Comparison of a Deterministic and a Genetic Algorithm applied to Structural Acoustic Optimization

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Introduction

For certain applications it can be useful to drive the natural frequencies of a structure as far away as possible from a given frequency f^* in order to avoid resonance. The thickness distribution of the structure's finite element (FE) model is optimized by means of numerical optimization techniques such that the difference between f^* and those two natural frequencies f_n and f_{n+1} that enclose f^* (i.e., $f_n < f^* < f_{n+1}$) is maximized. The performance of two different optimization algorithms, i.e., the deterministic algorithm COBYLA [1] and the genetic algorithm PIKAIA [2], was tested. The results of this comparison are presented in this paper.

Cost Function and Constraints

The cost or objective function $F(\boldsymbol{x})$ that is to be maximized can be formulated as $F(\boldsymbol{x}) = \min(f^* - f_n, f_{n+1} - f^*)$, which in the end leads to a "symmetric" solution $f^* - f_n \approx f_{n+1} - f^*$. The design variables $\boldsymbol{x}$ are the local thickness values at the surface nodes of the FE model. A minimum and a maximum allowable thickness are the constraints to the design variables; the total mass of the structure and its mean level of structure borne sound (MLS), which is defined as follows [3, 4], are not permitted to exceed their initial values. The mean squared transmission admittance [5] at frequency f

$$h_{\mathrm{t}}^2(f) = \frac{1/n \sum_{i=1}^{n} v_{\perp \mathrm{rms}i}^2(f)}{F_{\mathrm{rms}}^2(f)} \qquad (1)$$

is calculated from the number of FE nodes at the structure's surface n, the rms normal surface velocity $v_{\perp \mathrm{rms}i}(f)$ at surface node i determined from an FE analysis, and the excitation force $F_{\mathrm{rms}}(f)$. The so-called level of structure borne sound

$$\mathrm{LS}(f) = 10 \lg \frac{S\, h_{\mathrm{t}}^2(f)}{S_0\, h_{\mathrm{t}0}^2} \ \mathrm{dB} \ , \qquad (2)$$

where S is the surface area of the sound radiating surface and $S_0\, h_{\mathrm{t}0}^2 = 2.5 \cdot 10^{-15} \ \mathrm{m^4/(N^2\,s^2)}$ is a standardized reference value, can be interpreted as the vibrational sensitivity of a structure when subjected to some excitation force. The MLS is a frequency-independent average of the LS in the frequency range of interest $f_{\min}$ through $f_{\max}$

$$\mathrm{MLS} = 10 \lg \frac{\int_{f_{\min}}^{f_{\max}} \left[S\, h_{\mathrm{t}}^2(f)/S_0\, h_{\mathrm{t}0}^2 \right] \ \mathrm{d}f}{f_{\max} - f_{\min}} \ \mathrm{dB} \ . \qquad (3)$$

Optimization Procedure

The thickness at the corner nodes of the FE elements at the surface of the structure (see Fig. 1) can be varied by the optimization algorithm. Thus, the optimization problem is defined by a total of 220 design variables and 442 constraints. The following two algorithms were tested against each other. The COBYLA algorithm (**C**onstrained **O**ptimization **BY** **L**inear **A**pproximations) is a deterministic, constrained, derivative-free, nonlinear, sequential trust-region optimization algorithm based on the simplex algorithm. For further details the reader is referred to [1]. In contrast, PIKAIA is a genetic algorithm, i.e., a stochastic one. The original version is unconstrained, but it was adapted to incorporate constraints by means of penalty functions. Further details can be found in [2].

Finite Element Model

Figure 1 shows the FE model of the structure to be optimized, which consists of two plates that are joined at 90° (front plate: 240 mm × 190 mm, 12×10 elements; top plate: 240 mm × 230 mm, 12×12 elements; initial thickness: 4 mm; total: 276 quadratic 20-node solid elements, 2110 nodes, 6330 DOFs). The plates are made of steel
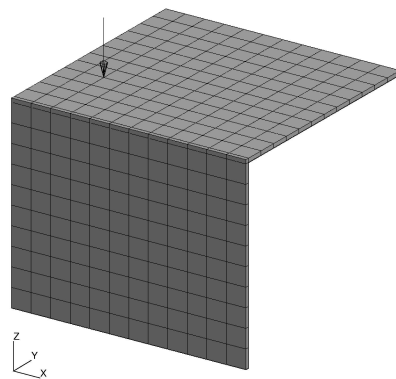

Figure 1: Initial FE model of the two plates joined at 90°.

($\rho = 7850 \ \mathrm{kg/m^3}$, $E = 2.04 \cdot 10^{11} \ \mathrm{N/m^2}$, $\nu = 0.3$), the damping is assumed to be frequency-independent (0.4%). The two plates are simply supported along their edges and are excited by a harmonic force in the frequency range 0–3000 Hz at the location denoted by the arrow. The thickness of the plates is allowed to be varied in the range 1–10 mm, the mass and the MLS of the optimized structure are not allowed to exceed those of the initial one, i.e., 3.195 kg and 81.5 dB, respectively. The frequency f^* is chosen to be $f^* = 974$ Hz, since this is ap-

proximately halfway between the fourth and the fifth natural frequency of the initial structure, namely, 964.3 Hz and 983.4 Hz, respectively (see Fig. 4). Thus, the initial minimum frequency difference is 9.4 Hz.

