Optimisation of correlation function models for statistical aeroacoustic noise prediction

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Introduction

The problem of predicting the noise generated by aeroacoustic systems can be dealt with in a variety of ways, the cheapest of which is to use a statistical description of the aerodynamic phenomena, coupled with an acoustic analogy. This kind of approach is straightforward provided good analytical models are available for the said phenomena. A difficulty arises however, related to the extremely low acoustic efficiency of the mechanisms of sound production. In a jet for example Lighthill [1] gives the ratio of acoustic to turbulence power as, $\eta = 10^{-4}M^5$, where M is the Mach number. This means that the errors in the models can be orders of magnitude greater than the actual acoustic energy generated. The statistical models need therefore to be extremely accurate. In this work an effort is made to improve this accuracy, a temporal correlation coefficient function being proposed which depends on only two physical parameters, the integral time scale and the Taylor microscale. The function shows better agreement with experimental data than previously used functions, and, being dependent on real physical quantities, its dependence on these gives rise to some interesting questions concerning the physics of noise generation by turbulence, in particular: what is the role of the smaller turbulence scales ?

Theory

Lighthill [1] showed how the Navier-Stokes equations can be reformulated such that the acoustic field generated by an unbounded turbulence field is expressed in terms of the dynamic of that field. The acoustic intensity per unit volume of turbulence can thus be written as

$$P(\theta,\phi) = \frac{\rho_o x_i x_j x_k x_l}{16\pi^2 x^4}\int R_{ijkl}(\mathbf{r},\mathbf{r}')\frac{\partial^4 g(\tau)}{\partial^4 \tau}d^3\mathbf{r} \qquad (1)$$

where $R_{ijkl}(\mathbf{r},\mathbf{r}')$ and $g(\tau)$ are the spatial and temporal components of the fourth order two-point velocity correlation tensor. In terms of the physics of sound production the spatial and temporal components of the velocity correlation tensor are related to, respectively, the efficiency with which the turbulence energy is converted into acoustic energy, and, the dynamic of that conversion. While accurate modelling of both of these quantities is crucial for good sound prediction, the more critical of the two is the temporal component. This difference is related to the subsequent mathematical operations required for computation of the acoustic intensity. While the spatial part of the correlation undergoes a spatial integration, the temporal part is differentiated four times and then Fourier transformed in order to give the frequency dependence of the sound field. Errors in the spatial part will lead thus to varying degrees of inaccuracy in the overall sound levels predicted (either too large or too small a volume will be integrated), while errors in the temporal part will be amplified by the four differentiations and the result then Fourier transformed. This paper will therefore focus on the temporal part of correlation tensor and its optimisation using physically meaningful quantities.

The temporal correlation function

The temporal correlation function of the turbulence, viewed in a convected frame of reference, is what governs the dynamic of the acoustic field generated by a jet. The principal physical quantity involved is the integral time scale - a characteristic time over which the turbulence in this moving frame becomes decorrelated. However it is not simply the time taken for the said decorrelated which will govern the nature of the sound field generated, but also the character of the decay. Different functions have been used in the past to model this decay; Gaussian, Inverse Hyperbolic and Exponential to name a few (see figure 1). While the Gaussian and Inverse Hyperbolics give the desired shape very close to $\tau = 0$ (i.e. a slope of 0), the exponential best captures the decay (cf. Jordan and Gervais 2003 [2]) However the exponential function has slope

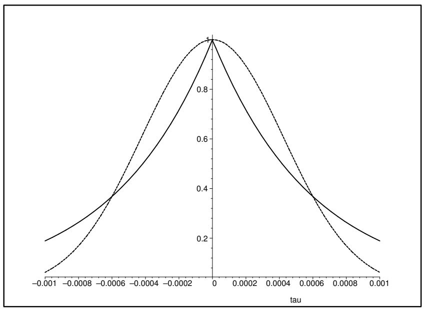

Figure 1: Gaussian (dotted) and Exponential (solid) functions

equal to one at $\tau = 0$, and this is a source of significant difficulty when it comes to evaluating the spectral character of the acoustic field, the error incurred in the initial model becoming amplified with each of the aforementioned differentiations.

An improved model

An exponential function of the form $e^{-|\tau|/\tau_c}$ gives a good description of the turbulence decay, and requires only a single physical parameter, τ_c (the integral time scale), to achieve this. There is a real interest in analytical models which use only physically meaningful parameters, as it can be hoped that through the application of such models accurate noise prediction will be possible from a simple description of the jet geometry and its exit conditions. We therefore wish to retain the exponential form and use it as the basis of a better adapted function.

The property which is lacking is a slope of zero at $\tau = 0$.

