T.G.V. disk brake squeal : a dynamic instability ?

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Introduction

Disconfort problems due to the noise emittence of braking systems in trains have suggested recently several mechanical analyses. This paper gives some of our results obtained from the numerical modeling of TGV brakes in relation with some experimental data. The numerical discussion is based upon the Coulomb's law of contact with a constant coefficient of friction. A dynamic stability analysis enables us to show the loss of stability by flutter of the steady sliding response of the pad on the brake disks.

The braking system

The braking system of a TGV disk brake is mounted on the bogie. It is composed of two symmetric plates with lining under the form of pads. Four disks are mounted on an axle. The applied normal force on a disk is about 19 kN at maximum.

Experimental data

The experimental data, obtained at the train station, are different squeal spectrum. For this, a microphone has been mounted near a disk. The noise spectrum (fig 1) shows the existence of 7 vibration frequencies which merge from the background noise. They correspond to the frequency interval *6 kHz to 20 kHz*. A modal analysis has also been performed from the dynamic response of a bogie at rest. The figure 1 gives the two surperposed spectra with two FRF.

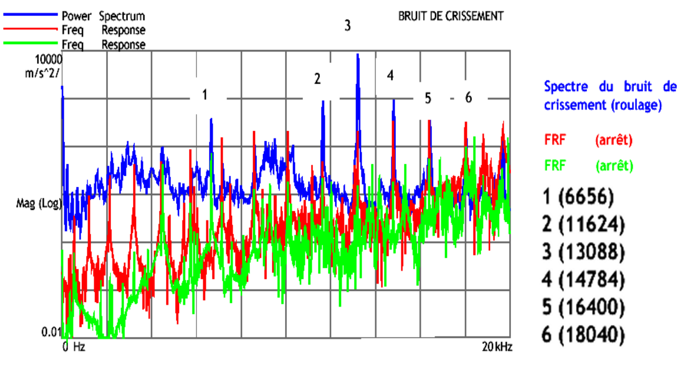

Figure 1: Superposed noise spectra

It is clear that some modal frequencies of the disk coincide with some measured squeal frequencies.

Mechanical modeling

The problem suggests a mechanical modeling by deformable solids. In small deformation, the model of linear, homogeneous and isotropic elasticity is assumed to describe the system of a disk with two brake linings. The contact is unilateral following Coulomb's friction with a constant friction coefficient f.

Dynamic equations of the brake

The displacement u of the disk D in frictional contact on $\partial\Omega_C$ must satisfy the virtual work equation:

$$\int_D \sigma(u) : \epsilon(u^*) d\Omega + \int_D \rho\gamma . u^* d\Omega = \int_{\partial\Omega_F} F_d . u^* dS$$

$$+ \int_{\partial\Omega_C} (N.[u_n^*] + T.u_t^*) dS \qquad (1)$$

$$\forall u^* \in U_{ad_0} \text{ with } U_{ad_0} = \{u/u = 0 \text{ on } \partial\Omega_U\}$$
$$u = U_d \text{ on } \partial\Omega_U$$

$$\begin{cases} [u_n^*] = (u^* - u_G^*).n \\ u_t^* = u^* - u_n^*.n \end{cases} \text{ and } \begin{cases} N = n\sigma n \\ T = \sigma n - Nn \end{cases}$$

n is the external normal to the disk D and u_G the lining displacement.

The constitutive equations are:

$$\sigma(u) = K : \epsilon(u) \text{ in } D$$
$$\begin{cases} N \leq 0 \\ [u_n] \leq 0 \\ N.[u_n] = 0 \end{cases} \text{ and } \begin{cases} |T| \leq -fN \\ Tw - fN|w| = 0 \end{cases} \text{ on } \partial\Omega_C$$

w the relative sliding spead is: $w = v + \dot{u} - \dot{u}_G$ where v denotes the associated rotation velocity of the disk.
In the same spirit, u_G must satisfy a similar variational equation (1).

Assumption H1: The squeal occurs at slow speed, the rotation terms can be neglected in the expression of the acceleration. Thus, the approximation $\gamma = u_{,tt}$ can be introduced in (1) (cf. to [1] for a more complete expression of γ).

The steady sliding equilibrium

At equilibrium and under the assumption H1, $w = v$ and $\gamma = 0$. The equilibrium displacement u satisfies from (1):

$$\int_D \epsilon(u) : K : \epsilon(u^*) d\Omega = \int_{\partial\Omega_F} F_d.u^* dS + \int_{\partial\Omega_C} (N.[u_n^*] + T.u_t^*) dS \tag{2}$$

$$\forall u^* \in U_{ad_0} \text{ with } U_{ad_0} = \{u/u = 0 \text{ on } \partial\Omega_U\}$$
$$u = U_d \text{ on } \partial\Omega_U$$

$$\text{with} \begin{cases} N \leq 0 \\ [u_n] \leq 0 \\ N.[u_n] = 0 \end{cases} \quad \text{and } T = f.N.\frac{v}{|v|} \text{ on } \partial\Omega_C$$

The same equations must be written for u_G. The discretized equations are:

$$\mathbf{K}.U = R, \ \mathbf{K^G}.U^G = R^G$$

respectively for the disk and for the lining. With the notation $U = (Y, U_N, U_T)$, Y denotes the d.o.f. associated with the nodes outside the contact zone. Then: $R = (F, N, f[\phi]N)$ where $[\phi]$ is an appropriate matrix. (U, U^G) and (R, R^G) must satisfy the unilateral contact condition.