Optimization Results

The optimized thickness distributions using COBYLA on the one hand and PIKAIA on the other hand can be seen in Fig. 2. The results look very different, which can be explained by the fact that one or both algorithms found only a local optimum. Interestingly, COBYLA reduces the structural mass by 1.012 kg (-31.7%) although this is not the objective of the optimization. Figure 3 de-
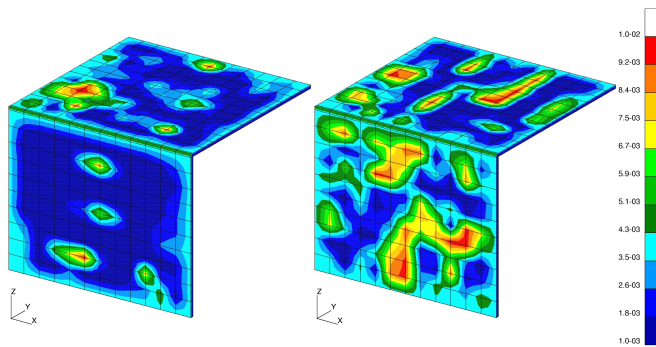

Figure 2: Optimized thickness distribution; left: COBYLA's result, right: PIKAIA's result (spectrum values in m).

picts the iteration history for both optimization runs. COBYLA (red line) reaches its convergence criterion after $10\,884$ iterations and 115.6 h (≈ 4.8 d) of CPU time (Pentium 4, 2.533 GHz, 1.5 GB RAM). None of the constraints are violated, and the difference between $f^* = 974$ Hz and its neighboring natural frequencies increases from 9.4 Hz to 242.6 Hz ($+233.2$ Hz, $+2481\%$). PIKAIA does not feature an explicit convergence crite-
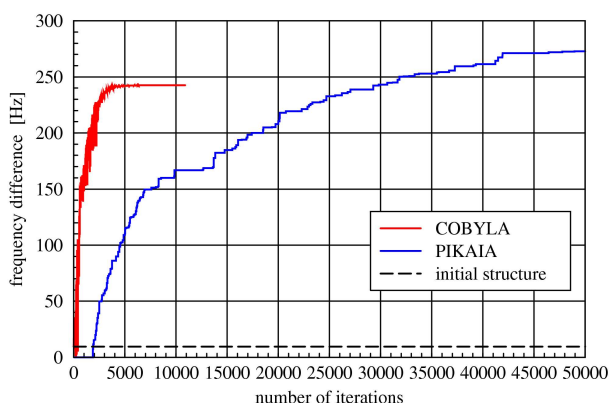

Figure 3: Iteration history for both optimization runs.

rion and is therefore stopped manually after $50\,000$ iterations (659 h $\approx$ 27.5 d real time on 5 computers in parallel) after some asymptotic behavior was observed (blue line). The frequency difference increases from 9.4 Hz to 272.8 Hz ($+263.4$ Hz, $+2802\%$).

From the LS spectra in Fig. 4 the increase of the frequency difference is clearly visible. For the initial struc-

ture the natural frequencies $f_4^{\text{initial}} = 964.3$ Hz and $f_5^{\text{initial}} = 983.4$ Hz are rather close to $f^* = 974$ Hz (marked by the vertical dot-dashed line), whereas for the optimized geometries the respective natural frequencies are $f_5^{\text{COBYLA}} = 731.4$ Hz, $f_6^{\text{COBYLA}} = 1216.6$ Hz, $f_3^{\text{PIKAIA}} = 701.2$ Hz, and $f_4^{\text{PIKAIA}} = 1248.2$ Hz.
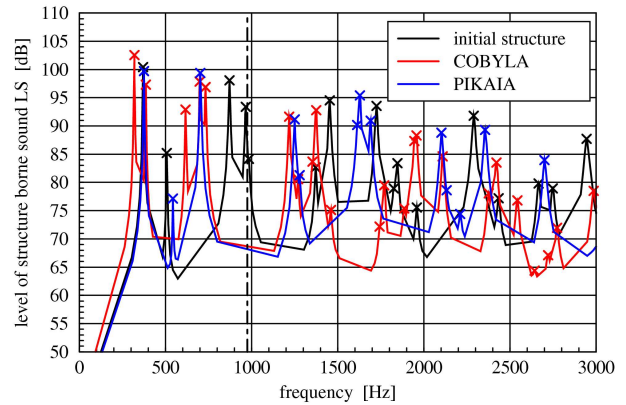

Figure 4: LS spectra (dot-dashed line marks $f^* = 974$ Hz).

Conclusions and Future Work

For this particular optimization problem, COBYLA is more efficient than PIKAIA since it is able to generate a substantial improvement of the objective function in a reasonable amount of time. Conversely, PIKAIA is more effective than COBYLA because it is able to increase the objective function about 13% more than COBYLA. However, to do so PIKAIA takes almost four times as many iterations and almost six times as much time.

For the future it is planned to combine passive structural optimization with active structural acoustic control to further improve the vibrational behavior of mechanical structures.

References

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