A means of manipulating the exponential form to produce a slope of zero at $\tau = O$, whilst maintaining the global decay as governed by the integral time scale, is to perform a convolution of the Exponential with a Gaussian function. In fact this Gaussian, the first two terms of whose Taylor series give the osculating parabola, is governed by the temporal Taylor microscale (λ_t) of the turbulence - the additional parameter is thus again a physical one and no arbitrary constants have been added. The new correlation function is written

$$g(\tau) = \int_{-\infty}^{\infty} e^{-\frac{\epsilon^2}{\lambda_t^2}} e^{-\frac{|\tau-\epsilon|}{\tau_c}} d\epsilon, \qquad (2)$$

a graphical representation of which is shown in figure 2. It

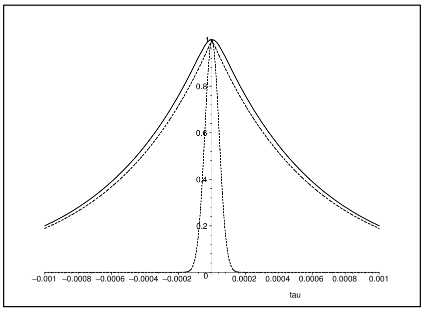

Figure 2: Exponential - dash-dot; Gaussian corresponding to osculating parabola - dotted line; $g(\tau)$ (Equation 2) - solid line

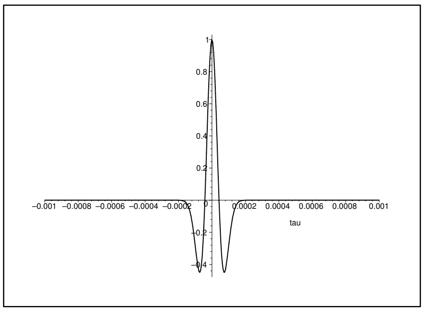

Figure 3: Fourth derivative of $g(\tau)$

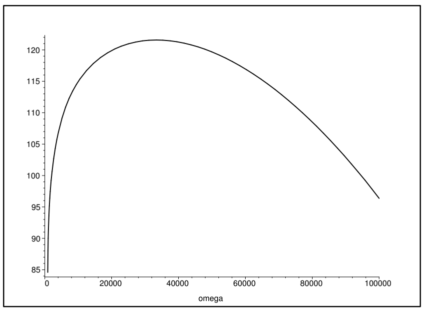

Figure 4: Acoustic spectrum (Fourier transform of figure 2(b))

can be seen how the decay, as modelled by the exponential, has been retained, while the behaviour close to $\tau = 0$ is now characterised by a slope of zero.

The function expressed by equation 2 and shown in figure 2 is representative of the turbulence dynamic. The relationship between this dynamic and the acoustic dynamic is given explicitly by equation 1. The result of the four differentiations is shown in figure 3, and the corresponding acoustic spectrum, obtained by means of a Fourier cosine transform, shown in figure 4.

Discussion

The most striking characteristic of this result is the seemingly strong influence of the small turbulence scales on the overall acoustic dynamic. As illustrated by figure 3, the region where the fourth derivative of the turbulence correlation function is significant is at small values of τ, which means that its shape (representative of the acoustic dynamic) has been significantly contributed to by the shape of the turbulence correlation function in this region (solid line in figure 2), which is governed by the smaller turbulence scales. This gives rise to interesting questions concerning the physics of the sound production, because it appears to imply that these small scales play in fact an important role in the generation of sound. This goes against the trend in recent years which attributes the generation of aerodynamic sound to the large turbulent structures. It is true that these structures play a principal role in the turbulence dynamic, but they also carry the smaller scales, to which they transfer their energy. To this day, the exact nature of the aerodynamic sound generation mechanism remains unclear, and so the importance of these smaller scales cannot be ruled out. The mechanism is certainly a complex one, which most likely involves the entire turbulence dynamic - the large scales which globally control the flow and carry most of its energy, and the smaller scales which play an important role in the spatiotemporal decorrelation of the turbulence.

These are interesting questions, the answers to which will require a more in-depth analysis. It should be noted however that this model constitutes a very recent result, which for the moment has not been validated using experimental or numerical data. It will in fact be very difficult to validate experimentally because the scales which must be measured generally remain beyond the reach of current measurement technology. Validation using a high precision numerical tool will thus be necessary (a compressible DNS for example).

Conclusion

A model has been developed for the moving-frame temporal correlation coefficient function, important in the statistical description of aerodynamic noise production by free jets. The model is generated by convolving a Gaussian function (function of the Taylor microscale) with an Exponential function (function of the Integral time scale). From this, expressions for the correlation and spectral functions governing the acoustic dynamic are derived. Results indicate that the small turbulence scales may play an important role in the generation of aerodynamic noise, but further analysis is required to verify this.

References

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