0^i = 0 \tag{1}$$

Then, the modified structure is considered (using "1" subscript), and a residual force is built from the residual force and the flexibility matrix of this structure, using the displacement field of the reference structure, like indicated on equation 2.

$$F_i^r = \left(K_1 - \omega_0^{i2} M_1\right) U_0^i \qquad R^i = K_1^{-1} F_i^r \tag{2}$$

Finally this residual is considered by its kinetic or potential energy, and more precisely by the element contribution (for a FE analysis, "e" is the element number) to the total energy:

$$I^i = \sum_e I_e^i \qquad I_e^i = R^{iT} M_1^e R^i \tag{3}$$

Residual expressions

The residual expression proposed above is not the only one that can be considered: many expressions could be used to build this indicator. One propose here to consider 4 expressions detailed in equation 4, and to compare their efficiency in the localization process.

$$\begin{array}{ll} R_1^i = K_1^{-1} F_i^r & I_1^{i,e} = R_1^{iT} M_1^e R_1^i \\ R_1^i = K_1^{-1} F_i^r & I_2^{i,e} = R_1^{iT} K_1^e R_1^i / \omega_0^{i2} \\ R_2^i = M_1^{-1} F_i^r / \omega_0^{i2} & I_3^{i,e} = R_2^{iT} M_1^e R_2^i \\ R_2^i = M_1^{-1} F_i^r / \omega_0^{i2} & I_4^{i,e} = R_2^{iT} K_1^e R_2^i / \omega_0^{i2} \end{array} \tag{4}$$

Test structure

The test structure used in this analysis is a flexural beam (steel, 5 mm radius, 1 meter long), which is blocked at both ends. The considered finite element model contains 60 beam elements.

In order to test efficiency of the residuals, the considered changes in the structure are mass or stiffness reductions: the changed property of element 20 is reduced to 50% of its initial value.

Localization efficiency

In this part a localization efficiency index (LE_p) will be used, as defined in equation 5: it is evaluated using the value of indicator for mode "i" on modified element "em" (em=20) divided by the global energy residual for the same mode. Then the mean index is calculated on a frequency range including n modes (n=10). "p" indicates the number of the considered residual expression (here p=1 to 4).

$$LE_p = \frac{1}{n} \sum_{i=1}^{n} \frac{I_p^{i,em}}{\sum_e I_p^{i,e}} \tag{5}$$

Another index is also used to avoid the elimination of indicators presenting a shift or "overflowing" phenomena: in some situations the element with the most larger residual value is adjacent to the modified one. In these cases, considering the adjacent elements "edaj" leads to the expression of the modified localization efficiency index (LEA_p):

$$LEA_p = \frac{1}{n} \sum_{i=1}^{n} \frac{\sum_{eadj} I_p^{i,em}}{\sum_e I_p^{i,e}} \tag{6}$$

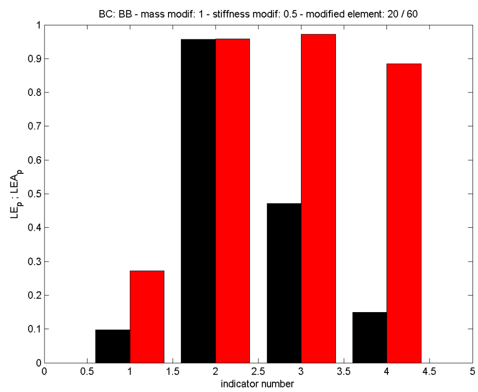

Figure 1: Localization efficiency indexes for stiffness modification of element 20.

Figure 1 shows the resulting LE_p and LEA_p when the stiffness of element 20 is reduced: one can observe that the LE_p index indicates that only the second residual expression leads to perfect localization: for expressions 3 and 4, the adjacent elements have to be considered, while the first expression does not give satisfactory results in this situation. Figure 2 shows the results of a similar analysis performed considering a mass reduction of the element 20. One can observe that only the third expression leads to almost perfect localization, while using adjacent elements, indicator 4 gives also convenient results.

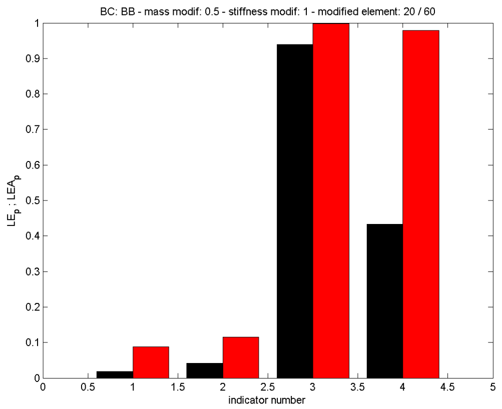

Figure 2: Localization efficiency indexes for mass modification of element 20.

There is no clear best choice between the proposed expressions for the localization: it mostly depends on the type of considered change. In practical situations, confident localization results could be obtained only if the nature of the changes are known, which should be the case using this method since it is purely numerical.

Hierarchization efficiency

The previous results concern only the localization efficiency, but do not take into account the hierarchization of the modes: a change in structural properties could induce small changes on some eigen modes, while other ones could be completely shifted.

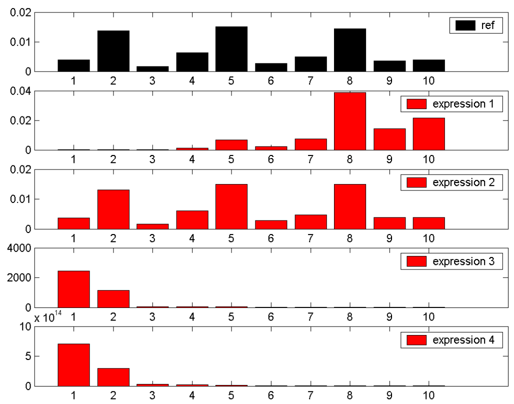

Figure 3: Comparison of hierarchization efficiency for the 10 first eigenmodes: stiffness modification.

Using the same cases as above, the total energy residual of the structure is compared for the 4 expressions to the frequency shifts of eigenmodes. One can observe on figures 3 and 4 that the conclusions of the previous part are still valid: residuals expressions built from the inversion of stiffness matrix work quite well for stiffness changes but not for mass shifts, while the opposite conclusion is valid for residuals built from inversion of the mass matrix.

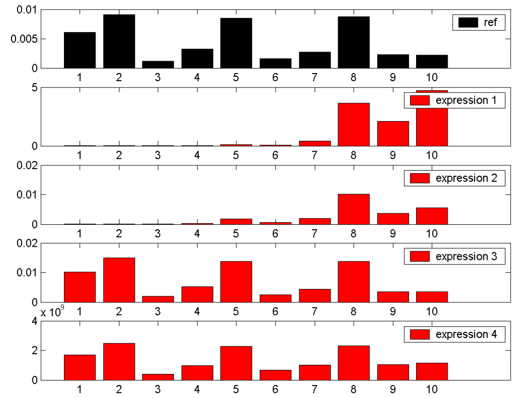

Figure 4: Comparison of hierarchization efficiency for the 10 first eigenmodes: mass modification.

Conclusion

Some energy residuals expression for the localization of structural zones inducing sensitive behaviour have been compared here. Unfortunately no universal choice has been found, since the efficiency of a given indicator depends on the type of changes which have been made on the structure. Nevertheless, when this method is applied, these changes are well known, since this method is based only on numerical analyses: in that situation, one can easily choose which indicator is the best choice for a given problem. Moreover, most of changes in structures concern stiffness, while mass properties are usually well known and present only small variations.

References

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