Acoustical energy diffracted around a building

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Introduction

Acoustical diffraction is a current phenomena which occurs when sound waves hit obstacles: this phenomena brings acoustical energy into shadows area that are prevented from direct incidence. Using the spirit of ray tracing, Keller [1] developped in the 1960's the Geometrical Theory of Diffraction (GTD) which extends geometrical acoustics including diffraction phenomena. In particular, he introduced a diffraction coefficient to give formulations for diffracted rays asymptotically valid at high frequencies. Kouyoumjian and Pathak [2] proposed under the name of Uniform Theory of Diffraction (UTD) an improvement of the GTD. They gave a more accurate form of the diffraction coefficient which is still valid in the transition regions between illuminated and shadow areas and respects the continuity of the diffracted field at these boundaries.
This paper presents study of diffraction by means of an original method based on energy concepts [3, 4, 5]. This method relies on the equation of the energy radiative transfer, and is well-suited for high frequency applications. This method has given rise to a specific software called CeReS developped at the LTDS. In the proposed approach, diffraction phenomena is treated by introducing diffraction sources whose amplitudes are calculated applying the power balance between all acoustical sources. Some comparative studies between this method and some standard methods (BEM, GTD) have previously been performed [6]. In this context, the aim is to apply the method in a more complicated case of diffraction around a building. The problem is considered as two-dimensional. In this context, diffraction is a major phenomena responsible for noise in the backyard of the building.

Theoretical formulation

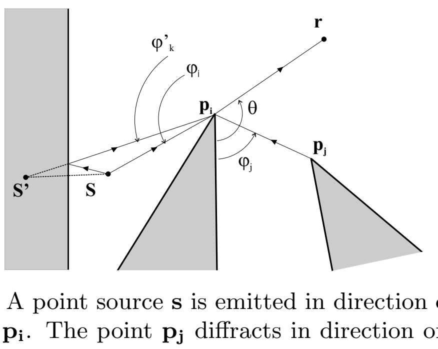

Figure 1: A point source $\mathbf{s}$ is emitted in direction of diffraction point $\mathbf{p_i}$. The point $\mathbf{p_j}$ diffracts in direction of $\mathbf{p_i}$. $\mathbf{s}'$ is the image source of $\mathbf{s}$, $\mathbf{r}$ is the receiver point.

The method consists in introducing fictitious sources at diffraction points $\mathbf{p_i}$. Their amplitudes σ_i depend on the direction of emission θ. To determine these amplitudes, an energetic diffraction coefficient D is defined as the ratio of the emitted power per unit solid angle P_{emit} in the direction θ and the incident intensity I_{inc} stemming from the direction of incidence φ for the two-dimensional case:

$$P_{\text{emit}} = D(\varphi, \theta) I_{\text{inc}} \tag{1}$$

D can be written in terms of the diffraction coefficient d introduced by the GTD [7] or by the UTD [2] as:

$$D(\varphi, \theta) = |d(\varphi, \theta)|^2 \tag{2}$$

Consider the case where there is only one acoustical source $\mathbf{s}$ of amplitude ρ_0. Amplitudes of the diffraction sources are calculated by applying the power balance at each diffraction point. To this end, the radiative transfer method relies on the assumption that all sources are uncorrelated so that the superposition principle is applied to energy fields. Thus, the incident intensity at each point $\mathbf{p_i}$ is the sum of several contributions which differ according to the point. Using equation 1, the power balance at point i is written in the general following form:

$$\begin{aligned} \frac{\sigma_i(\theta)}{2\pi} &= \rho_0 D(\varphi_i, \theta) H(\mathbf{s}, \mathbf{p_i}) \\ &+ \sum_k D(\varphi'_i, \theta) \sigma'_k(\varphi'_k, \theta) H(\mathbf{p'_k}, \mathbf{p_i}) \\ &+ \sum_{j \neq i} D(\varphi_j, \theta) \sigma_j(\varphi_j) H(\mathbf{p_j}, \mathbf{p_i}) \end{aligned} \tag{3}$$

The left-hand-side member of equation 3 is the emitted power per unit solid angle in the direction θ. φ_i, φ'_k and φ_j are respectively the incidence angle of incident, reflected and diffracted rays at point i (Figure 1).The first term of the right-hand-side member is the direct contribution coming from the primary source written in terms of the kernel function H: assuming no atmospheric attenuation $H(\mathbf{s}, \mathbf{r}) = \mathbf{u}/2\pi s$, $\mathbf{u}$ is the source-receiver unit vector, and s is the source-receiver distance. The second term is the reflected contribution evaluated using the image-source method: $\mathbf{p'_k}$ are the symmetric of the sources with respect to reflection planes and σ'_k (=ρ_0 or σ_k) are the amplitudes of reflection sources. The last term sums contributions from diffraction points $\mathbf{p_j}$ which diffract in the direction of $\mathbf{p_i}$. In case of multiple diffraction, it reduces to a linear set of equations on amplitudes of diffraction sources. Finally, the complete acoustical field at any receiver point $\mathbf{r}$ is simply the sum of all contributions, that is the incident field, the reflected field and all fields diffracted in direction of $\mathbf{r}$.

