Field fluctuation spectroscopy in a reverberant cavity with moving scatterers

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1 Introduction

Recently interest increases about the interaction of sound inside a strong reverberant cavity filled with moving scatterers. In such a configuration, the motion of scatterers induces fluctuations of the scattered and reverberated sound field. In a theoretical point of view, a statistical model based on a diffusive field assumption has been proposed[1]. It has been shown that the field's autocorrelation that quantify the field's fluctuations is directly linked to global parameters of the system such as the scatterers speed, the cavity volume or the total elastic cross section of the scatterers. Different experimental works confirm this model[2, 1]. These researches lead to a new technique: the Diffusive Reverberant Acoustic Wave Spectroscopy (DRAWS). DRAWS has been successfully applied in order to measure the elastic cross section or the speed of fish swimming in a water tank[3, 1]. Recently Conti et al.[4] have proposed an extension of DRAWS in order to also measure the inelastic total cross section (i.e., the scatterers absorption). The method has been applied to human body[4]. In this paper, we propose to confirm theoretically the Conti et al. method in the framework of diffusive model. Experimental results obtained inside a 1.5 liter reverberant water tank at 900kHz central working frequency with spheres made of materials (steel, gelatine, Ureol,...) of different absorptivity are presented. Especially an original illustration of the extinction paradox is highlighted.

2 Theory

The time dependence of the pressure wave field, $\phi(t)$, can be decomposed into an infinite sum of $\phi_n(t)$ wave fields. Here $\phi_n(t)$ represents the wave field which is scattered n times and t is the propagation time of the wave after its emission from the source (at $t = 0$). Under the diffuse field assumption, the mean squared fields obey to a set of the balance equations between the different scattered order field intensities:

$$\frac{d\left\langle \phi_n^2(t)\right\rangle}{dt} = -\left\langle \phi_n^2(t)\right\rangle \left(\frac{1}{\tau_E} + \frac{1}{\tau'_A}\right) + \frac{\left\langle \phi_{n-1}^2(t)\right\rangle}{\tau_E} \qquad (1)$$

where τ_E is the elastic mean free time and τ'_A the inelastic one. Eq. 1 results from the following consideration: within time dt, the elastic scattering of the $(n-1)^{th}$ order scattered wave field feeds the n^{th} order field. However at the same time, n^{th} order field decreases due to its elastic scattering and its absorption. Obviously the *feeding* term in Eq. 1 is not present for the field that is not scattered ($n = 0$). The absorption is the sum of the inelastic scattering and the cavity absorption (boundaries+water dissipation), i.e. $1/\tau_{A'} = 1/\tau_A + 1/\tau_C$. Moreover, it has been shown under the dilute approximation that $\tau_E = V/N\sigma_E$ in a reverberant cavity of a volume V filled with N scatterers[1]. Straightforwardly, $\tau_A = V/N\sigma_A c$ where σ_A is the scatterers' inelastic cross section. We assume that the different scattered field orders are decorrelated from one to one, therefore $\left\langle \phi^2(t)\right\rangle = \sum_n \left\langle \phi_n^2(t)\right\rangle$. Two fields, $\phi(t)$ and $\phi'(t)$ recorded while the scatterer positions have completely changed are only correlated through the part of the field that is not scattered, i.e. $\langle \phi(t)\phi'(t)\rangle = \left\langle \phi_0^2(t)\right\rangle$. Using Eq. 1, it comes

$$\langle \phi(t)\phi'(t)\rangle / \left\langle \phi^2(t)\right\rangle = e^{-t/\tau_E} \qquad (2)$$

Moreover summing equations 1 for all n yields:

$$d\left\langle \phi^2(t)\right\rangle/dt = -\left\langle \phi^2(t)\right\rangle/\tau'_A \qquad (3)$$

$$d\left\langle \phi_C^2(t)\right\rangle/dt = -\left\langle \phi_C^2(t)\right\rangle/\tau_C \qquad (4)$$

ϕ_C corresponds to the reverberated field recorded without scatterers. Finally it comes from the two previous equations:

$$\left\langle \phi^2(t)\right\rangle / \left\langle \phi_C^2(t)\right\rangle = e^{-t/\tau_A} \qquad (5)$$

Hence the experimental measure of $\left\langle \phi^2(t)\right\rangle$, $\left\langle \phi_C^2(t)\right\rangle$ and $\langle \phi(t)\phi'(t)\rangle$ gives an estimate of scatterers' elastic and inelastic cross sections σ_E and σ_A by the mean of Eqs. (2) and (5).

3 Experiments

The one sphere experimental setup is shown in Fig. 1. A generator emits a few 900kHz periods. Synchronously,

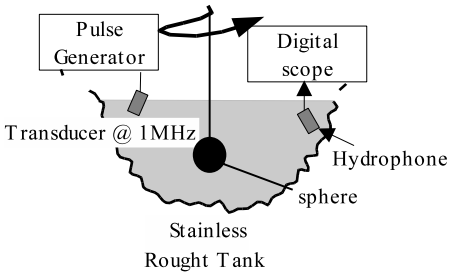

Figure 1: Experimental set-up.

a digital scope records the scattered-reverberated pressure field on an hydrophone. A parallel port leads to transfer the pressure response into a computer. This response is then recorded on the hard drive of a computer by the way of the parallel port. The acquisition is repeated 200 times, between each acquisition the ball position is changed. From the bank of responses, $\left\langle \phi^2(t)\right\rangle$

and $\langle\phi(t)\phi'(t)\rangle$ are estimated. As for ϕ_C, it is obtained from the record of the pressure field response with no ball in the tank. In Fig. 2 are plotted the two ratios (Eqs. (2) and (5)) with respect to the propagation time. Plots are obtained from two 19mm-diameter spheres: one made of Ureol and the other one made of steel. From linear fits, σ_E and σ_A are deduced. Experiments are also performed with plasticine and gelatin materials. The experimental cross-sections are presented on Table 1. They are compared to their theoretical values[5] when acoustic properties of materials are known.

