Application of a novel wave based prediction technique for acoustic cavity analysis

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Introduction

Numerical prediction techniques have become common practice in automotive industry for vehicle interior acoustics analysis. The finite element method (FEM), the boundary element method (BEM) and the statistical energy analysis (SEA) method are most commonly used to analyze the interior acoustic pressure field.

The FEM [1] is a deterministic prediction technique which discretizes the problem domain into a large number of small elements. Within these elements, the dynamic pressure response is described in terms of simple, polynomial shape functions. Since these shape functions are no exact solutions of the governing Helmholtz equation, a very fine discretization is required to obtain reasonable prediction accuracy. This leads to very large numerical models, whose size grows with frequency. Computational limitations regarding memory and CPU time restrict the practical applicability of the FEM to problems in the low-frequency range [2].

The BEM [3] is a deterministic prediction technique which is based on a boundary integral formulation of the problem, so that only the boundary of the considered domain has to be discretized. Within these boundary elements, some acoustic boundary variables are expressed in terms of simple, polynomial shape functions. Since only the boundary of the problem domain has to be discretized, the numerical models become smaller than FE models. Moreover, the method does not require a volume meshing of the acoustic cavity. However, drawbacks of this method are the fully populated, frequency dependent, complex and not always symmetric system matrices which lead to computationally demanding calculations. In this way, the smaller model size does not result in an enhanced computational efficiency, so that the practical use of the BEM is also restricted to low-frequency applications.

The SEA method [4] is a statistical prediction technique which divides the considered problem into a number of components which are interconnected by coupling loss factors. Expressing for each individual component the power balance between input power, internal dissipation and power flow towards the other components, and subsequent solution of the obtained system of equations, yield an average energy level for each component. The resulting numerical models are small and easy to solve, so that computational load is no restriction on the applicability of the method. However, it is assumed that each component has a high modal overlap. This limits the use of the technique to problems in the high-frequency range.

Between the high-frequency limit of the element based methods and the low-frequency limit of the SEA method, there is a mid-frequency gap for which neither the element based methods nor the SEA method are applicable. At present, no single method has been successful in bridging the gap.

A recently developped wave based method (WBM) [5], which adopts an indirect Trefftz approach [6], may provide a solution for problems in the mid-frequency range. Like the element based techniques, the WBM is a deterministic technique, but in contrast to the element based methods, the new technique expands the dynamic pressure response in terms of wave functions which are exact solutions of the governing Helmholtz equation. In this way, no fine discretization is required and model sizes become much smaller, which results in an enhanced computational efficiency such that its practical frequency limitation can be shifted towards the mid-frequency range.

This paper discusses the experimental validation of the WBM for the acoustic analysis of a three-dimensional (3D) car-like cavity and compares its applicability with that of the FEM.

Problem definition

Figure 1 shows a car-like cavity. An air-filled cavity is surrounded with concrete walls Ω_0, which can be considered acoustically rigid. The system is excited by two volume velocity sources. These sources are considered in the numerical models as normal velocity boundary excitations $\bar{v}_n$ applied on a small surface Ω_v corresponding to the loudspeaker membrane. The pressure inside the cavity is denoted as $p(x, y, z)$.

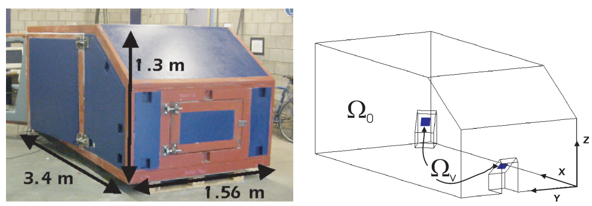

Figure 1: Concrete car-like cavity, excited by 2 loudspeakers

Assuming that the system is linear, inviscid and adiabatic, the steady-state acoustic pressure $p(x, y, z)$ inside the cavity is governed by the homogeneous Helmholtz equation

$$\nabla^2 p(x, y, z) + k^2 p(x, y, z) = 0 \qquad (1)$$

with $k = \frac{\omega}{c}$ the acoustic wave number, c the speed of sound, ω the circular frequency of excitation and ∇^2 the

Laplace operator.

