# The Bouncing of Balls: Influences of Spectral and Temporal Variations on Perception 

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## Introduction

Sound radiated from vibrating objects contains information about the mechanics of the objects. In this psychomechanical study we investigate the temporal and spectral information used by listeners in the perception of bouncing events. We will analyse the bouncing events and present two perception experiments.

In order to generate bouncing sounds, we dropped metal balls with a diameter of 10 to 20 mm on a plate made of MDF, which consists of wood particles pressed and glued together. The plates had surfaces of $50 \times 120 \mathrm{~cm}$ and thicknesses of 6,12 or 18 mm .

When a ball is released from some height above the plate, it will reach the plate at some speed, say $v_{i n}$. After the contact it will have a lower speed $v_{\text {out }}$ in the opposite direction. We will call the fraction $v_{\text {out }} / v_{\text {in }}$ the restitution coefficient. A high restitution coefficient, close to one, means that the ball will keep on bouncing for a long time. In the case of a low restitution coefficient, close to zero, it will come to rest on the plate after a few bounces.

## Analysis

To investigate the restitution coefficient, we look at the the ball and plate are in contact. When a ball is pressed against a plate, the deformation of the ball and plate will cause the midpoints to become closer than the normal undeformed minimum distance, this we will indicate with $\alpha$. The variation of $\alpha$ during a bounce is governed by a nonlinear differential equation, published by Zener [1], in a somewhat different form

$$
\begin{equation*}
\frac{d^{2} \alpha}{d t^{2}}=\kappa \alpha^{1 / 2}\left(\frac{3}{2} \beta \frac{d \alpha}{d t}+\frac{\alpha}{m_{s}}\right), \quad \kappa=\frac{\sqrt{R}}{D} \tag{1}
\end{equation*}
$$

where $\kappa$ is the contact stiffness. In this equation, the mass of the sphere $m_{s}$ and its radius $R$ represent the sphere. The plate parameters $\beta$ and $D$ are somewhat less evident. The plate point admittance $\beta$, can be calculated as follows: (Cremer et al.[4])

$$
\begin{equation*}
\beta=\frac{1}{4 h^{2}} \sqrt{\frac{3\left(1-\mu^{2}\right)}{E \rho}} \tag{2}
\end{equation*}
$$

where $h$ is the thickness of the plate, $\rho$ its density, and $\mu$ and $E$ the elastic parameters of the plate. There exists a large difference in stiffness between the material of the steel ball $(E=215 \mathrm{GPa})$ and that of the wooden plate $(E \approx 5 \mathrm{GPa})$. This implies that the plate deforms much

[^0]more than the ball during the bounces. Due to this effect $D$ is supposed to be a property of the plate only:
\[

$$
\begin{equation*}
D=\frac{3}{4}\left(\frac{1-\mu^{2}}{E}\right) \tag{3}
\end{equation*}
$$

\]

Having identified the equation and its parameters, it can be shown that there are only two variables that govern the ball-plate contact. If the plate is not too thin, we need not to take the plate admittance into account when calculating the the contact time, $t_{c}$ (Chaigne \& Doutaut [2])

$$
\begin{equation*}
t_{c}=3.22\left(\frac{m_{s}}{\kappa}\right)^{2 / 5} v_{i n}^{-1 / 5} \tag{4}
\end{equation*}
$$

which is the more important parameter for the spectrum of the sound. The other variable is the inelasticity parameter $\lambda$ as introduced by Zener:

$$
\begin{equation*}
\lambda=\beta \frac{3.22 m_{s}}{t_{c}} \tag{5}
\end{equation*}
$$

The restitution coefficient itself has been shown [1] to be only a function of $\lambda$. In Figure 1 we present the values for a 10 mm steel ball bouncing on a 12 mm wooden plate.
The restitution coefficient itself is found numerically from Equation 1, but it has been shown [1] to be a function of $\lambda$ only. The measured values for the bouncing of a 10 mm steel ball on a 12 mm MDF plate are shown in Figure 1. Also shown are values obtained by numerical calculations using Equation 1, and the given formula's for $\kappa$ and $\beta$ as well as directly measured values, in a somewhat similar way as done by Chaigne\&Doutaut [2] In general, different plates will radiate different sounds when subjected to the same forces. The restitution coefficient is not uniquely determined by the factors described by Equation 1 neither. Our analysis up to now is valid for the center of the plate. Eichler [5] showed that the admittance at the edge of the plate is about 0.29 times the admittance in the middle, which results in a lower restitution coefficient, and a different sound spectrum for each bounce. We found good agreement between measurements and calculation, using the admittance as given in Equation 2 up to a few centimeters from the border.

