Radiation resistance of a transducer: application to transducer calibration

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Introduction

To date, little attention was paid to experimental measurements of the acoustical radiation resistance. However this property appears very appropriate for the determination of the noise factor or the efficiency of a piezo-electric transducer. This concept is also suitable to easily make an absolute self-calibration of hydrophone or immersion transducer in pulse echo mode.
In this paper, we propose a simple method to determine the reciprocity relationship [1], based on power and thermal noise consideration in a piezoelectric transducer. We introduce the equivalent radiation resistance, which is the link between the emission and the reception mode. For the first time, the calculations were performed for modulus and the phase of the signal.

Theory

Let us consider a piezoelectric transducer, with it front face loaded by a propagating medium. The electric impedance can be split up into the reactance, *X(f),* the imaginary part of the electric impedance as a function of the frequency, the acoustic radiation resistance, $R_a(f)$, and the loss resistance, $R_{loss}(f)$. These latter's are related respectively to the power dissipated in the propagating medium and into the transducer components by losses (figure 1).

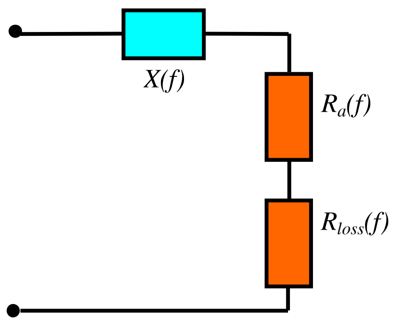

Figure 1: equivalent serial scheme of the input electrical impedance of piezoelectric transducer.

This radiation resistance can be expressed as function of the transducer characteristics, it is used to determined the pulse/echo sensitivity of the transducer

Emission mode

For a transducer in emission mode, when it is excited by a harmonic voltage, $V_{ex}(f)$, the average power dissipated, $\langle P \rangle$, in $R_a(f)$ is equal to that dissipated in the propagating medium equation (1):

$$\langle P \rangle = \frac{1}{2}.\left|V_{ex}(f)\right|.\frac{R_a(f)}{\left|Z_{elec}(f)\right|^2} = \frac{1}{2}.\frac{S.\left|P_{out}(f)\right|^2}{Z_m} \quad (1)$$

Where $Z_{elec}(f)$ is the electrical impedance, Z_m is the propagating medium acoustic impedance, $P_{out}(f)$ is the pressure of the acoustic wave, S is the surface of the transducer, and f is the frequency. The modulus of the emission transfer function, $H_t(f)$ is then equation (2):

$$\left|H_t(f)\right| = \left|\frac{P_{out}(f)}{V_{ex}(f)}\right| = \sqrt{\frac{Z_m}{S}}.\frac{\sqrt{R_a(f)}}{\left|Z_{elec}(f)\right|} \quad (2)$$

Reception mode

For a transducer in reception mode, on the front face of the transducer, the thermal noise from the propagating medium produce a rms free field noise pressure, P_{noise} (Pascal per $Hz^{1/2}$) [2, 3]. This acoustic noise produces, on the electrical port, a rms noise voltage V_{noise} (Volt per $Hz^{1/2}$). The modulus of the noise reception transfer function thus express as equation (3):

$$\left|\frac{V_{noise}}{P_{noise}}\right| = \frac{\sqrt{4.k.T.R_a(f)}}{1/2.\sqrt{4.k.T.Z_m/S}} = 2.\sqrt{\frac{S}{Z_m}}.\sqrt{R_a(f)} \quad (3)$$

Where k is the Boltzmann constant, and T is the temperature in Kelvin. This reception transfer function, the origin of which is related to the thermal in the propagating medium, is the same as for the determined signal, $\left|H_r(f)\right|$. Combining the two previous equations, the radiation resistance can be calculated as function of the pulse echo transfer function $H_{tr}(f)$ equation (4).

$$R_a(f) = 1/2.\left|H_{tr}(f)\right|.\left|Z_{elec}(f)\right| \quad (4)$$

Where $\left|H_{tr}(f)\right| = \left|H_r(f)\right|.\left|H_t(f)\right|$
And the relation between the emission and the reception transfer function equation (5):

$$\left|H_r(f)\right| = \frac{2.S.\left|Z_{elec}(f)\right|}{Z_m}.\left|H_t(f)\right| \quad (5)$$

Phase consideration

A piezoelectric transducer consisting in a piezoelectric plate glued on a backing material with one matching layer can be modelled using one-dimensional model such as the Mason model [4]. The piezoelectric plate is modelled as an electrical port connected to mechanical propagation line, the matching layer is modelled as a propagation line and the backing and the front medium are terminating loads. The transducer is then equivalent to an electrical and acoustical lumped parameter network. The superposition theorem for the complex electric and acoustic amplitude is written for the phase equation (6):

$$\arg[H_t(f)] + \arg[Z_{elec}(f)] = \arg[H_r(f)] \qquad (6)$$

Where arg is the argument of a complex number.
We can thus rewrite the equations (5) in modulus and phase equation (7).

$$Z_a(f) = 1/2.H_{tr}(f).Z_{elec}(f) \qquad (7)$$

And equation (4) becomes equation (8)

$$H_r(f) = \frac{2.S.Z_{elec}(f)}{Z_m}.H_t(f) \qquad (8)$$

Where $Z_a(f)$ is now the electrical acoustic radiation impedance of the transducer.

