Room Acoustical Simulation at Low Frequencies Using the Scattering Element Method

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Introduction

Arguably all known room acoustical simulation packages assume geometrical, i.e. "ray-like" sound propagation and are thus limited to high frequencies and, if diffuse reflections are adequately incorporated, to diffuse sound fields (large rooms). Apparently, these approaches work well in a frequency range where the wavelength is shorter than a fraction of a typical room dimension, for instance $f > 10\,c/\sqrt[3]{V}$ [1].

One possibility to cope with lower frequencies is to use so-called scattering methods. These methods have evolved from electromagnetic theory, where they are known as "Transmission Line Matrix" methods [2]. Under certain conditions, they are equivalent to "Finite-Difference-Time-Domain" methods [3], which in turn have a long history in both electromagnetics and meteorology. Applied to acoustical problems, these methods are often called "Digital Waveguide" methods [4]. Because these terms, established in different fields, often mean one and the same thing we dare to suggest "Scattering Element Method" (SEM) as a unifying terminology. For a recent in-depth treatment on scattering methods we refer the reader to [5].

Recently, a method was suggested to extend a Scattering Element System by a multiple-input-multiple-output boundary model, using a matrix of digital filters [6]. In the present work, this approach is supplemented by a stability criterion. The method is then used to predict room acoustical parameters at low frequencies in a reverberation room.

A Short Review of the SEM

Principle. The SEM operates on a spatial grid spanning the sound field to be analyzed. The most common topology for this grid is rectangular and equidistant. The distance Δ between the grid points (nodes) should be chosen to be smaller than the wavelength by a factor of at least ten, in order to avoid dispersion errors. Given Δ, the temporal sampling should take place at a rate $F_S = \sqrt{3}c/\Delta$, with c being the speed of sound.

With these assumptions, the principle operation of the SEM can be illustrated by imagining a system of tubes between the nodes, in which only plane waves propagate at a speed of $\sqrt{3}c$: Consider a wave propagating within a tube from one node to a second one, which, at a given time step $k-1$, is exactly at mid-distance between the two nodes. Seen from the second node, this is the incident wave from that particular direction at time step $k-1$, i_{k-1} (fig. 1 left). One time step later, this wave has propagated a distance Δ, thereby passing the node at which it is scattered into all 6 tubes leaving the node. Thus, at time step k, there are waves propagating from the node into all 6 directions, at mid-distance between the node and its adjacent nodes. These scattered waves are called s_k. The amplitudes of the s_ks are obtained from the impedance ratio at the node:

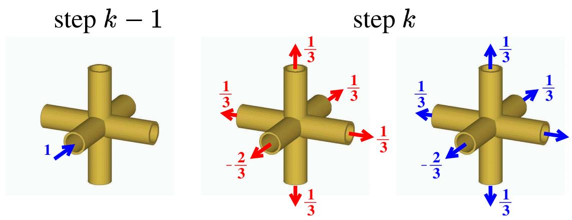

Figure 1: Principle of the SEM: scattering governed by the impedance ratio at the node.

minus two thirds of the amplitude of the incident wave for the wave propagating back into the tube where the incident wave came from, and one third of the amplitude of the incident wave for all others (fig. 1 center). These scattered waves can then be considered incident waves for the adjacent nodes (fig. 1 right), and so forth. The algorithm thus consist of only two steps which are continuously repeated:

1.) scattering: $i_{k-1} \rightarrow s_k$ 2.) shifting: $s_k \rightarrow i_k$

The sound pressure at any node one is interested in can be obtained by summing over all waves incident onto this node: $p_k^{\text{node}} = \frac{1}{3}\sum i_k$. The whole algorithm can be implemented very efficiently, such that most room-acoustical problems in the frequency range below $10\,c/\sqrt[3]{V}$ can be solved conveniently using an ordinary personal computer.

Boundary Conditions. As the SEM is based on reflection/scattering, simple boundary conditions (locally reacting, without memory) are best implemented using reflection factors, yielding a slightly modified step 2.) of the algorithm:

$$\text{2.) shifting: } i_k = s_k r, \text{ reverse direction,} \tag{1}$$

where r is the (real) reflection factor. For non-locally reacting boundaries with memory, this can be extended to

$$\begin{aligned} \text{2.) shifting: } \boldsymbol{i_k} &= \boldsymbol{B_0} \cdot \boldsymbol{s_k} + \ldots + \boldsymbol{B_M} \cdot \boldsymbol{s_{k-M}} \\ &- a_1 \cdot \boldsymbol{i_{k-1}} - \ldots - a_M \cdot \boldsymbol{s_{k-M}}, \\ &\text{reverse direction,} \end{aligned} \tag{2}$$

where the $\boldsymbol{i}$s and $\boldsymbol{s}$s are vectors of incident and scattered waves of all nodes at the boundary, and the as and $\boldsymbol{B}$s contain filter coefficients of the reflection factor matrix $\boldsymbol{R}(z) = \boldsymbol{B}(z)/a(z)$. Thus, in the z-domain,

$$\boldsymbol{i}(z) = \boldsymbol{R}(z)\boldsymbol{s}(z). \tag{3}$$

Stability Issues

To ensure numerical stability, both the SE system and boundary models must be stable and passive. This is inherent in the SE system [5], but not necessarily for the boundary models.