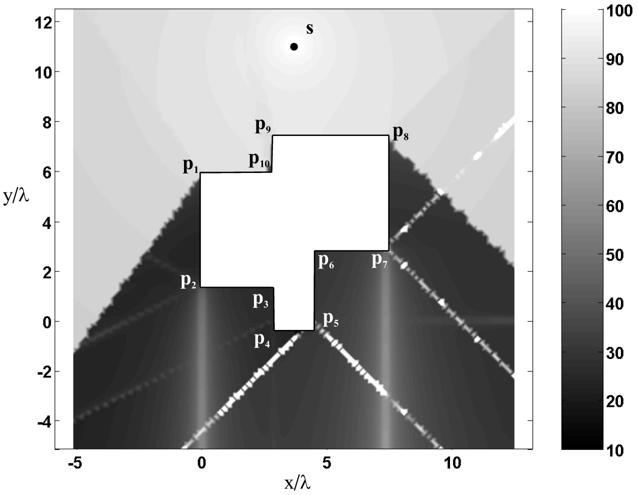

Figure 2: Map of the SPL (dB - ref: 2.10^{-5} Pa) around the building. A point source $\mathbf{s}$ is emitting in front of the building. Ten diffraction sources are introduced at corners $\mathbf{p_i}$ of the building.

Diffraction around a building

In order to demonstrate the feasibility of the method, we apply the case of diffraction around a building (Figure 2). A point source located at $(5,15)$m is emitting in front of the building. The main width and length of the bulding are 10m. Ten diffraction sources are introduced at corners of the building. This problem illustrates the case of multiple diffraction. Equation 3 is written for the ten diffraction points, and is applied for the particular directions of diffraction involved in the expressions of σ_i. In this equation, no reflected field must be taken into account as reflected waves never impinge on diffraction points. It leads to a linear system which is numerically solved by Gaussian elimination. To evaluate the reflected contributions at point $\mathbf{r}$, the image-source method is employed. Reflection sources are introduced at points $\mathbf{s}'$, $\mathbf{s}''$, and $\mathbf{p}'_\mathbf{i}$ $(i = 1, 2, 4, 5, 7)$ which are the symmetrics of points $\mathbf{s}$ and $\mathbf{p_i}$ towards reflection planes. The acoustical intensity can thus be written in the following way:

$$
\begin{aligned}
\mathbf{I}(\mathbf{r}) \; &= \; \rho_0 \mathbf{H}(\mathbf{s},\mathbf{r}) + \rho_0 \mathbf{H}(\mathbf{s}',\mathbf{r}) \\
&+ \; \rho_0 \mathbf{H}(\mathbf{s}'',\mathbf{r}) + \sum_{i=1,2,4,5,7} \sigma_i(\theta'_i)\mathbf{H}(\mathbf{p}'_\mathbf{i},\mathbf{r}) \\
&+ \; \sum_{i=1}^{10} \sigma_i(\theta_i)\mathbf{H}(\mathbf{p_i},\mathbf{r})
\end{aligned} \qquad (4)
$$

where θ_i and θ'_i are the emission angles at points $\mathbf{p_i}$ and $\mathbf{p}'_\mathbf{i}$ towards $\mathbf{r}$. Remind that depending on the receiver position, some of these fields are stopped by the obstacle and must not be taken into account in the sum.

Numerical result

The SPL around the building is calculated on the third-octave band centered on f=250Hz (Figure 2). With regard to the size of the building, it ranks among medium or even high frequency range ($\lambda/L = 0.14 < 1$). Boundaries between illuminated and shadow zones appear as white lines: the radiative transfer method as the GTD lead to unphysical results in these areas because the field cannot be described in terms of rays. Besides, interference effects are not described because of the uncorrelation assumption between sources. All orders of diffraction are simultaneously taken into account compared to the infinite number of rays necessary in the classical ray-tracing method. Calculation times are seriously reduced and do not depend on the frequency as the number of sources involved in calculations do not depend on the frequency.

Conclusion

This work improves the use of the radiative transfer method including acoustical diffraction. The originality of the approach mainly based on ray-tracing concepts consists in introducing energy sources at diffraction points to determine the diffracted field. The amplitudes of these sources depend on the direction of diffraction, and are calculated by defining an energetic diffraction coefficient. A linear system where the unknowns are the amplitudes of the diffraction sources in particular directions of diffraction is solved. The method is applied here in the case of diffraction around a building. The application of the approach for three-dimensional problems is under development, as well as the implementation in the software CeReS.

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