Numerical Description of the Generation of Modal Waves by Bounded Acoustic Beam

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Introduction

The propagation of modal waves in materials is of great interest for characterization and defect detection in multilayered structures. Surface waves, such as Rayleigh or Stoneley waves, are well suited to inspect the behavior of material interfaces [1]. Lamb waves are used with success to detect defects in plates [2] and to control pipes for gas or petroleum.

A number of technics exist to generate such modal waves. Lamb waves may be excited by interdigital transducers at the surface of piezoelectric crystals or on piezoelectric films deposited on a nonpiezoelectric substrate.[3]. In non destructive testing devices, these modes may be generated by 'liquid wedge trensducers'[4], or by 'air-coupled ultrasonic transducers'[5].On a theoretical poin of view, several authors have studied the generation of surface or guided wave; one may mention Victorov [6], Miklowitz [7], Achenbach [8]]

The aim of this paper is to present a numerical simulation tool in order to describe the generation of modal waves by a bounded acoustic beam incident on a layered structure. Cartographies and cross sections for stress fields and displacements are obtained which show the generation of (generalized) Lamb waves in a plate immersed in a fluid, as well as the Rayleigh waves along the fluid/solid interfaces and the Stoneley wave for the case of a solid/solid interface.

The case of Lamb waves

A monochromatic bounded beam, described by a field variable φ_i, is incident on a solid elastic plate (s) with thickness h, immersed in a fluid (f). If θ denotes the incidence angle, the wave number k_x in the direction x of the layer is given by

$$k_x = K_0 \sin \theta \qquad (1.1)$$

where K_0 is the wave number in the fluid (f) at the given frequency.

Strictly speaking, the Lamb waves exist for a plate in vacuum. Writing that the normal stress is zero on each free surface $(T_{zz} = T_{zx} = 0$ for $z = 0, h)$ one obtains the dispersion relation which splits in two equations for the wave number k_x of the symmetric 'S' and antisymmetric 'A' modes. Now, in the case of the immersed plate, if we choose a pair (K_0, k_x) for the incident beam such as the representative point be situated on a Lamb dispersion branch ('S' or 'A'), one may expect that the corresponding mode will be generated in the plate.

The acoustic fields in (s) and (f) are calculated by using a plane wave decomposition and a convenient procedure to evaluate numerically the corresponding Fourier integrals. Figure (1) shows a cartography in the physical plane for the stress field in the case of the mode S1.
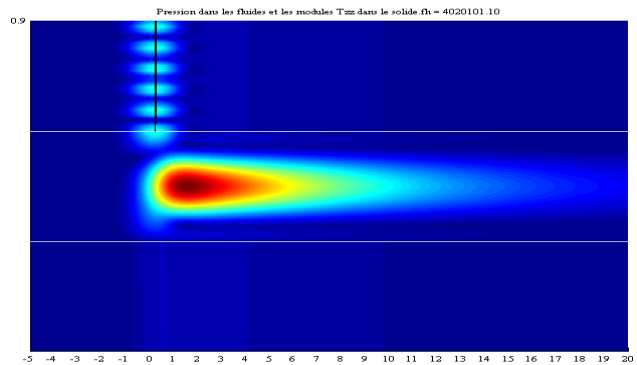

FIG 1 : Lamb wave generation (mode S1) by a bounded beam (cartography of normal stress)

The case of interface waves

Rayleigh waves

Again for the solid plate, but at high frequency, the modes Ao and So join together to give the Rayleigh surface wave. For such a choice of the pair (K_0, k_x), one may expect to generate the (generalized) Rayleigh wave along the incident interface $(f)/(s)$. This is clearly visible on the cartography of figure (2). Actually, the cross sections for stress components given on figure (3) show that a second Rayleigh wave is generated along the second interface $(f)/(s)$, which is an interesting result for control perspectives.
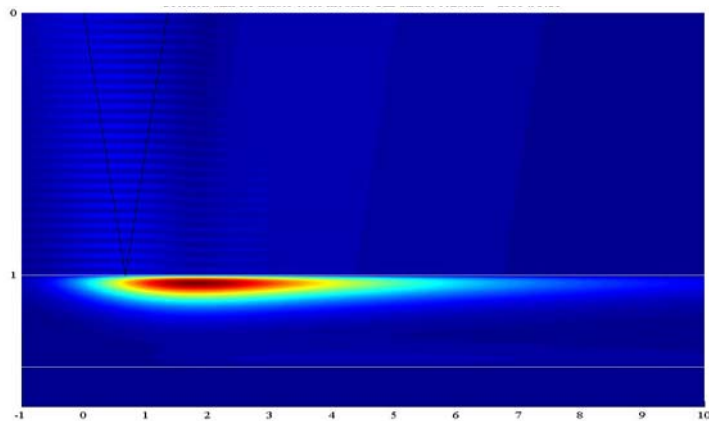

