Characterization of porous materials at audio frequencies.

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Introduction

The acoustic characterization of porous materials saturated by air such as plastic foams, fibrous or granular materials is of great interest for a wide range of industrial applications. These materials are frequently used in the automotive and aeronautics industries and in the building trade. One important parameter which appears in theories of sound propagation in porous materials at audio frequencies is the permeability k_0. The permeability intervenes in the description of the viscous coupling between the fluid and the structure. As such, in studies of acoustical properties of porous materials, it is extremely useful to be able to measure this parameter. The permeability k_0 is related to the flow resistance. The flow resistance of porous material is defined as the ratio between the pressure difference across a sample and the velocity of flow of air through that sample; the flows being considered are steady and nonpulsating. This is quite analogous to the definition of electrical resistance as the ratio between voltage drop and current. The specific flow resistivity σ of a porous material is defined as the flow resistance per unit cube. The relation between permeability (k_0) and specific flow resistivity (σ) is given by: $k_0 = \eta/\sigma$, where η is the fluid viscosity.

Model

In the acoustics of porous materials, one distinguishes two situations according to whether the frame is moving or not. In the first case, the dynamics of the waves due to the coupling between the solid skeleton and the fluid is well described by the Biot theory[1]. In air-saturated porous media the structure is generally motionless and the waves propagate only in the fluid. This case is described by the model of equivalent fluid [2] which is a particular case of the Biot model, in which the interactions between the fluid and the structure are taken into account in two frequency response factors: the dynamic tortuosity of the medium $\alpha(\omega)$ and the dynamic compressibility of the fluid included in the porous material $\beta(\omega)$. In the frequency domain, these factors multiply the density of the fluid and its compressibility respectively and represent the deviation from the behavior of the fluid in free space as the frequency changes. The range of frequencies such that viscous skin thickness $\ell = (2\eta/\omega\rho_f)^{1/2}$ (ω is the angular frequency) is much larger than the radius of the pores r, $\frac{\ell}{r} \gg 1$, is called the low-frequency range. For these frequencies, the viscous forces are important everywhere in the fluid. At the same time, the compression-dilatation cycle in the porous material is slow enough to favor the thermal exchanges between fluid and structure. At the same time, the temperature of the frame is practically unchanged by the passage of the sound wave because of the high value of its specific heat: the frame acts as a thermostat. In this case the isothermal compressibility id directly applicable. In the time domain, the dynamic tortuosity and compressibility of the fluid included in the porous material act as operators and in the viscous domain (low frequency approximation) their expressions are given [2] by

$$\tilde{\alpha}(t) = \frac{\eta\phi}{\rho_f k_0}\partial_t^{-1}, \qquad \tilde{\beta}(t) = \gamma\,\delta(t).$$

In these equations, $\delta(t)$ is the Dirac operator and ∂_t^{-1} is the integral operator $\partial_t^{-1} g(t) = \int_0^t g(t)dt'$, η and ρ_f are, respectively, the fluid viscosity and the fluid density and γ the is the adiabatic constant. The relevant physical parameters of the model are the static permeability $k_0 = \eta/\sigma$, σ the specific flow resistivity. In this framework, the basic equations of our model can be written as

$$\rho_f \tilde{\alpha}(t) * \frac{\partial v_i}{\partial t} = -\nabla_i p, \qquad \frac{\tilde{\beta}(t)}{K_a} * \frac{\partial p}{\partial t} = -\nabla . v,$$

where * denotes the time convolution operation, p is the acoustic pressure, v is the particle velocity and K_a is the bulk modulus of the air. The first equation is the Euler equation, the second one is the constitutive equation. Along the $x-$axis , these equations become :

$$\frac{\eta\phi}{k_0} v(x,t) = -\frac{\partial p(x,t)}{\partial x}, \qquad \frac{\gamma}{K_a}\frac{\partial p(x,t)}{\partial t} = -\frac{\partial v(x,t)}{\partial x}.$$

The Euler equation is reduced to the Darcy's law which expresses the balance between the driving force of the wave and the drag forces $\eta\phi v/k_0$ due to the flow resistance of the material. The fields which are varying in time, the pressure, the acoustic velocity, etc., follow a diffusion equation [2],

$$\frac{\partial^2 p(x,t)}{\partial x^2} - D\frac{\partial p(x,t)}{\partial t} = 0, \qquad D = \frac{\eta\phi\gamma}{k_0 K_a}. \qquad (1)$$

The diffusion constant D is a damping term due to the viscous and thermal effects which take place in the porous material. The amplitudes of reflected and transmitted waves from a slab of porous materials can be determined by the relevant boundary conditions [3].

