Including Anisotropic Turbulence into Outdoor Sound Propagation

Bernhard Schwarz-Röhr

Universität Oldenburg, D26111 Oldenburg, Germany, Email: bernhard@aku.physik.uni-oldenburg.de

Introduction

Experiments were performed to study the decay of the acoustic coherence due to turbulent fluctuations in the atmosphere. The experiment took place at clear sky and an average wind of 3.9 m/s on a sports lawn. A horn loudspeaker at a height of 5 m was driven by bandwidth limited (approximately 1..10 kHz) pulses. At a distance of 106 m in downwind direction the acoustic coherence was determined by a lateral array of microphones (height 5 m). Time windowing was used to minimize ground reflections. Temperature and velocity fluctuations were monitored by a hot wire probe and an ultrasonic anemometer located at the filled microphone in Figure 1.

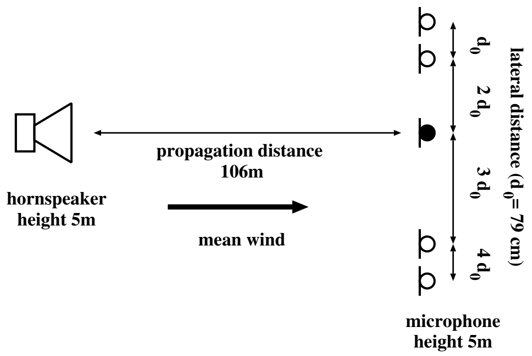

Figure 1: Sketch of the setup.

Velocity data

Temperature fluctuation were neglegible for sound propagation in this experiment. The ultrasonic anemometer sampled the three velocity components at a rate of 1.03 Hz for two hours. From the time series the one-dimensional spatial power spectra of the velocity fluctuations along the mean wind vector were determined. Introducing co-ordinates with the x-axis aligned with the mean wind, the y-axis in the horizontal plane and the z-axis oriented vertically, Figure 2 shows the resulting spectra $F_r(k)$, $F_y(k)$ and $F_z(k)$ for the velocity components v_x, v_y, v_z respectively. For large values of k all spectra show a $k^{-5/3}$ behaviour, indicating an inertial subrange (ISR). But it appears that this ISR is not isotropic, since for isotropic, incompressible turbulence the condition

$$F_y(k) = F_z(k) = \frac{4}{3}F_x(k)$$

should hold which is not met by the data.

By fitting the appropriate one-dimensional von Kármán -spectrum to $F_r(k)$, the parameters $\hat{C}_v^2 = 0.52\ \mathrm{m/s^2\ m^{-2/3}}$ and $\hat{k}_0$= 0.03 m^{-1} were obtained. It should be noted that since $F_r(k)$ and $F_y(k)$ coincide in Figure 2, the transverse spectrum $F_y(k)$ is well fitted by the radial spectrum with the same parameters.

In Figure 3 the transverse spectra for isotropic von Kármán turbulence are fitted to $F_y(k)$ and $F_z(k)$. In order to model the behaviour in the ISR, the structure constants had to be adjusted by a factor 3/4 for both transverse spectra (Figure 3). Since the height of the sensor (5 m) is much smaller than $\hat{k}_0^{-1}$, the outer scale for the vertical component is smaller than that for the horizontal components, a value of $k_0 = 2{\cdot}\hat{k}_0$ for $F_z(k)$ was found. For $F_y(k)$ a value of $k_0 = 1/1.7 \cdot \hat{k}_0$ was obtained.

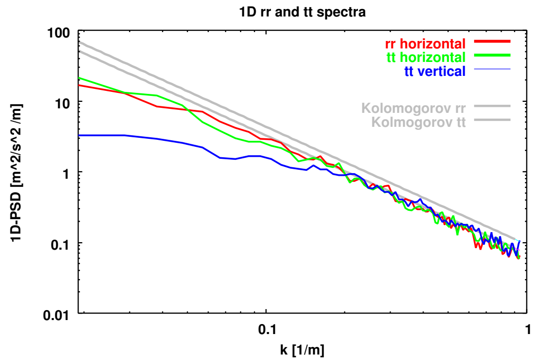

Figure 2: One-dimensional spectra. At high wave numbers the spectra show a -5/3 behaviour (Kolmogorov rr). But the transverse spectra are not shifted by a factor of 4/3 (Kolmogorov tt).

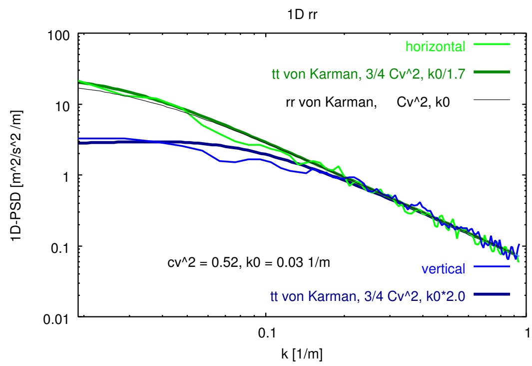

Figure 3: Fits of the one-dimensional von Kármán spectrum functions to $F_y(k)$ and $F_z(k)$. The structure constant had to be adjusted by a factor of 3/4. Both spectra have different values for the outer scale.

Acoustic coherence

The acoustic coherence

$$\gamma^2(\rho) = \frac{|\langle p_1(\omega)p_2^*(\omega)\rangle|^2}{\langle p_1(\omega)p_1^*(\omega)\rangle\,\langle p_2(\omega)p_2^*(\omega)\rangle}$$

was determined, here $p_1(\omega)$ and $p_2(\omega)$ denote the spectra measured at two microphones at a lateral distance ρ, ω is the angular frequency, $\langle\rangle$ denotes averaging. By choosing appropriate pairs of microphones, the coherence could be determined for $\rho = d_0, 2d_0, ...8d_0$, where d_0=79 cm is the distance between the two outer microphone pairs (Figure 1).