Basic concepts of the WBM

The WBM adopts an indirect Trefftz approach [6] in that the dynamic response variable $p(x, y, z)$ is approximated by an expansion $\widehat{p}(x, y, z)$ in terms of acoustic wave functions $\Phi_a(x, y, z)$

$$p(x, y, z) \simeq \widehat{p}(x, y, z) = \sum_{a=1}^{n_a} p_a \Phi_a(x, y, z) \qquad (2)$$

Since each acoustic wave function $\Phi_a(x, y, z)$ satisfies the Helmholtz equation (1), the expansion (2) is also an exact solution of the governing dynamic equation (1). Reference [5] elaborates on the choice of these functions. The wave function contributions p_a are the unknowns. These contributions are merely determined by the acoustic boundary conditions, imposed at the surfaces of the cavity Ω_0 and Ω_v. Since these boundary conditions are defined at an infinite number of boundary positions, while only finite sized prediction models are amenable to numerical implementation, they are transformed into a weighted residual formulation yielding a set of n_a algebraic equations in the unknown contributions p_a. Solution of the resulting matrix equation and substitution of the wave function contributions p_a into the field variable expansion (2) results in the approximation $\widehat{p}(x, y, z)$ of the desired response variable $p(x, y, z)$.

In contrast to the FEM, the resulting frequency dependent WBM system matrix is not sparse and it does not have a banded structure. The advantage of the WBM is, however, that the system matrix is substantially smaller because there is no need for a fine element discretization. This property, combined with the fast convergence of the WBM, make it a less computationally demanding method for dynamic response calculations, and make it possible to tackle problems also in the mid-frequency range.

Results and comparison with FEM

The case is considered of the car-like cavity, shown in figure 1. The cavity is filled with air ($c = 346.1485 + 0.346i$ $\frac{m}{s}$, $\rho_0 = 1.163$ $\frac{kg}{m^3}$). All the walls of the cavity are rigid. The system is excited by the front loudspeaker, which is considered as a normal velocity boundary excitation in the numerical models.

To validate the WBM and to compare its performance with that of the FEM, both methods are applied for the considered problem. An FE model (25890 dofs) is solved with LMS/SYSNOISE Rev.5.6 using the iterative QMR solver. A WB model (1116 dofs) is implemented in a C++ code. All calculations are performed on a HP-C3000 UNIX workstation (400 MHz single processor, 2.5 Gb RAM memory, SPECint95=31.8, SPECfp95=52.4). Both models are chosen such that both involve a similar computational load (i.e. 5000 CPU seconds for 200 frequency lines). The CPU time for the WB model includes both assembly time and solution time of the system of equations, since the WBM system matrices are

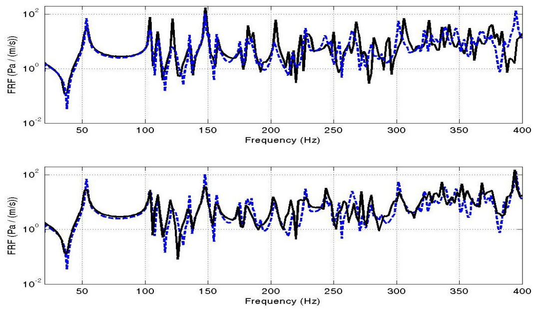

Figure 2: Pressure FRF (amplitude): comparison of FE (solid line, top figure) and WB (solid line, bottom figure) predictions with measurements (dashed line)

frequency dependent, while the FEM CPU time includes only the time for solving the FE matrix system. Figure 2 compares the prediction results for the pressure frequency response function (FRF) in a point inside the cavity $(x, y, z) = (1m; 0.3m; 0.8m)$, calculated with the aforementioned models, with measurement results. The figure indicates that the WBM, unlike the FEM, does not suffer from numerical dispersion, in that there is no (significant) shift between the measured resonance frequencies and those obtained with the WBM solution, even at higher frequencies. The FE results start to deteriorate at 250 Hz because of this dispersion, while the WBM predictions are still accurate in the mid-frequency range.

Conclusions

This paper discusses the validation of the WBM for the uncoupled acoustic analysis of a 3D car-like cavity. It is shown that, unlike the FEM, the novel WBM suffers less from numerical dispersion errors and that the technique can tackle also problems in the mid-frequency range because of its enhanced computational efficiency.

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