Numerical simulation of sonic boom

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From Whitham function to CFD

Sonic boom is the aerodynamical field of a supersonic body at very great distances (several hundred times the fuselage length) in an heterogeneous atmosphere. Sonic boom modeling distinguishes the *aerodynamical flow* around the aircraft (the “nearfield”) from the *atmospherical propagation* down to the ground (the “farfield”), though there is no fundamental physical difference between the two. The nearfield flow is considered as the source term for the acoustical approximation used in the farfield. This source term can be computed independantly from the sonic boom propagation. Relying on four approximations (slender body, linearized supersonic flow, farfield approximation and locally axisymmetrical flow), Whitham [1] proposed a self-consistent matching between aerodynamics and geometrical acoustics for a body of revolution. Whitham “F” function has been generalized by Walkden [2] to an non-axisymmetric body with lift. Given the progresses in computational fluid dynamics, the asymptotic Whitham formulation is nowadays superseded by CFD simulations of the flow around the aircraft down to a “sufficient” distance (a few times the fuselage lengths) tested practically by convergence tests at the ground level. Then the resulting pressure field is used directly as an input to the propagation code. However, some smoother matching can be realized. Indeed, in Whitham theory, only monopolar (representing thickness fuselage) and dipolar (representing lift) contributions are taken into account, contrarily to CFD simulations. Page and Plotkin [3] developed the so-called Multipole Matching Method for extracting Whitham function from CFD results. Other improved matching methods are reviewed by Plotkin and Page [4], and could be important in practice for low boom configurations.

Rays and meteorology

Geometrical acoustics is a high frequency approximate formulation of acoustics applicable to a 3D heterogeneous, moving medium for short waves. It states that, at first approximation, sound propagates along curves known as rays and defined so as to make the propagation time between the sound source and the receiver extremal (Fermat’s principle). Propagation equations reduce to a differential system of 6 equations, governing the position of a wavefront point and the wavefront normal vector along a given ray. The first application of ray acoustics to sonic boom is due to Esclangon [5] following researches during World War I to separate sonic boom of supersonic shells from blast waves from gun muzzles. Considering only rays launched downwards, their intersection with the ground defines the geometrical carpet, delineated by limiting rays that turn up exactly when reaching the ground (in the case of an upward refracting atmosphere). Beyond this cut-off, inside the shadow zone, sonic boom decays rapidly (see below). Flight at Mach 2 yields a carpet about 90 km wide in the standard atmosphere, but this width is very sensitive to meteo data and fluctuates a lot [6]. Moreover, in case of a downward refracting atmosphere, there is no geometrical cut-off and sonic boom does not decay as fast as in a shadow zone.

Nonlinear distorsion

In linear acoustics, amplitude is determined by an energetic principle (Blokhintsev invariant [7]), according to which sound amplitude is inversely proportional to the square root of a geometrical quantity (the ray-tube cross area) measuring the rate of convergence of adjacent rays. This quantity can be evaluated efficiently according to the method of Candel [8]. This results into a total set of 18 differential equations to be solved along each ray (12 for a stratified atmosphere). However, for sonic boom, this procedure is not sufficient, as high amplitudes and long propagation distances induce strong nonlinear effects, such that the sound speed is (slightly) dependant on the instantaneous wave amplitude. This leads to the formation and evolution of weak acoustic shock waves. The second key contribution of Whitham [9] was to recover nonlinearities, so that the pressure field satisfies an inviscid Burgers equation along each ray. Nonlinear effects explain the evolution of the temporal waveform of the pressure field, from a rather complex shape in the aircraft nearfield reflecting details of the aircraft geometry and lift distribution, to the ultimate “N” wave shape at the ground level. Guiraud [10] extended Whithams’s analysis to an heterogeneous and thermoviscous fluid with wind, while it was fully applied to the effective numerical evaluation of sonic boom by Hayes *et al.* [11]

