A Journey of Sound Radiation, from the Baffled Plate to the Infinitely Long Cylinder

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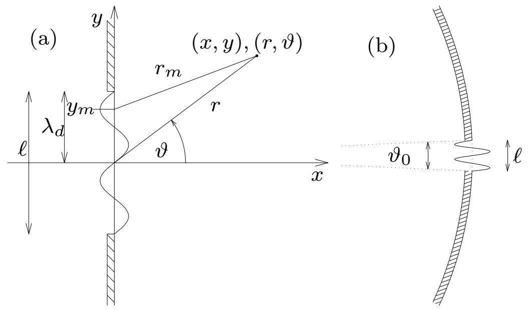

Figure 1: Baffled plate and "infinite" cylinder, coordinate system and variables

Introduction

In this paper, the influence of added curvature, on the sound radiation will be investigated in two dimensions ($\frac{\partial}{\partial z} = 0$), by comparing the radiation of a baffled plate and a cylinder with various radii.

Baffled Plate (BP)

The "baffled plate" is a plate of height ℓ built into a infinite rigid wall (see Fig. 1a). The plate is allowed to vibrate in a numerous variety of modes, whose velocity distribution can be described by a sum of orthogonal sine and cosine modes:

$$\frac{v(y)}{v_0} = \begin{cases} {}^{\cos}_{\sin}(k_d y - d\frac{\pi}{2}), & |y| \le \frac{\ell}{2} \\ 0, & \text{otherwise} \end{cases}, \quad \text{with } k_d = \frac{d\pi}{\ell},$$

where d is the order of the mode, and v_0 the peak value of velocity. The sine-modes have nodes at the edges, whereas the cosine-modes have anti-modes or loops, each leading to different radiation characteristics. The radiated pressure, in general and in the far-field, is the weighted sum of all monopole "sources" $H_0^{(2)}(k_0 r_m)$ on the plate[1]:

$$p(x, y) = \frac{1}{2}\rho c k_0 \int_{-\infty}^{\infty} v(y_m) H_0^{(2)}(k_0 r_m) dy_m, \qquad (1)$$

$$p_{\text{far}}(r, \vartheta) = \rho c \sqrt{\frac{k_0}{2\pi r}} e^{-j(k_0 r - \frac{\pi}{4})} V(k_y = -k_0 \sin\vartheta).$$

The law of cosine was used to approximate r_m (see Fig. 1) by $r - y_m \cos(\frac{\pi}{2} - \vartheta)$ in the exponential term of the asymptotic expansion of the Hankel-function[2] $H_0^{(2)}(k_0 r_m) \approx \sqrt{\frac{2}{k_0 r_m \pi}} e^{-j(k_0 r_m - \pi/4)}$, and the radius r_m was replaced by r in the non-phase dependent term, leaving the far-pressure proportional to the Fourier transform of the plate velocity $V(k_y = -k_0 \sin\vartheta)$ in y-direction.

The directivity pattern (DP), $|D(\vartheta)|^2$, is the angle dependent part of the Intensity ($I_{\text{far}} = \frac{1}{2\rho c}|p_{\text{far}}|^2$) in the far-field:

$$D(\vartheta) = e^{j\alpha_0} e^{-jd\frac{\pi}{2}} \operatorname{sinc}\left((\frac{\lambda_d}{\lambda_0} \sin\vartheta + 1) d\frac{\pi}{2} \right) \qquad (2)$$
$$+ \; e^{-j\alpha_0} e^{jd\frac{\pi}{2}} \operatorname{sinc}\left((\frac{\lambda_d}{\lambda_0} \sin\vartheta - 1) d\frac{\pi}{2} \right),$$

