Simple and multi-reflections using the PE method with a complementary Kirchhoff approximation

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Introduction

Noise impact of road and railway infrastructures are more and more severely regulated by national laws: acceptable thresholds in emission and reception are decreasing. This implies a prediction for longer distance of propagation [1] where meteorology and terrain effects (impedances discontinuities, uneven ground) can no more be ignored.

The parabolic equation (PE) is one of the powerful numerical methods used to solve complex outdoor sound propagation [2]. Efficient for many configurations, it gives incorrect results when backscattering can not be neglected.

This paper aims at presenting a new method able to integrate backscattering in the GFPE (Green's Function Parabolic Equation) method. In this approach, reflections on vertical obstacle are considered by using a complementary Kirchhoff approximation called GFPE-Kirchhoff. Thus, this new method allows solving multi-reflections problems with the GFPE method.

Theoretical approach

The Parabolic Equation (PE)

Starting from the Helmholtz equation in cylindrical (r,z) coordinates for the sound pressure $P(r,z)=\frac{1}{\sqrt{r}}u(r,z)e^{jk_r r}$:

$$\left(\frac{\partial^2}{\partial r^2}+\frac{1}{r}\frac{\partial}{\partial r}+\frac{\partial^2}{\partial z^2}+k(r,z)^2\right)P(r,z)=0 \qquad (1)$$

An initial field is propagated step by step from the source to the receiver. After many developments described in Gilbert article [3], the field at $u(r+\Delta r,z)$ is solving by:

$$u(r+\Delta r,z)=\left[\frac{1}{2\pi}\int_{-\infty}^{+\infty}\left(U(r,k')+R(k')U(r-k')\right)\times e^{j\Delta r\left(\sqrt{k_r^2-k'^2}-k_r\right)}e^{jk'z}dk' + 2j\beta\times U(r,\beta)\times e^{j\Delta r\left(\sqrt{k_r^2-\beta^2}-k_r\right)}e^{-j\beta z}\right]\times e^{j\frac{\Delta r\,\delta k^2(z)}{2k_r}} \qquad (2)$$

$U(r,k)=\int_0^{+\infty}e^{-jkz'}u(r,z')dz'$, $\beta=\frac{k_r}{Z_g}$, Z_g the normalized ground impedance.

The GFPE-Kirchhoff method

In order to take reflection on an impedant vertical surface into account, the GFPE-Kirchhoff method is used. This approach is "complementary" to the one considered for the diffraction by a straight barrier. An image-source S' is built relatively to barrier vertical plane. The sound pressure at any calculation point above the obstacle is set to zero (Table 1). The ones on the screen are multiplied by the plane wave reflection coefficient calculated from the barrier impedance and then propagated to the receiver.

We are dealing here with a wind blowing from the source to the receiver. To insert the atmospheric refraction in the GFPE-Kirchhoff method, an upwind sound speed profile is built. This profile, symmetric relatively to c_0 of the downwind sound profile is applied for the propagation from the image source S' to the obstacle. Then the "initial" downwind sound speed profile is used for the propagation from the obstacle to the receiver (see Table 1).

The total pressure at the receiver is the sum of the "free field above ground" calculation (a) and the one obtained in case (b).

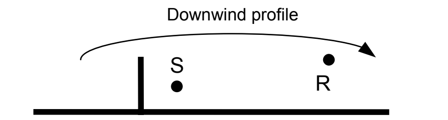
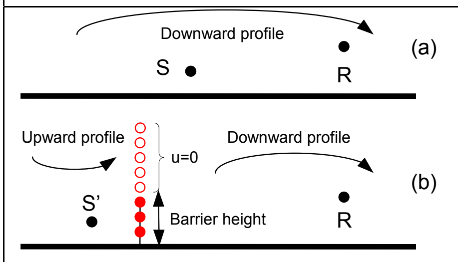
	Real configuration
	GFPE-Kirchhoff method : $P_{total}=P_{(a)}+P_{(b)}$

Table 1: GFPE-Kirchhoff method applied to a barrier located behind the source with a downwind profile

The multi-reflection

The GFPE-Kirchhoff method can be extended to multi-reflections due for instance to the presence of 2 parallel barriers. The calculation for an order of reflection of 2 is described Table 2. The total pressure at the receiver is the sum of (a), (b) and (c) calculations.

Numerical simulations and validation

A realistic road traffic noise configuration with two parallel and absorbent barriers ($\sigma=180$ cgs, Delany and Bazley's formulation [4]) is studied with and without meteorological effects (see Figure 1). A strong sound speed gradient described by $c(z)=c_0\left(1+4.9\times10^{-3}z\right)$ corresponding to a wind blowing from source to receiver is chosen to point out the influence of refraction.