## Perception experiments

First, we would like to see if listeners recognize the natural relation between the restitution coefficient and the spectrum of the sound. Second, we tested if listeners would, in their judgment of the size of the ball, trust more upon the temporal cue, the restitution coefficient, or upon the spectrum of the individual sounds.
For this experiment we recorded the sounds of three steel balls of different weights bouncing on an MDF plate


Figure 1: The restitution coefficient as function of the impact velocity. The upper line represent the values obtained by numerical simulation using Equation 1 and the given formulae for $\beta$ and $\kappa$. The lower line is calculated with Equation 1, and measured values for $\beta$ and $\kappa$. The dots represent the data from three recorded bouncing sequences. The velocity decreases from bounce to bounce and thus we move from right to left in the figure during a bouncing sequence.
of 120 x 50 x 1.8 cm . By changing the silent interval between the bounces, and the amplitude correspondingly, the restitution coefficients of the bouncing series were changed in seven steps. Step 1 corresponds to the lowest restitution coefficient, step 7 to the highest. Since the weights of the balls differed, the restitution coefficients of the original recordings differed correspondingly. This was step 3 for the largest ball, step 4 for the middle ball and step 5 for the smallest ball. Only for the largest ball it was not possible to obtain the highest two restitution coefficients, because it needed to be stretched too much, resulting in unnatural sounds. These two steps were omitted, for this ball.

To answer the first question we asked 13 participants to choose the most natural sound out of two presented. The two sounds always originated from the same recording, but differed in their manipulated restitution coefficients. In total 104 pairs were presented in random order. The results of these 6 subjects are shown in Figure 2. In total 6 out of 13 participants showed to recognize the correlation between the restitution coefficient and the spectral properties of the bounces. Four other subjects always prefered the highest restitution coefficient as the most natural. In a second part of the same experiment, the subjects were asked to judge the size of the bouncing ball. Here the two presented sounds were recordings from dif-


Figure 2: The naturalness of different sounds compared. A value of +1 means that the sound was always preferred over the others, for -1 it is always rejected. This data is averaged for the 6 participants that appear to recognize the correlation between restitution coefficient and the spectrum of the sound.
ferent balls, and the restitution coefficient was also different for both. The results are shown in Table 1. In some cases, the information in the restitution coefficient pointed at one ball as the largest while the spectral information indicated the opposite. These cases correspond to the left upper half of the table, where on the diagonal the restitution coefficient was the same for both sounds. We can see that in these cases the listener is less sure what to choose, but on the average the listener will more often follow the spectral information.

|  |  | smallest ball |  |  |  |
| :---: | :--- | :--- | :--- | :--- | :--- |
|  | RCS | 1 | 3 | 5 | 7 |
|  | 7 | 0.38 | 0.15 | 0.31 | 0.62 |
| largest | 5 | 0.28 | 0.38 | 0.69 | 0.82 |
| ball | 3 | 0.49 | 0.74 | 0.92 | 0.95 |
|  | 1 | 0.72 | 0.92 | 0.87 | 0.92 |

Table 1: Responses of the subjects when asked for the largest ball. The restitution coefficient step, indicated with RCS, is artificially changed for both sounds. If the value is bigger than zero, as is always the case, the participants have chosen, on the average, correctly considering the spectrum of the sound.

## Discussion

We have analyzed the differential equation governing the restitution coefficient of a bouncing event and pointed out a relation between the spectral and temporal properties of the bouncing behavior. In a perception experiment, we found two types of responses, one group was able to identify this correlation, and another group always chose the highest restitution coefficient as the most natural. A similar division in two groups was found by Canévet et. al. [3] when asking subjects for the pleasantness of single impact sounds. The number of participants in our experiment was too small to reliably estimate the relative sizes of the groups.

Second, we argued that both the spectral and temporal cues are also influenced by other parameters also. We tested whether the subjects would follow the spectral or the temporal information when asked to compare the sizes of the balls and these two sources of information were on conflict. In our experiment the subjects clearly followed the spectral cues.

## References

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