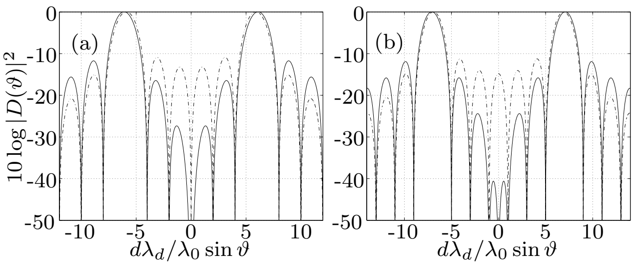

Figure 2: DP in dB over $d\lambda_d/\lambda_0 \sin\vartheta$ with $\lambda_d/\lambda_0 = 2$ of cosine- (-) and sine-mode (- ·) of order: a) $d = 6$, b) $d = 7$.

with $\alpha_0 = 0$ for cosine-modes and $\alpha_0 = -\frac{\pi}{2}$ for sine-modes. In Fig. 2 the DP, governed by the two sinc-functions, is displayed over $d\lambda_d/\lambda_0 \sin\vartheta$, for cosine- and sine-modes of even and odd order $d = 6, 7$. The maxima lie at $\pm d$, which expressed in angles are $\sin\vartheta_{1,2} = \lambda_0/\lambda_d$ ($\vartheta_{1,2} = \pm 30°$). This depiction also shows that for low frequencies ($\lambda_d \ll \lambda_0$) the DP has dipole characteristics (imagine mirror image) for all mode-cases, except in the case of the odd sine-mode, for which it has monopole characteristics.

The DP can be explained physically by investigating the phase of the sources. As depicted in Fig. 3, all points on surface ($\overline{AC}$), which is perpendicular to the angle of investigation ϑ, have the same distance to a point in the far-field at this angle. This means that the pressure in the far-field at ϑ, can readily be calculated, if the phase between the "virtual" sources on $\overline{AC}$ is known, by summing up the influence of all these sources as if in one point.

Being that all lines leading to a point in the far-field are parallel, each virtual source on the stretch $\overline{AC}$ corresponds to one source on the plate $\overline{AB}$, having firstly the phase shift α, relative to point A, caused by the modal excitation (local phase shift), and secondly the phase shift β (see Fig. 3a)), due to the distance the wave travelled in the fluid (chronological phase shift). The total phase $\gamma = \alpha + \beta$, where the phase $\alpha = k_d y$ and $\beta = k_0 \chi$, with $k_d = d\pi/\ell$ and $\chi = y \sin\vartheta$. The significant part of the total normalized pressure in the far-field at ϑ is then:

$$\int_{-\frac{\ell}{2}}^{\frac{\ell}{2}} e^{-j\gamma} dy = \int_{-\frac{\ell}{2}}^{\frac{\ell}{2}} e^{-jky} dy, \quad \text{with } k = k_d + k_0 \sin\vartheta$$
$$\sim \operatorname{sinc}\left((\frac{\lambda_d}{\lambda_0} \sin\vartheta - 1) d\frac{\pi}{2} \right). \qquad (3)$$

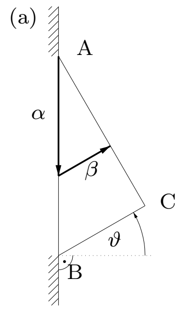

Figure 3: Plate and fluid-wave vibration with $\lambda_d/\lambda_0 = 2$ and $d = 7$; b) $s = 7$, c) $s = 5$

Taking into account the second wave on the plate, propagating in opposite direction, leads to the second sinc-function as seen in Eq. 2. Depicted in Fig. 3b) is the case of the main peak of DP, where the local phase α and chronological phase β cancel each other out, making all virtual sources on $\overline{AC}$ in phase. The total number of half wavelengths $\lambda_d/2$ on the plate $d = 7$ and the number of half fluid wavelengths $\lambda_0/2$ over $\overline{BC}$ $s = 7$. A minimum of DP (Fig. 3c)) occurs, e.g., when $d = 7$ and $s = 5$, or whenever $|d-s|$ is a multiple of 2π, yet not zero, which leads to the maximum; then all virtual sources on $\overline{AC}$ have a counterpart with opposite phase, which cancel each other out.