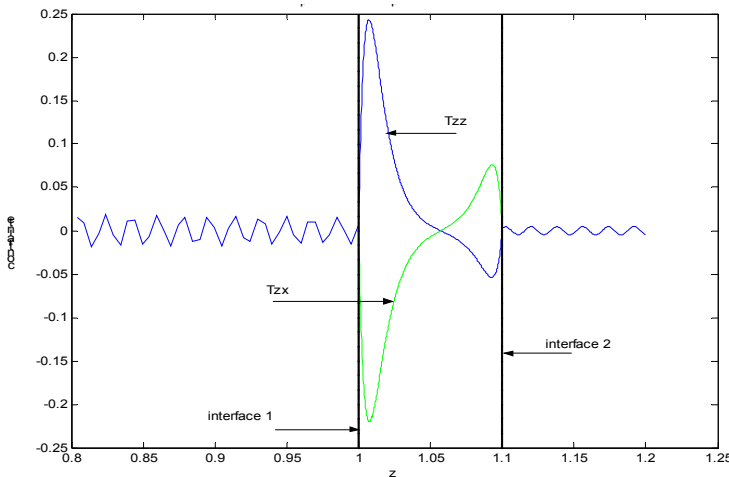FIG 2 : Rayleigh wave generation by a bounded beam (cartography of normal stress)

FIG 3: Real parts of stress components showing the generation of two Rayleigh waves by the bounded beam

Stoneley wave.

This last result encourages to study the case of a fluid (f)/ solid (s_1)/ solid (s_2) structure with perfect bonding conditions along the $(s_1)/(s_2)$ interface. When the two solids are semi-infinite and their physical properties are conveniently choosen, it is known that a Stoneley wave may propagate along the interface.

On figure (4) we draw the cross sections of the displacement components for the exact Stoneley wave and for the case of the structure $(f)/(s_1)/(s_2)$ excited by a bounded beam.

The generation of a Stoneley wave along the $(s_1)/(s_2)$ interface is obviously obtained (here in the case of physical parameters for a tungsten/aluminium interface).
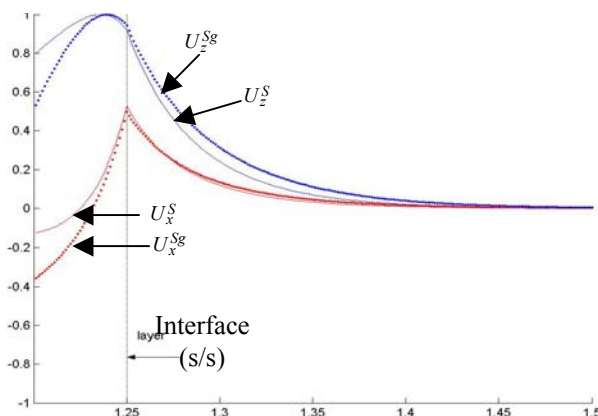

FIG 4: Generation of a Stoneley wave by a bounded beam
- continuous line : exact Stoneley wave
- dotted line : generalized Stoneley wave

Conclusion

By a numerical procedure based on the Fourier integral representations of the acoustic fields, it is possible to simulate the generation of modal waves by a bounded acoustic beam in a layered structure. Cartographies for the stress or displacement components may be obtained. The generation of the desired modal wave is clearly proved by drawing the cross section for these variables accross the structure. This numerical simulation tool may be used to study the perturbation induced by a finite defect on an interface of the structure.

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