The boundary model from eq. 3 is stable if all poles of $\boldsymbol{R}(z)$ are within the unit circle. Passivity, on the other hand, requires that for any possible excitation, the power reflected from the boundary must be not greater than the power entering the boundary, i.e. in the frequency domain,

$$\begin{aligned} \left\|\boldsymbol{i}(\omega)\right\|_2^2 &\leq \left\|\boldsymbol{s}(\omega)\right\|_2^2 \\ \left\|\boldsymbol{R}(z)\big|_{z=\mathrm{e}^{j\omega}}\boldsymbol{s}(\omega)\right\|_2^2 &\leq \left\|\boldsymbol{s}(\omega)\right\|_2^2. \end{aligned} \tag{4}$$

Using the sub-multiplication property of the 2-norm,

$$\left\|\boldsymbol{R}(z)\big|_{z=\mathrm{e}^{j\omega}}\boldsymbol{s}(\omega)\right\|_2^2 \leq \left\|\boldsymbol{R}(z)\big|_{z=\mathrm{e}^{j\omega}}\right\|_2^2 \left\|\boldsymbol{s}(\omega)\right\|_2^2, \tag{5}$$

one ends up with the requirement that

$$\left\|\boldsymbol{R}(z)\big|_{z=\mathrm{e}^{j\omega}}\right\|_2 \leq 1. \tag{6}$$

In other words, the largest singular value of the reflection factor matrix must be not greater than one for any frequency.

Simulation of a Reverberation Room

The SEM was used to predict room acoustical parameters (T, G, C_{80}) at 5 microphone positions in a reverberation room ($V = 195$ m^3) excited by one speaker in a corner of the room.

The room was spatially sampled at $\Delta = 0.2$ m, resulting in $F_S = 2970$ Hz. It was equipped with 4 plate absorbers (each 0.6×1.2 m^2) whose $\boldsymbol{R}(z)$ were estimated on the basis of measurements of the complete 18×18 mobility matrix [6]. In order to limit the complexity (filter order) of the absorber models, simulations were performed in 3rd octave bands.

As the SEM (in the form presented and used here) represents a lossless system, it was necessary to add artificial damping in order to account for the finite reverberation time of the empty room. This was done by assigning a first order low-pass to all wall nodes, using the scalar form of eq. 2. The parameters of these low-pass filters (overall gain and damping at high frequencies) were adjusted such that the predicted reverberation times were within $\pm 25\%$ with respect to the values observed experimentally.

One simulation (one 3rd octave band, 16384 time steps) took a few minutes on a modern PC (Pentium IV).

In fig. 2, measured parameters are plotted versus predicted ones, for every 3rd octave band and every microphone position. It should be pointed out that the shown error bars represent maximum/minimum values (for different assumptions regarding the damping present in the empty room). The agreement between prediction and measurement compares favorably to the results found in recent round robin tests: Averaged over all measurement positions, the maximum difference in any 3rd octave band is 2.5 dB for C_{80} and 1 dB for G.

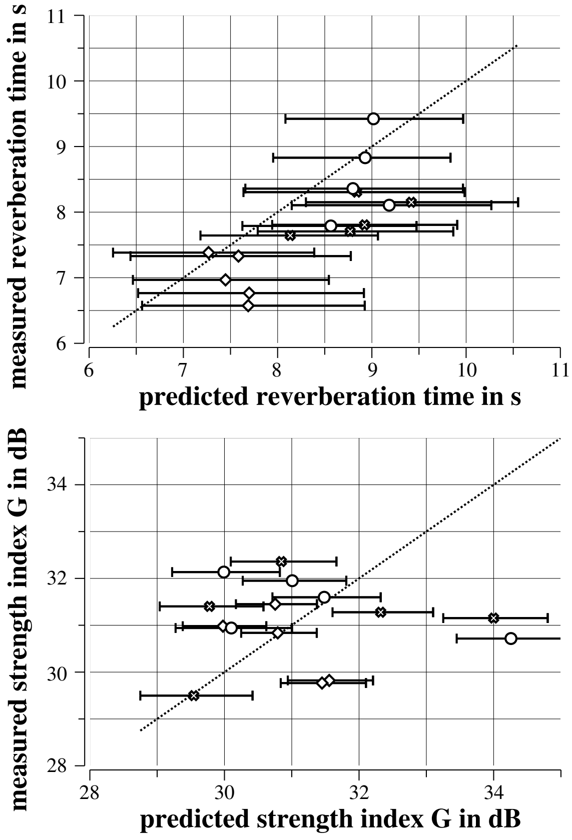

Figure 2: Reverberation room: prediction and measurement. Shown are means (circles, crosses, diamonds) and maxima/minima (error bars) across various assumed values of the wall damping, for 5 microphone positions and 3 3rd octave bands (circles-125 Hz, crosses-160 Hz, diamonds-250 Hz).

Summary and Future Work

In this work, the so-called scattering element method (SEM) was used to predict the acoustical behavior at low frequencies of a reverberation room equipped with plate absorbers. In order to ensure numerical stability of the simulation, the absorber models (developed previously in [6]) were supplemented with a passivity criterion. Resulting predictions of room acoustical parameters agree rather well with measured values.

Future work should mainly concentrate on improving the boundary models (numerical and measurement methods to establish models, criteria to optimize expense vs. accuracy). Naturally, further validation in real-life applications is necessary.

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