Absorption by molecular relaxation

Weak shock theory predicts only ideal shocks with instanteneous pressure jumps, while sonic boom recordings show pressure jumps taking place over a finite time, from a few to several tens of milliseconds. This loosely defined “rise time” is an important parameter in controlling the subjective loudness of sonic boom. Thermoviscosity leads to rise times of the order or the microsecond, about 1000 times shorter than the observed ones. Indeed, at infrasonic and audible frequencies, the main source of sound absorption is the vibrational relaxation of diatomic molecules of nitrogen and oxygen. The key role of relaxation in the rise time of sonic boom has been outlined by Hodgson [12]. The assumption of steady shock allows a rather simple and numerically efficient analysis of the absorption effects on sonic boom (Kang and Pierce [13]), concentrated in a “shock structure”. The validity of this steady state approximation has been examined by Cleveland [14]. It leads (Hodgson

[12], Coulouvrat and Auger [15]) to the notion of “partially dispersed shock” : for low shock amplitudes, the nitrogen relaxation is dominant, with long rise times associated of the order of 10 ms or more, while for higher amplitudes, oxygen relaxation should also be taken into account. Then the shock structure is more complex and rise times are shorter. With a view to sonic boom minimization, it would be desirable to reduce boom amplitude below the critical value, so as to have long rise times dominated only by nitrogen relaxation. For mild temperatures, this fixes a goal of the order of 15 Pa, precisely the objective of the QSP program.

Shadow zone

For an upward refracting atmosphere, sound penetration inside the shadow zone is dominated by diffraction effects (Pierce [16]), with part of the energy of the limiting ray creeping along the ground and sheding off progressively diffracted rays that radiate energy inside the shadow zone. If the ground is of finite impedance, that decay is enhanced by ground absorption. This process has been applied to sonic boom penetration into the shadow zone and compared with reasonable agreement to Concorde recordings (Coulouvrat [17]). Results show that the rise time at the cut-off over a finite impedance ground is of the same order of magnitude or larger (10 to 30 ms) than the rise time induced by molecular relaxation or by atmospheric turbulence. Similar conclusions are found overseas (Boulanger and Attenborough [18]). Inside the shadow zone, the rise time increases almost linearly, up to values about 50 ms after a few kilometers. There, sonic boom is simply reduced to a low frequency rumble. As indicated by Concorde shadow boom heard along the coasts of Northern Brittany, it does not seem to be perceived anymore as annoying. The same kind of behaviour is expected for an unsteady flight at the carpet extremity during the deceleration phase.

Sonic boom focusing

The approximation of geometrical acoustics breaks down in the neighbourghood of surfaces called “caustics”, where the ray tube cross sectional area vanishes. Around caustics, the pressure field is amplified. As caustics are also zones of convergence of rays, diffraction effects, neglected in the geometrical approximation, must be reintroduced to limit the amplitude of the field there. Caustics are classified by the catatstrophe theory (Thom [19]), the simplest caustics being the fold “caustics” (the rainbow). Sonic boom focusing at fold caustics occurs at the ground level during flight acceleration or turns. Turn focusing can be avoided by preventing sharp turns at low Mach numbers, but acceleration focusing cannot be avoided by realistic manoeuvers. In case of shock waves (as for sonic booms), diffraction effects alone are unsufficient to obtain a bounded signal, and local nonlinearities are also essential. This lead Guiraud [10] to derive the mixed-type nonlinear Tricomi equation satisfied locally by the pressure field around the caustic. An approximate analytical solution limited to signals with well separated shocks, was proposed by Gill and Seebass [20]. A numerical procedure has been developed by Auger and Coulouvrat [21]. It was improved by Marchiano *et al.* [22] and applied to realistic estimations of sonic boom focusing coupled to matching with nearfield CFD computations. Either approximate or numerical simulations compare well with test flights (Wanner *et al.*, [23], Downing *et al.*, [24]) though a precise and quantitative validation of Guiraud’s theory by laboratory scale experiments is only very recent (Marchiano *et al.* [25]). Sonic boom focusing remains a critical issue, as it may prevent a supersonic aircraft to accelerate overland, even if designed at cruise speed for an acceptable low boom.

These aspects have been integrated into an advanced software “BANGV” dedicated to the simulation of the sonic boom of a manoeuvering aircraft in a stratified and absorbing atmosphere with wind over an absorbing ground.

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