Numerical Simulation of Bounded Acoustic Beams by the Fourier Integral Method

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Introduction

The numerical evaluation of the interaction between a bounded acoustic beam and a multilayered solid structure is of interest to simulate a number of ultrasonic technics in evaluation and defect detection of materials. Often these technics make use of the so-called modal waves (guided waves and interface waves) that may propagate on a long range along the structure. So it is important to have at hand numerical simulation tools that are able to describe the generation of modal waves by an incident bounded beam valid on a long range.

Numerical modelisation of acoustic beams: the classical methods

Besides local approximation methods such as those which use the inhomogeneous plane waves [1] or more recently the profiled plane waves [2], a bounded acoustic beam may be classically described by plane wave superposition. In the monochromatic case, the pressure fields are then obtained in the form of spatial Fourier integrals that represent the exact solution of the interaction problem. Analytical expressions for such integrals are out of reach in general and only asymptotic or numerical evaluations are available [3], [4], [5].

The numerical evaluation of Fourier integrals is usually performed by the FFT algorithm. This procedure requires to introduce a regular discretization of the integration variables and a corresponding set of values of the conjugate variables. The pressure field is thus obtained for a particular set of points in the space. Moreover, the regular discretization results in an artificial periodization of the solution. A direct numerical evaluation of Fourier integrals by classical methods such as Lagrange method avoids to prescribe specific positions of the points in the physical space, but the periodization phenomenon is not overcome.

So, unless the numerical accuracy is considerably increased, this procedure may be irrelevant to simulate long range modal wave generation.

The exponential integration method

The Fourier representations for the pressure fields usually appears as integrals in terms of a wave number variable. The periodization phenomenon is due to the regular sampling of the phase factor in terms of this integration parameter in the Fourier integral. In general, the corresponding exponential factor is running very rapidly in far field evaluations, which needs to increase the sampling accuracy.

The numerical method presented here propose to separate the function of the wave number to be integrated in two factors: an amplitude factor with slow phase variations and an exponential factor that represents the rapid phase running.

A polynomial interpolation of the amplitude function in terms of the wave number may be derived at any order n. Then, if the phase is a linear function of the integration variable, as supposed in equation (1), the Fourier integral may be easily calculated. If the phase function is not linear, as in equation (2), which is usually the case for most of the fields to be evaluated, a linear interpolation allows to compute the Fourier integral as preceedingly. the discrepancy between the phase function and its linear interpolation may be taken into account and introduced in the polynomial interpolation of the amplitude function.

$$p(x) = \int_{-\infty}^{\infty} \hat{A}(k_x)\, e^{i k_x x}\, dk_x \qquad (1)$$

$$p(x,z) = \int_{-\infty}^{\infty} \hat{A}(k_x)\, e^{i\varphi(k_x,x,z)}\, dk_x \qquad (2)$$

In practice, the complete integration interval is divided into Nk intervals for the variable kx in the integrals (1) or (2) and the procedure is applied to each partition interval.

Examples

As an example of the numerical evaluation of integrals of the form (1), we may reconstruct a Tukey profile of an emitter from its spectrum. Figures (1) and (2) show the difference between the classical complete discretization method and a method of exponential integration with interpolation of order 5.

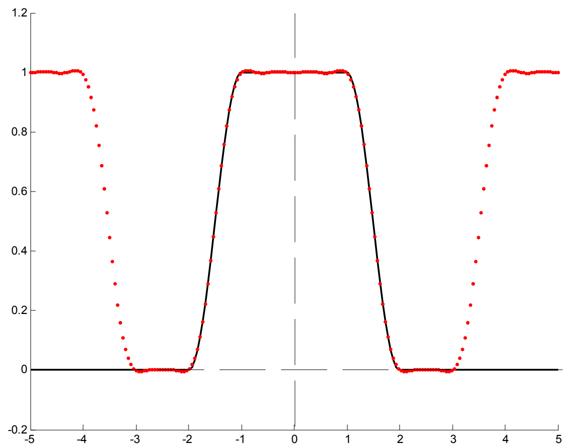

FIG.1 : Reconstruction of a Tukey profile: the classical discretization method

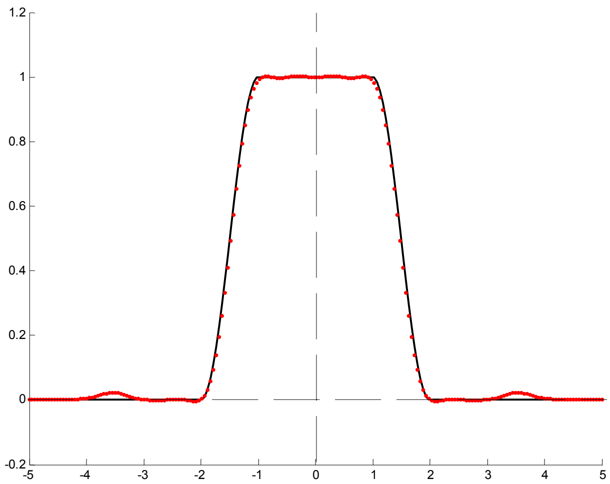

FIG.2 : Reconstruction of a Tukey profile:
exponential integration of order 5

As an example of the numerical evaluation of integrals of the form (2), we may compute the pressure field on an interface produced by a gaussian beam under some incidence angle (the integration is performed along the emitter plane). Again, figures (3) and (4) show the difference between the classical complete discretization method and a method of exponential integration with interpolation of order 3 here.