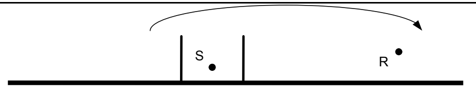
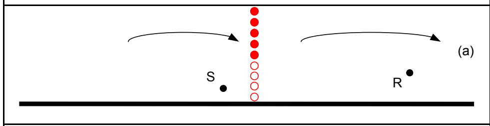
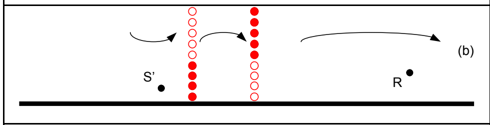
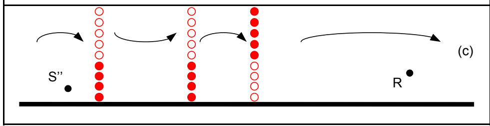
	Real configuration
	Diffraction
	Simple reflection + diffraction
	Double reflection + diffraction

Table 2: Principle of calculation with 2 parallels barriers at order 2 with downwind profile, ○ $u = 0$, ● $u = u(r,z)$

Numerical simulations and validation

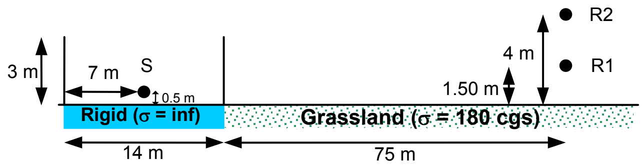


Figure 1: Studied configuration

A measurement campaign above 1/20 scale models has been undertaken to validate the theoretical results. The meteorological effects are introduced using the analogy between sound propagation above a flat surface along curved ray paths and sound propagation above a curved surface along straight ray paths [4] (see Table 3). Results in homogeneous atmosphere are also compared to a reference solution obtained with BEM [5] (Boundary Element Method) calculations. Scale model measurements are performed between 2000 Hz -14000 Hz which corresponds to a frequency range of 50 Hz – 700 Hz at scale 1. Grassland and barrier impedance are represented by a felt ($\sigma = 3600$ cgs) corresponding to $\sigma = 180$ cgs at 1/1 scale.

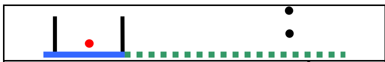
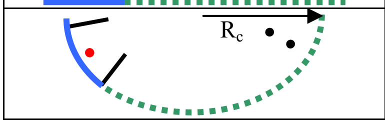
	$c(z) = c_0$
	$c(z) = c_0(1 + az)$ $a = \frac{1}{R_c} = 4.9 \times 10^{-3}$ m^{-1}

Table 3 : analogy between sound speed profile and curved surface

Results

Results discussion is almost the same for both receivers so that only results for the highest one are presented here. The agreement between calculations and measurements is very good either in homogeneous or inhomogeneous conditions. As expected especially in homogeneous case, the 24 Hz step which separates two interferences is well correlated with the distance between the 2 barriers. The comparison between Figure 2 and Figure 3 points out the importance of meteorological effects: the barriers efficiency decreases due to downward refraction.

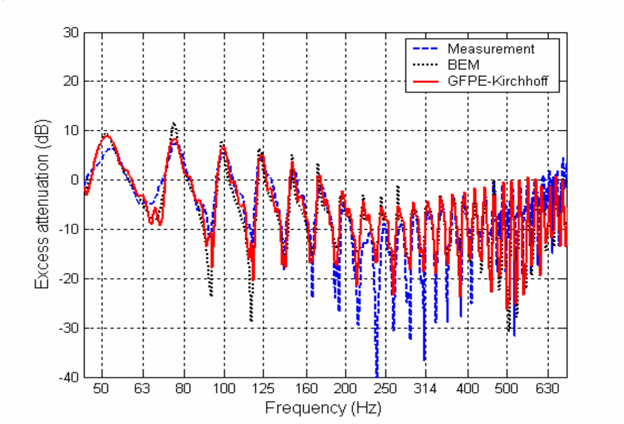


Figure 2: Comparison between GFPE, BEM and scale model measurements for the configuration detailed in Figure 1, $c(z) = c_0$, $\sigma_{barrier} = 180$ cgs

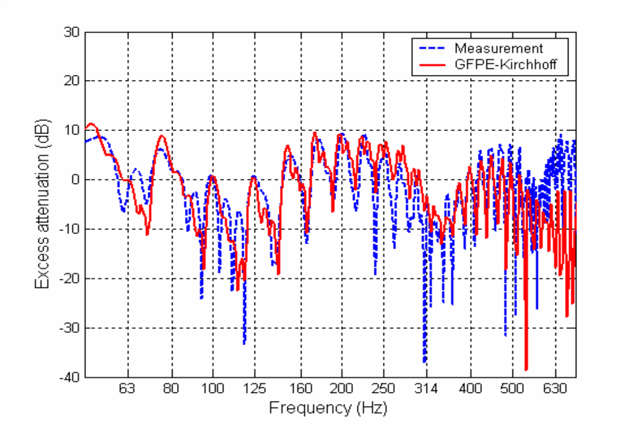


Figure 3: Comparison between GFPE and scale model measurements for the configuration detailed in Figure 1, $c(z) = c_0\left(1 + 4.9 \times 10^{-3} z\right)$, $\sigma_{barrier} = 180$ cgs .

Conclusion

Results show that the method presented here is adapted to solve backscattering problems. The GFPE-Kirchhoff method seems to be promising. Then, integration of multi-reflections effects created by impedant vertical obstacles is possible in a PE method. Scale model measurements have demonstrated that the approach is also available in a refracting atmosphere. Thus, this new technique allows solving realistic configurations coupling complex meteorological effects and backscattering. Works are in progress with more complex range dependant wind speed profiles.

References

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