Stability of a slowly diverging axisymmetric jet in the presence of a coflow

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Introduction

The stability theory has been a successful method for the understanding of supersonic jet-noise generation [8]. However, in the case of subsonic jets, noise mechanisms are still debated. Hence, the acoustical emission by vortex pairing [2] conflicts with the theory of superdirectivity [5] [4]. In this model, the acoustical source is supposed to result of an instability of global mode type [4] [3] .

In the present work, a global stability analysis is leaded on an axisymmetric jet of speed $\tilde{U}_j$ with a coflow of velocity $\tilde{U}_\infty$, see figure 1. Since the shear-layer of the jet thickens slightly in the streamwise direction, its stability characteristics vary from point to point along the x axis. The local stability of every radial velocity profile is thus calculated. The downstream evolution of fluctuations is then predicted on a large axial domain thanks to global stability criteria [6]. A comparison is made with experimental results [7].

Formulation of the problem

The mean velocity profile is described in $(\tilde{r}, \tilde{x}, \phi)$ cylindrical coordinates by:

$$\tilde{U}(\tilde{r}, \tilde{x}, \phi) = 2\tilde{U}_m \left[1 - R_u \tanh\left(\frac{\tilde{D}}{4\tilde{\delta}_\theta(\tilde{x})} \left(\frac{2\tilde{r}}{\tilde{D}} - \frac{\tilde{D}}{2\tilde{r}} \right) \right) \right]$$

with $\tilde{D}$ the jet diameter, $\tilde{U}_m = (\tilde{U}_j + \tilde{U}_\infty)/2$ the mean flow velocity, $R_u = (\tilde{U}_j - \tilde{U}_\infty)/(\tilde{U}_j + \tilde{U}_\infty)$ and $\tilde{\delta}_\theta$ the momentum thickness of the shear layer. The mean pressure $\tilde{p}_0$, the mean density $\tilde{\rho}_0$, $2\tilde{U}_m$ and $\tilde{D}$ are chosen as reference quantities. In the next, non-dimensional variables will be written without the sign $\tilde{}$.

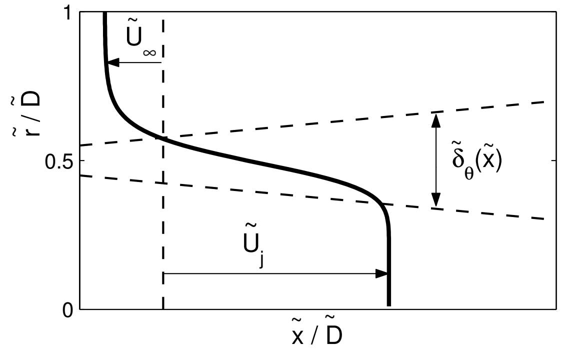

Figure 1: Mean axial velocity profile.

With the aim in view to predict the linear stability of such a flow, an inviscid and incompressible perturbation is added to every involved mean variable. For example, the pressure p is searched as $p = 1 + p_f(r, x, \phi, t)$ with $|p_f| \ll 1$.

The long time evolution of p_f defines the stable or unstable behavior of the jet. It is searched as a normal mode form $\exp(-i\omega t)$ with ω complex. If the imaginary part of ω, ω_i, is strictly positive (resp. strictly negative), then the fluctuation grows exponentially with time (resp. decreases) and the flow is said to be unstable (resp. stable) for this perturbation. The azimuthal dependence of p_f is calculated for each azimuthal mode $\exp(in\phi)$, with n an integer. Moreover, if the flow were invariant in the streamwise direction, then an evolution of normal mode form $\exp(ik(\omega, n)x)$ would be expected. Nevertheless, the wave number k must be here evaluated for each diverging velocity profile. In order to decouple the wavy evolution of the fluctuation with the dependence of k on x, we suppose that the thickening of the shear layer is slow. Precisely :

$$\frac{d\delta_\theta}{dx} \sim \epsilon \ll 1$$

The variable $X = \epsilon x$ scales with the slow evolution of the shear layer. Hence, $k = k(\omega, n, X)$. Finally, p_f is sought as :

$$p_f = g(r, X) \exp\left(in\phi - i\omega t + i \int_0^x k(X' = \epsilon x', \omega, n) dx' \right)$$

It is then possible to write a dispersion equation of Rayleigh type at the leading order $0(\epsilon^0)$ with the only variable p_f :

$$D\left(\frac{\partial}{\partial r}, \omega, k; n, \delta_\theta(X), R_u \right) g = 0 \qquad (1)$$

Local stability

The dispersion equation (1) is solved for fixed parameters n, R_u and at each axial position by imposing δ_θ. A shooting method is employed : (1) is integrated from $r = 0$ to the coflow domain where g must satisfy a condition of exponential decrease.

If ω is fixed, then two spatial branches of solutions $k^+(\omega)$ and $k^-(\omega)$ may exist. These solutions correspond to waves that develop upstream and downstream respectively of the considered axial position X [1]. Following the global stability theory, we are interested in the absolute mode (ω_0, k_0) defined as a pinching point of k^+ and k^- branches : $k^+(\omega_0) = k^-(\omega_0) = k_0$. The group velocity associated to this wave vanishes and its stable behavior is predicted by the sign of ω_{0i}.