Acoustic Measurements

Experiments are performed in a guide pipe (60 m) having a diameter of 5 cm. A sound source Driver unit "TOA" constituted by loudspeaker Realistic 40-9000 is used. Pulses are provided by synthesized function generator Standford Research Systems model DS345-30MHz. The signals are amplified (type 2610 Bruel&Kjaer) and filtered (model SR 650-Dual channel filter, Standford Research Systems). The signals are measured using one microphone (Bruel&Kjaer, 4190). The experimental setup is shown in Fig. 1. Consider a cylindrical sample of plastic foam (diameter 5 cm) with the following characteristics: thickness 2.5 cm, porosity $\phi = 0.9$ Fig. 2 shows the experimental incident signal (solid line) and experimental transmitted signal (dashed line). From Fig. 2, we can see that there is no delete between the incident and transmitted signals, the two waves have the same arrival time. This means that there is no propagation in the porous material. The transmitted wave is just attenuated with no significant dispersion comparing to the incident signal, the two signals have the same spectral bandwidth (Fig. 2). These results are in adequacy with the theory developed in the previous section, in which the propagation equation is reduced to a diffusive equation (Eq. 1). Using the experimental data of incident and transmitted waves, we solve the inverse problem using the least square method which minimizes $U(\sigma)$ defined by

$$U(\sigma) = \sum_{i=1}^{i=N} (p^t_{exp}(x, t_i) - p^t(x, t_i))^2,$$

where $p^t_{exp}(x, t_i)_{i=1,2,\ldots N}$ represents the discrete set of values of the experimental transmitted signal and $p^t(x, t_i)_{i=1,2,\ldots N}$ is the discrete set of values of the simulated transmitted signal. Figure 3 shows the variation of the cost function U with the specific flow resistivity. The obtained optimized value of σ is 38000 Nm^{-4}s, it permeability (k_0) can be deduced easily for the value of σ; $k_0 = 500$ Darcy. Fig. 4 shows the comparison between the simulated transmitted signal calculated with the optimized value of σ and experimental transmitted signal. The correspondence between theory and experiment is good, which leads us to conclude that this simple method is well appropriate for estimating the specific flow resistivity (and thus the permeability) of porous material with rigid frame.

References

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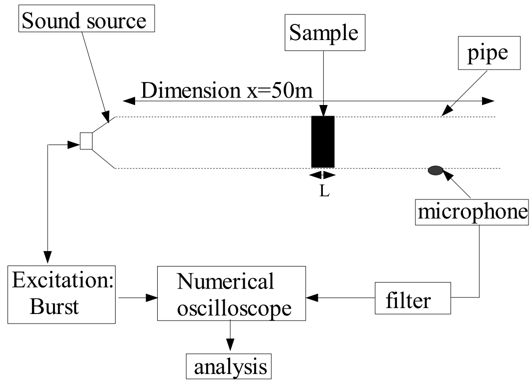

Figure 1: Experimental set-up for ultrasonic measurements.

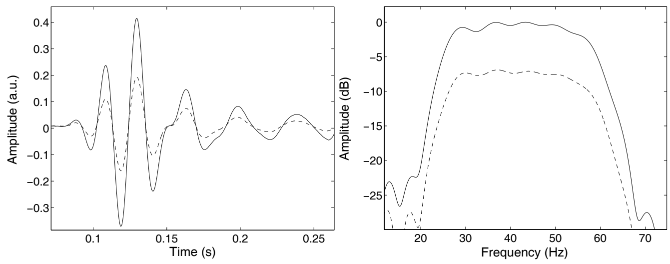

Figure 2: Incident signal (solid line) and transmitted signal (dashed line).

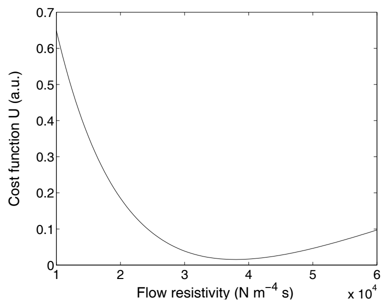

Figure 3: Variation of the cost function U with the specific flow resistivity σ.

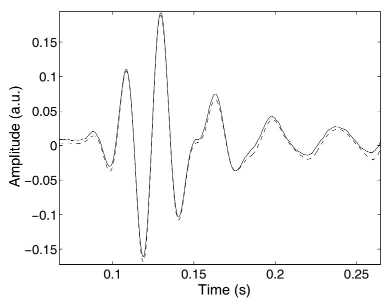

Figure 4: Comparison between simulated transmitted signal (solid line) and experimental transmitted signal (dashed line).