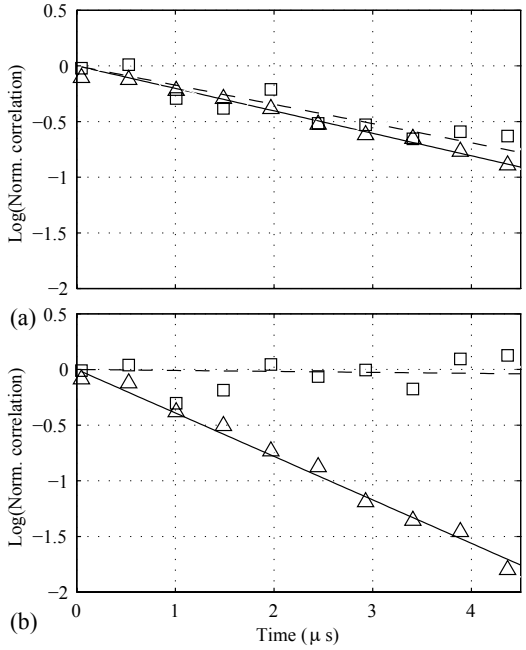

Figure 2: Experimental mean intensity ratio, $\left\langle\phi^2(t)\right\rangle / \left\langle\phi_C^2(t)\right\rangle$ (squares) and normalized correlation, $\langle\phi(t)\phi'(t)\rangle / \left\langle\phi^2(t)\right\rangle$ (triangles). Straight lines represents least-square linear fits. Plots (a) and (b) are obtained with 19mm diameter spheres made of Ureol and stainless steel, respectively.

	Steel		Ureol		Plasticine		Gelatin	
	σ_E	σ_A	σ_E	σ_A	σ_E	σ_A	σ_E	σ_A
E1	498	11	251	214	302	206	350	38
E2	543	20	256	214	296	203	311	23
E3	551	17	252	207	315	204	313	24
Th	581	0	282	235	x	x	x	x

Table 1: Table of tree measures (E1, E2 and E3) of σ_E and σ_A expressed in mm^2 with four materials. The row Th corresponds to the theoretical values when material properties are known.

4 Discussion

At $900kHz$, the wavelength equals $1.66mm$ in water. Hence a $19mm$ diameter ball is large compare to the wavelength. In such a case, one may assume that the ray theory is valid. The impedance of steel is about 40 times higher than the water one. Hence a steel-ball can be seen as a nearly perfectly reflecting sphere. Under the ray theory assumption, the elastic cross section of such a scatterer would equal its geometrical cross section (πR^2 where R is the sphere radius). For our 19mm diameter sphere, the geometric cross-section equals $283mm^2$. Nevertheless the experimental value for σ_E (see Table 1) is about twice larger than the geometrical one. This behavior is characteristic of the so-called "extinction paradox." Contrary to the specular reflection on the scatterer, the ray theory cannot model the forward-diffraction on the edge of the scatterer. Surprisingly the specular-reflection contribution and the diffraction contribution to the elastic cross section are equal. This has been theoretically justified. Here we give an experimental illustration. To this end, we exploit the results obtained with the 19mm-diameter ball made of Ureol (see Table 1). The impedance of Ureol is almost equal to water one. Moreover the intrinsic attenuation of Ureol is extremely strong ($240dB/m/MHz$). Therefore an Ureol ball acts as a nearly perfect absorbing scatterer. In such a case, all the "rays" previously reflected penetrate in the ball and are absorbed. Now rays contribute to the inelastic scattering whereas the diffraction process remains unchanged and still participate to the elastic cross-section. Therefore σ_E and σ_A would be almost equal to the geometric total cross-section, i.e. $283mm^2$. This is roughly experimentally observed, the difference originates the imperfect absorption, the slight mismatch of impedance between water and Ureol and finally the wavelength that is not infinitely small compare to the ball radius.

5 Conclusion

In conclusion, DRAWS is an original and simple technique in order to measure both the elastic and inelastic total cross sections. An experimental validation has been shown within the framework of the extinction paradox.

References

[1] Julien de Rosny, J. H. Page, Philippe Roux, and Mathias Fink. Field fluctuation spectroscopy in a reverberant cavity with moving scatterers. *Phys. Rev. Lett.*, 90:094302, 2003.

[2] D.A. Demer, S. Conti, J. de Rosny, and P. Roux. Absolute measurements of total target strength from reverberation in a cavity. *J. Acous. Soc. Am.*, 113:1387, 2003.

[3] J. de Rosny and P. Roux. Multiple scattering in a reflecting cavity application to fish counting in a tank. *J. Acoust. Soc. Am.*, 109(6):2587, 2001.

[4] S.G. Conti, P. Roux, D.A. Demer, and J. de Rosny. Measurement of the scattering and absorption cross sections of the human body. *App. Phys. Lett.*, 84:819, 2004.

[5] P.M. Morse and K.U. Ingard. *Theoretical acoustics.* Princeton university press edition, 1986.