In Figure 4 the measured coherence is shown as a function of frequency. For lateral distances greater than 4 d_0 the coherence was less than 0.1. The decay of coherence at low frequencies is due to the band-passed signal source.

The well-know Parabolic Equation Method (PEM) was used to predict the coherence. This theory in principle includes anisotropic turbulence, as can be seen from the general solution for the mutual coherence function given for instance in [1]. Rewritten to the coherence γ^2 this solution reads for homogeneous turbulence

$$\gamma^2(x, \vec{\rho}) = \exp\left(-\frac{\omega^2}{4c_0^2} x \int_0^1 d\beta\ [A(0) - A(\beta\ \vec{\rho})]\right) \quad (1)$$

here $\vec{\rho}$ is the separation vector between the two microphones and x the propagation distance. $A(\vec{\rho})$ is given by

$$A(\rho) = \int_{-\infty}^{\infty} dx\ B_\epsilon(x, \vec{\rho}) \quad (2)$$

where B_ϵ is the correlation function of the fluctuations of the effective index of refraction for a separation vector $(x, \vec{\rho})$. Note that only the correlation function in the plane of the microphones is required in this equation.

The PEM predicts a decay of the coherence proportional to $\exp(-\omega^2 \ldots)$. This prediction was tested by fitting straight lines to a plot of $5 \log 10(\gamma^2)$ vs. ω^2 in Figure 5. The small deviations from the straight line are probably due to reflections between the microphones.

Inserting an isotropic von Kármán spectrum into equations 1 and 2 leads to [2]

$$\gamma^2(x, \rho) = \exp\left(-\gamma_v\ \frac{4x}{k_0\rho}\ R(k_0\rho)\right) \quad (3)$$

Here γ_v is a constant depending on C_v^2, k_0 and ω^2. $R(k_0\rho)$ is a function of $k_0\rho$ only. The coherence obtained from this equation is shown in Figure 4. While equation 3 predicts the measurements for small values of the lateral distance ρ fairly good, the calculated coherence for large values of ρ is much greater than the observed one. A more sensitive test is obtained from equation 3 by calculating the ratio

$$\ln(\gamma^2(\rho_1))\ /\ \ln(\gamma^2(\rho_2)) = R(k_0\rho_1)\ /\ R(k_0\rho_2). \quad (4)$$

From this equation an apparent value of k_0 can be determined uniquely for each pair of measured coherences. Inserting the slopes of the fits in Figure 5 into equation 4, values ranging from 0.12 m^{-1} to 0.001 m^{-1} were obtained for the apparent k_0. The deviation from $\hat{k}_0$ increases with ρ_1 and ρ_2. The huge variation indicates that the decay of coherence is not properly described by the isotropic theory.

Calculating $A(\rho)$ from equation 2 from measured turbulence data requires the knowledge of B_ϵ in the whole (x, ρ)-plane, which is not available from our measurement. Since the radial and transverse velocity spectra are both well fitted by the radial spectrum function it was assumed that $B_\epsilon(x, \rho)$ in equation 2 depends on the magnitude $\sqrt{x^2 + \rho^2}$ only. The predictions of these modell are even worse than that of the isotropic theory.

Conclusion

In this paper radial and transverse spectra of the velocity fluctuations were calculated directly from anemometer data. These data show an anisotropic turbulence. Surprisingly these data indicate that even in the inertial subrange the well known relationship between radial and transverse spectra does not hold.

The parabolic equation method was used to predict the decay of acoustic coherence. The basic property of PEM solution, the $\exp(-\omega^2 \ldots)$-decay of the coherence was confirmed by our data. The solutions for isotropic turbulence were not in very good agreement with the measured data.

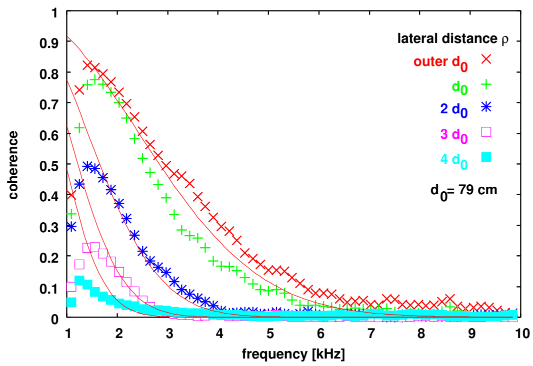

Figure 4: Measured acoustic coherence for different lateral distances ρ. The solid lines are the theoretical values for isotropic turbulence using the parameters from the wind measurement.

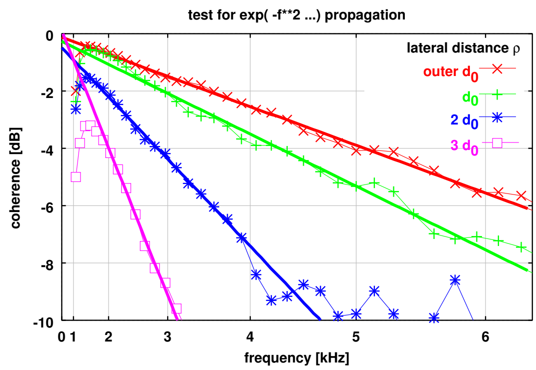

Figure 5: Measured acoustic coherence vs. ω^2 and straight line fits.

References

[1] Akira Ishimaru: Wave propagation and Scattering in Random Media. Academic Press, New York 1978

[2] V. E. Ostashev: Acoustics in Moving Inhomogeneous Media. E& FN Spon, London, 1997