The figure 2 presents the calculated evolution of the temporal amplification rate ω_{0i} with $\delta_\theta(X)$ for different parameters R_u and n. A good agreement is found with Cooper & Crighton's [3] results for $n = 0$ and $R_u = 1$ corresponding to the free jet case for which $\tilde{U}_\infty = 0$. This flow is absolutely stable at every axial position. As R_u is increased, ω_{0i} grows and an absolutely unstable domain expands for $R_u > 1.191$, that is $\tilde{U}_\infty/\tilde{U}_j < -0.16$, and $n = 0$. The first absolute helical mode $n = 1$ is unstable for $R_u > 1.388$. Furthermore, $\omega_{0i}(X, R_u, n)$ decreases with increasing n at least for the presented δ_θ and R_u variations and $n \leq 3$. The axisymmetric mode is the less stable absolute instability.

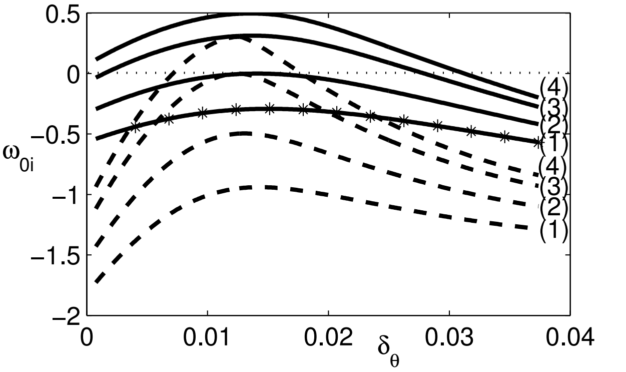

Figure 2: Evolution with $\delta_\theta(X)$ of the temporal growth rate of the absolute mode.
— : $n = 0$; - - - : $n = 1$; (1) : $R_u = 1$; (2) : $R_u = 1.191$; (3) : $R_u = 1.388$; (4) : $R_u = 1.5$;
* Cooper & Crighton's analytical results [3].

Global stability

A global mode is a self-fluctuation of a large flow domain, either open ($-\infty < X < +\infty$) or half-open ($0 < X < +\infty$), and whose pulsation ω_g is independent of X. It is of only hydrodynamic origin. The flow resonance does not involve an acoustic feedback through a pressure wave which could be radiated by the flow.

For a flow be globally unstable, a large enough absolutely unstable X-domain must prevail. Hence, a free cold jet, $R_u = 1$, is globally stable.

A jet flow may be considered as a half-open domain that is bounded by a nozzle located in $X = 0$. In this case, the global pulsation ω_g is estimated by $\omega_g = \omega_0(X = 0) + O(\epsilon^0)$ [6]. The figure (3) presents the neutral stability curve $\omega_{0i}(\delta_\theta, R_u) = 0$ (a) and the corresponding Strouhal number $St_{\tilde{D}} = \omega_{0r}/(2\pi)$ (b). We report as well Strykowski & Niccum's experimental data $\delta_\theta(R_u)$ measured close to the nozzle exit [7]. As $R_u < 1.35$, the jet flow is absolutely stable in the vicinity of the nozzle exit and no self-sustained global mode rises. This threshold value agrees well with the critical $Ru = 1.32$ observed by Strykowski & Niccum. For supercritical R_u, these authors notice self-oscillations of the shear layer at a discrete frequency which could be attributed to the instability of a global mode type. Hence, a low frequency is observed in their hot-wire spectra, see figure 8 of the paper. The corresponding Strouhal number is in good agreement with the predicted value $St_D = 0.68$ but no quantitative comparison is available.

Conclusion

A stability study was dealt and the process leading to the rise of a global mode in an axisymmetric jet was enlightened. The acoustical emission of such an instability is the subject of a future work.

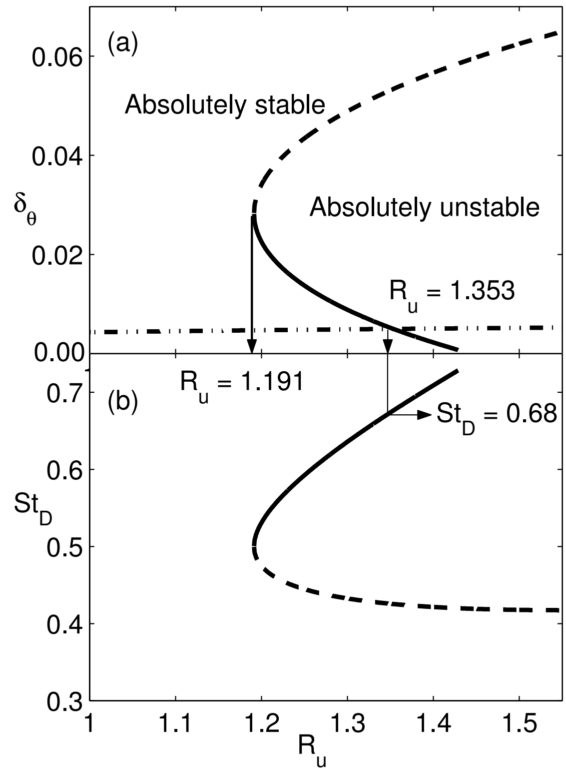

Figure 3: (a) Absolute stability curve $\omega_{0i}(\delta_\theta, R_u) = 0$ of the axisymmetric mode $n = 0$. ...: $\delta_\theta(R_u)$ at the initial development of the jet shear layer studied by Strykowski & Niccum [7].
(b) Corresponding Strouhal number $St_{\tilde{D}} = \omega_{0r}/(2\pi)$.

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