

# Properties of acoustic waves guided by internal interfaces in piezoelectric crystals

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## Introduction

Plane interfaces separating two solids can guide non-uniform waves whose amplitude decays with distance from the interface inward the solids. The existence of interface acoustic waves (IAW) depends on the physical properties of the media and the type of contact. Difficulties in analyzing IAW propagation are due to the fact that the problem cannot be solved completely analytically even for Stoneley waves in isotropic solids. Nevertheless using a special mathematical approach allows one to avoid explicit calculations and draw a number of general conclusions regarding IAW in non-piezoelectric and piezoelectric bi-crystals at different conditions on the contact [1-3]. The present communication discusses the existence of IAWs in two piezoelectric structures of arbitrary symmetry: 1) infinitesimally thin metallic layer inserted into a crystal; 2) rigidly bonded crystals whose piezoelectric moduli differ by sign while the other material constants are identical ("180°-domain wall", 180DW). It is worth noting that the results to be exposed cannot be deduced directly from the results obtained in Refs. [2,3].

## "Subsonic" IAW

### Metallic insertion

By referring to the algebraic relations derived in [4] the dispersion equation can be cast into the form

$$Q_{\Phi}(v) \equiv 2i \sum_{\alpha=1}^4 \Phi_{\alpha}^2 = 0, \quad (1)$$

where  $\Phi_{\alpha}$  is the potential that the partial plane mode labeled by the subscript  $\alpha$  produces, the subscripts  $\alpha = 1, \dots, 4$  being assigned either to non-uniform modes with  $\text{Im}(p_{\alpha}) > 0$  or to uniform reflected modes.

Consider the "subsonic" velocity interval where all partial modes are non-uniform; i.e. the velocity  $v$  is supposed to be smaller than the limiting velocity  $\hat{v}_1$  of slow quasi-transverse bulk waves. The properties of the function  $Q_{\Phi}(v)$  (1) at  $v < \hat{v}_1$  have been studied in relation with the existence theorems for SAW [4]:  $Q_{\Phi}$  is real when  $v < \hat{v}_1$ ,  $Q_{\Phi} > 0$  at  $v = 0$ , steadily decreases with increasing  $v$ , and  $Q_{\Phi} \rightarrow -\infty$  as  $v \rightarrow \hat{v}_1$  if the potential  $\Phi_{LBW}$  of the limiting bulk wave (LBW) at  $v = \hat{v}_1$  does not equal zero. However, if  $\Phi_{LBW} = 0$ , then  $Q_{\Phi}(\hat{v}_1) > 0$ . Thus we arrive at the following conclusions.

- Metallic layer can guide at most one subsonic IAW.
- Subsonic IAW exists obligatory if  $\Phi_{LBW} \neq 0$ .
- Subsonic IAW does not exist if  $\Phi_{LBW} = 0$ .

It can be proved that the space of the non-existence of IAW is a set of 2D surfaces in the 3D space of angles specifying the interface and the direction of IAW propagation in it.

### "180°-domain wall"

180DW structures can be constituted of two crystals (of the same substance) possessing the plane of symmetry. One of the crystals must be rotated through 180° about the perpendicular to the plane of symmetry before both the crystals are cut along the equivalent planes and the upper (lower) part of crystal 1 is joint to the lower (upper) part of crystal 2. If the symmetry group does not contain planes of symmetry, then one can utilize left- and right-hand crystal types of a substance.

The equation on the IAW velocity reduces to

$$Q_F Q_{\Phi} - S_F^2 = 0, \quad (2)$$

where  $Q_F(v) = 2i \sum_{\alpha=1}^4 D_{\alpha}^2$ ,  $S_F(v) = i[2 \sum_{\alpha=1}^4 \Phi_{\alpha} D_{\alpha} - 1]$ ,  $D_{\alpha}$  is the normal projection of electric.

Within the "subsonic" interval  $Q_F$  and  $S_F$  are real,  $Q_F$  being negative. The analysis of (2) reveals that two IAW can exist as in general cases [2]. However, only one IAW exists in the case of weak piezoeffect. The appearance of IAWs depends on the material constants. Besides,

- No IAWs exist on 180DW if  $\Phi_{LBW} = 0$ .
- Any piezoelectric media can be used to fabricate 180DW structures supporting subsonic IAW. It is required that the "domains" be specially oriented.

Note that the slowest IAW on 180DW is faster than IAW on "metallic insertion" oriented like 180DW.