Experiment

In order to confirm these theoretical results, experiments were carried out on a 5 MHz commercial immersion transducer (Technisonic ILD-0506-HR) excited by an Accutron 1035 pulser receiver. The loop gain, $H_{tr}(f)$, was measured by making a pulse-echo on an iron target. Diffraction and 50 Ω electric environment corrections were done. The predicted displacement (frequency domain) on the transducer front face is given by equation (9):

$$U_{out} = \frac{V_{ex}}{i\omega}.\sqrt{\frac{1}{2.S.Z_m}}.\sqrt{\frac{H_{tr}}{Z_{elec}}} \qquad (9)$$

Where V_{ex} is the exciting voltage (frequency domain).
This result was compared to direct measurements of the front face of the transducer by the mean of a laser heterodyne interferometer. Figure 2 and 3 show the modulus and the phase of the mechanical displacement on the front face of the transducer as function of frequency by the two methods. In the case of the laser measurement, a very homogeneous displacement field was measurement along the radius of the transducer. The curves presented in figure 2 and 3 are average displacement curves.
Excepted for low frequency, the agreement between the two measurement methods is very good on both modulus and phase. At 4 MHz both method give a displacement of 7 nm on the front face of the transducer. In low frequency, the division by the pulsation can explain this discrepancy.

Conclusion

Thanks to noise analyse and phase consideration we have determined an explicit relation between the emission and reception transfer function in modulus and phase. Comparison with laser measurements of the absolute displacement gives similar results. In consequence the method proposed here can be used to carry out experiment where absolute calibration of transducer are required.

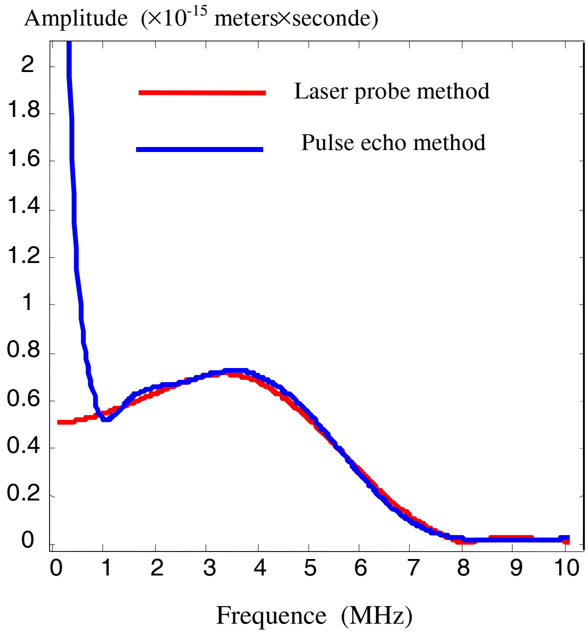

Figure 2: modulus of the displacement spectrum of the front face of the transducer. The two methods of determination are represented.

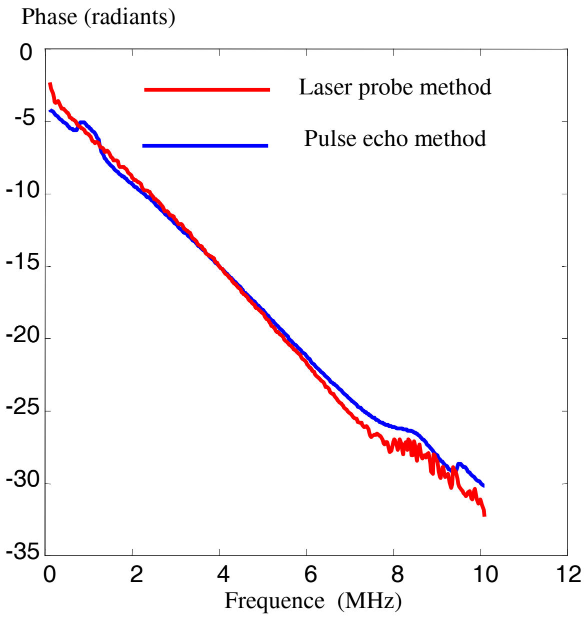

Figure 2: phase of the displacement spectrum of the front face of the transducer. The two methods of determination are represented.

References

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