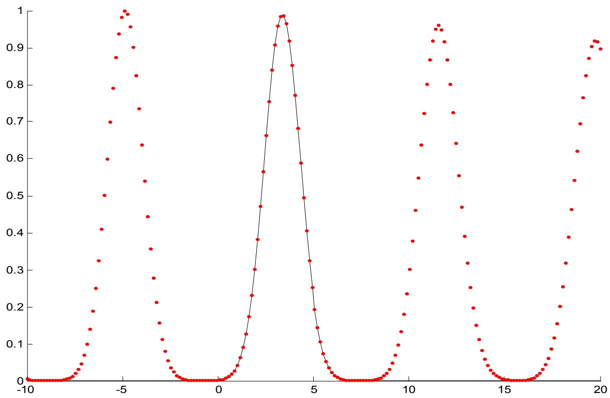

FIG.3 : Pressure field on an interface
the classical discretization method

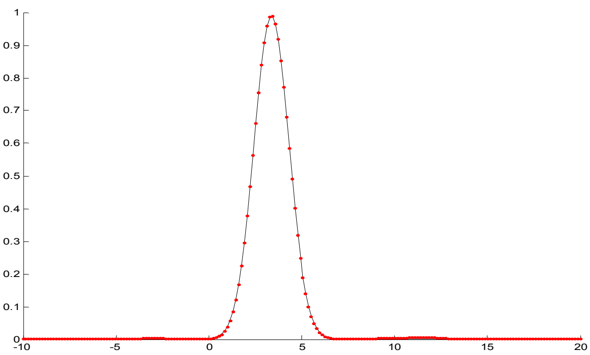

FIG.4 : Pressure field on an interface
exponential integration of order 3

The third example on figure (5) is relative to the application of the method of exponential integration (here of order 1) to the reconstruction of a three-dimensional gaussian beam. The three dimensional cross section presents only some extra protuberances that do not exist for the exact gaussian profile precisely where the classical complete discretization method would give full patterns of periodization.

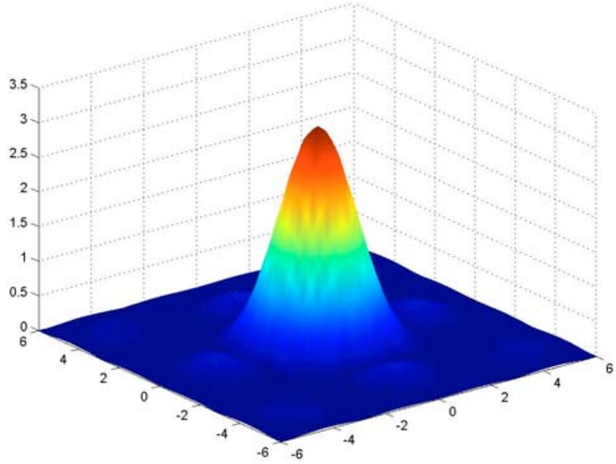

FIG.5: Reconstruction of a 3-D gaussian beam by the
exponential integration method of order 1

Conclusion

We have proposed an exponential integration method with interpolation of any order that allows to compute field representations in Fourier integral form without periodization of the field. Those technics may be usefull for the simulation of long range generation of modal wave in a layered structure by a bounded acoustic beam.

Bibliography

[1] J.M. Claeys, O. Leroy, "Reflection and transmission of bounded sound beams on half-spaces abd through plates", J. Acoust. Soc. Am. 72(2), 585-590, 1982

[2] N.F. Declercq, J. Degrieck, O. Leroy, "Profiled plane waves as the framework for the interaction of 3D bounded beams with anisotropic materials", WCU 2003, Paris, september 7-10, 2003

[3] M. Rousseau, Ph. Gatignol, "Short wave analysis for the reflection of bounded acoustic beams onto liquid-solid interfaces at the Rayleigh incidence", J. Acoust. Soc. Am. 78(5), 1859-1867, 1985

[4] M. Rousseau, Ph. Gatignol, "Asymptotic analysis of the non-specular effects for the reflection and the transmission of a Gaussian acoustic beam incident on a solid plate", J. Acoust. Soc. Am. 80(1), 325-332, 1986

[5] D.E. Chimenti, J.G.Zhang, S. Zeroug, L.B. Felsen, "Interaction of acoustic beams with fluid loaded elastic structures", J. Acoust. Soc. Am. 95(1), 45-59, 1994