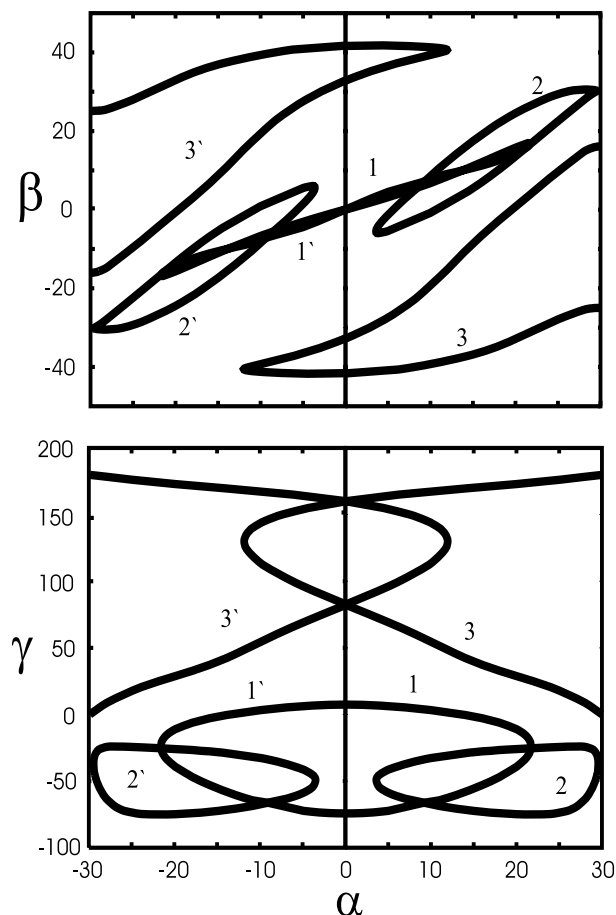
Consider now four orientation families: the direction of IAW propagation is 1) perpendicular to the plane of symmetry or 2) parallel to the even-fold symmetry axis; the interface is 3) the plane of symmetry or 4) perpendicular to the even-fold symmetry axis. In these cases  $S_F(v) \equiv 0$  (see [5]) so that (2) decomposes into  $Q_{\Phi} = 0$  and  $Q_F = 0$ . Since  $Q_F(v) = 0$  has no root at  $v < \hat{v}_1$ , for the above orientations

- at most one subsonic IAW exists on 180DW, this wave being similar to IAW on "metallic insertion".

## Leaky and "supersonic" IAW

In the range  $v > \hat{v}_1$  uniform partial modes appear so that IAWs ordinarily turn out to be leaky. The functions  $Q_{\Phi,F}(v)$  and  $S_F(v)$  become complex and the existence of

IAWs can be analyzed only in the limit of weak piezo-effect considering that IAWs arise from LBWs due to piezoelectric coupling. The result is that in the case of "metallic insertion" IAWs arise independently of the material constants if the slowness surface of bulk waves is convex and do not arise if the slowness surface is concave. The occurrence of IAWs on 180DW depends on the material constants for both the shape of the slowness surface.



**Figure 1:** Lines of "supersonic" IAW in LiNbO<sub>3</sub>. For the sake of convenience the curves  $\gamma(\alpha)$  are depicted within a 360° rather than 180° interval of angles  $\gamma$ .

However, the imaginary component of leaky-IAW velocity can vanish at particular geometries of propagation. This vanishing occurs independently of the crystallographic symmetry. The analysis reveals that the orientations supporting such "supersonic" IAWs on "metallic insertion" generally can be represented as 1D lines in the 3D space of orientation angles. Figure 1 shows the lines of "supersonic" IAW on "metallic insertion" in LiNbO<sub>3</sub>. The real component of the IAW velocity falls into the so-called first intersonic interval  $\hat{v}_1 < v < \hat{v}_2$ , where  $\hat{v}_2$  is the limiting velocity of fast quasi-transverse bulk waves (there is one pair of uniform modes in this interval). In Fig. 1 the angles  $\alpha$ ,  $\beta$ , and  $\gamma$  specifying the geometry of propagation measure the successive rotations about the z-, y-, and x-axis, respectively. The z-axis coincides with the three-fold symmetry axis, x-axis is along the propagation direction, and y-axis is perpendicular to the sagittal

plane. The dependence of  $\beta$  and  $\gamma$  on  $\alpha$  is depicted within the sector  $-30^\circ \leq \alpha \leq 30^\circ$ . In the sector  $30^\circ \leq \alpha \leq 60^\circ$  one has  $\beta(\alpha) = \beta'(60 - \alpha)$  and  $\gamma(\alpha) = -\gamma'(60 - \alpha)$ , where  $\beta'(\alpha)$  and  $\gamma'(\alpha)$  denote the values of functions within  $0^\circ \leq \alpha \leq 30^\circ$ . In the sector  $-60^\circ \leq \alpha \leq 0^\circ$  one has  $\beta(\alpha) = -\beta''(-\alpha)$  and  $\gamma(\alpha) = \gamma''(-\alpha)$ , where  $\beta''(\alpha)$  and  $\gamma''(\alpha)$  are the values of functions within the sector  $0^\circ \leq \alpha \leq 60^\circ$ . In the other 120°-degree sectors these curves are of exactly the same shape.

"Supersonic" IAWs on 180DW generally can exist only for isolated geometries of propagation, i.e. at certain points of the space of orientation angles. The same holds for "supersonic" IAWs on "metallic insertion" with the velocity exceeding  $\hat{v}_2$ .

In conclusion, note that the leaky-IAW branch affects strongly the reflection of the bulk waves from the interface, leading to specific resonance features in the behavior of the coefficients of mode conversion (see, e.g., [6, 7]).

## Acknowledgment

A. Darinskii thanks Deutsche Forschungsgemeinschaft (grant 436RUS 113/645/3-2) and the Institute for Solid State and Material Research Dresden for financial support. He also thanks Russian Foundation for Basic Research for partial financial support. M. Weihnacht is grateful to Dr. R. Wobst for his programming work.

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