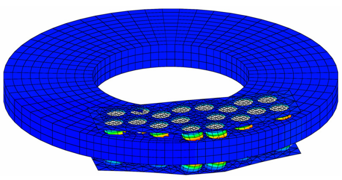

Figure 2: The steady sliding equilibrium of the brake

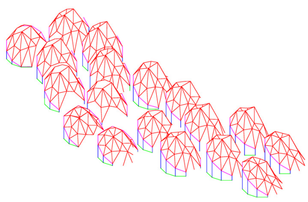

Figure 3: Normal contact reaction

Stability analysis of the equilibrium

The evolution of a small perturbed motion is now considered near the steady sliding equilibrium. For a sliding motion without stick regime,

$$u = u_e + \hat{u} \text{ with } [\hat{u}_n] = 0 \text{ on } \partial\Omega_C^e$$

where u_e is the equilibrium displacement and $\partial\Omega_C^e$ is the contact zone at equilibrium.
The sliding condition leads to

$$T = f.N.\frac{w}{|w|}$$

The stability can be discussed by the linearization method :

$$\hat{T} = f.\hat{N}.\frac{v}{|v|} + \frac{f.N_e}{|v|}\left(\hat{w} - \frac{v.\hat{w}}{|v|^2}v\right) \tag{3}$$

with $\hat{w} = [\hat{u}_{,t}]$ and N_e the normal contact reaction at equilibrium. The perturbed motion $\hat{u}$ is governed by the following variational equation under H1:

$$\int_D \rho\hat{u}_{,tt}.u^* d\Omega + \int_D \epsilon(\hat{u}) : K : \epsilon(u^*) d\Omega = \int_{\partial\Omega_C} (\hat{N}.[u_n^*] + \hat{T}.u_t^*) dS \tag{4}$$

Note that $N_e < 0$ on $\partial\Omega_C^e$.
The discretization of (4) gives after the elimination of $\hat{N}$:

$$M.\ddot{\hat{U}} + C.\dot{\hat{U}} + K.\hat{U} = 0 \tag{5}$$

M and K are *non symmetric* matrices. The displacement are searched in the form $\hat{U} = Xe^{\lambda.t}$. The high dimension of the considered matrices is a source of difficulty. A projection of the mass, rigidity and damping matrices on a troncated basis ϕ of vibration modes without friction is thus introduced. It follows *the generalized eigenvalue problem*:

$$[\lambda^2\tilde{M} + \lambda\tilde{C} + \tilde{K}].Z = 0 \tag{6}$$

with $\tilde{M} = \phi^T M\phi$, $\tilde{C} = \phi^T C\phi$, $\tilde{K} = \phi^T K\phi$ and $X = \phi Z$. The un-symmetry of the considered matrices leads to complexe eigenvalues and eigen-modes. A mode is unstable if $Re(\lambda) > 0$. On figure (4), it is given in blue the real parts of the eigenvalues in function of the eigen frequency. The associated displacements are given for some unstable modes. On the same figure, the red vertical lines are related to the measured squeal frequencies (fig 1).

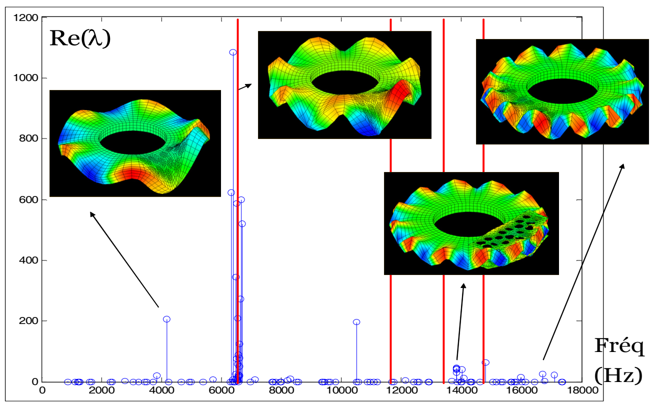

Figure 4: Unstable modes of the braking system

Conclusion

The given stability analysis gives a satisfactory modeling of the squeal emittence. The modal basis proposed for the numerical simulation is interesting since the squeal modes are very closed to the vibration frequencies.

References

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[2] X. Lorang ,2003, Analyse dynamique des freins de T.G.V. en vue des études de crissement, *Rapport de D.E.A.*