Analytical Cylinder (AC)

The normal velocity on the cylinder as seen in Fig. 1b), is set up of the same modes as on the baffled plate, yet angle dependent,

$$\frac{v_d}{v_0} = \begin{matrix}\cos\\ \sin\end{matrix}\left(d\frac{\pi}{\vartheta_0}\vartheta - d\frac{\pi}{2}\right), \quad |\vartheta| \leq \frac{\vartheta_0}{2}. \tag{4}$$

The Ansatz for pressure and velocity is the following,

$$\begin{aligned}\frac{p}{\rho c v_0} &= \sum_g a_g^c H_g^{(2)}(kr)\cos(g\vartheta) + a_g^s H_g^{(2)}(kr)\sin(g\vartheta),\\ v_n &= \frac{j}{\omega\rho}\hat{n}\cdot\nabla p,\end{aligned} \tag{5}$$

where $H_g^{(2)}(kr)\cos(g\vartheta)$ is a multipole of order g. The coefficients can be obtained analytically by exploiting orthogonality of the functions[3], leading to the directivity pattern given as

$$D(\vartheta) = \left|\sum_g j^g\left[a_g^c\cos(g\vartheta) + a_g^s\sin(g\vartheta)\right]\right|^2. \tag{6}$$

Fig. 4 shows the DP at a low frequency ($\lambda_d = 0.03\lambda_0$), for cosine-modes (upper row), sine-modes (lower row), of even order (left column), and odd order (right column), of the BP and AC for different radii. In all four mode cases the radiation towards the rear increases very quickly for decreasing cylinder size. Diffraction toward the rear is a logical consequence of a cylinder of small radius, especially of rigid surface. The DP of the even cosine-mode reacts the most to the reduction of size, changing from a dipole characteristic to a monopole characteristic.

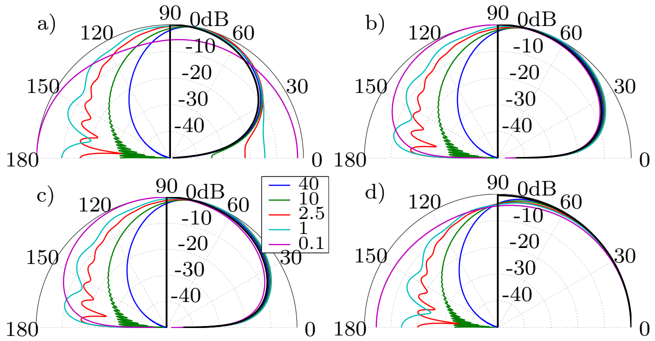

Figure 4: Comparison of DP(ϑ) calculated for infinite cylinder or rather BP (thick, black curve) and finite cylinder with radius $b = 40, 10, 2.5, 1, 0.1\lambda_0$ at $\lambda_d = 0.03\lambda_0$ for: a) cosine-mode $d = 6$, b) cosine-mode $d = 7$, c) sine-mode $d = 6$, d) sine-mode $d = 7$

For all sizes, the DP is zero at $\vartheta = 0$ for the cases of odd cosine- and even sine-mode, because the phase of the sources on the cylinder is asymmetrical to the origin ($\gamma(y) = -\gamma(-y)$), and therefore cancel each other out over the whole x-axis. In the case of the even cosine-mode this cancellation only takes place in the far field for the infinite cylinder, because then, all points of the vibrating surface have the same distance to the field point, and the sources half of a wavelength apart can cancel each other out. However by reducing the size of the cylinder, curvature is added, also adding a new phase between the sources. The phase distribution of the virtual sources on a stretch perpendicular to the observed direction of radiation $\vartheta = 0$, is no more linearly distributed, but distorted by the propagation through the fluid (chronological phase shift).

Although the DP changes extremely through reduction of the cylinder radius, while keeping λ_d/λ_0 constant, the modal radiation efficiency (RE), which is the ratio of radiated power from the plate or cylinder mode, to the power radiated from a section (length ℓ), of an infinitely long piston, which has the same mean square velocity, hardly changes. Even at a radius of $r = \lambda_0$, the energy is only redirected.

References

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[3] B. Zeitler and M. Möser. Sound radiation of a discontinuous structure calculated with various semi-analytical and numerical methods. In *Euronoise*, 2003.