Time response of a plate excited by a short-duration point force

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Introduction

The aim of this paper is to predict the time response of a thin elastic plate immersed in a fluid and excited by a transient force. The method is based on the expansion of the displacement in a series of the resonance modes of the fluid/plate system. An experimental study has been carried out for a clamped plate located between two anechoïc rooms. This paper briefly describes the numerical method and presents some comparisons between experimental and numerical results - the first 30 or 60 ms of the signals of acceleration and sound pressure, for several types of excitation forces.

Time response of the fluid/plate system

The thin elastic plate is surrounded on both sides by a homogeneous fluid at rest. It is excited by a short-duration force $f(t)$. The response of the system is the displacement $u(M,t)$ and the sound pressure in the fluid $p(Q,t)$. The method used to evaluate the two unknown functions has been described in previous papers [1, 2]. It is based on the expansion of the displacement in a series of the resonance modes W_n of the fluid/plate system. The resonance frequencies ω_n are complex, with a small imaginary part corresponding to the loss of vibratory energy in the fluid. This gives a decreasing behaviour of acceleration and sound pressure versus time. The resonance modes correspond to the free oscillations of the plate. The displacement $u(M,t)$ can be written as :

$$\begin{aligned} u(M,t) &= f(t) * Y(t) \sum_{n=1}^{\infty} \Big[\alpha_n W_n(M) \mathrm{e}^{-\imath\omega_n t} \\ &\quad - \alpha_n^* W_n^*(M) \mathrm{e}^{+\imath\omega_n^* t} \Big] \end{aligned} \tag{1}$$

In the case of a thin elastic plate, the expressions of coefficients α_n are obtained explicitly.

Computation of the resonance modes and frequencies

The resonance modes $W_n(M)$ and frequencies ω_n are the solutions of a homogeneous equation which is discretised and replaced by a linear system of the general form :

$$A\,W = \omega^2(\epsilon B(\omega) + Id)\,W \tag{2}$$

A is a matrix which corresponds to the energy of the plate itself. B is a matrix which corresponds to the coupling between the fluid and the plate. It depends on ω. The parameter ϵ is equal to the ratio of the fluid density to the surface density of the plate. ϵ is small in the case of a light fluid. This is an eigenvalue problem which is not classical because B depends on ω. Several methods have been tested to evaluate the coefficients of B and to obtain the resonance modes and frequencies. They are described in [1].

Comparison between numerical and experimental results

Two series of experimental studies have been carried out for a thin elastic plate located between two anechoïc rooms. For both studies, acceleration and sound pressure signals have been recorded during the first 30 or 60ms.

Plate excited with an impact hammer

In this first study, the excitation on the plate is provided by an impact hammer which can be equipped with three types of head : rubber, plastic and metal. Two acceleration signals have been measured, on the hammer itself and at one point close to the corner of the plate. The sound pressure signals have been measured at two microphones located in front of the plate at 54cm and 1m respectively from the plate. Measurements have been made with a sampling frequency equal to 262144 Hz. For the computation, an analytical model of the excitation has been used. For example, in the case of the rubber head, the excitation is approximated by :

$$\begin{aligned} f(t) &= 1 - \cos(\frac{\pi t}{T_1}) \text{ for } 0 < t < T_1 \\ &= 1 + \cos\frac{\pi(t - T_1)}{T_2} \text{ for } T_1 < t < T_1 + T_2 \end{aligned} \tag{3}$$

with $f(t)$ equal to zero for t negative or t larger than the total length $(T_1 + T_2)$. T_1 and T_2 are the two parameters used to define the model. They have been identified for each striking. For the metal and plastic heads, the excitation model is similar and characterized by the same time parameters. Figure 1 presents three examples of excitation, obtained for the three types of head (rubber, plastic, metal). For rubber, the total length $(T_1 + T_2)$ is roughly equal to 3 or 4 ms. For plastic and metal, the total length is of the order of 0.6ms to 0.9ms. Figure 2 presents the acceleration obtained on the plate with the impact hammer equipped with a plastic head. The impact point and the measurement point are both located close to a corner of the plate. Figure 3 presents a sound pressure signal obtained with the impact hammer equipped with a rubber head. The impact point is located more closer to the center of the plate and the microphone is located 1m from the plate. For both figures, the agreement is quite good. Both acceleration and sound pressure have been computed with 300 modes. This corresponds to a maximum frequency equal to 3kHz. At 3kHz, the levels of the pressure spectrum for the rubber head have decreased by more than 20dB ; the main part of the spectrum is between 0 and 800Hz. On the contrary, for the plastic head, the main part of the spectrum is between 0 and 5kHz. This is why in Figure 2, both signals are quite similar except that the numerical curve is softer than the experimental one: The modes higher than 3kHz are not taken into account in the computation.

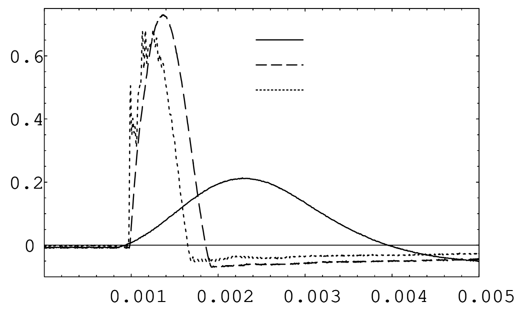

Figure 1: Excitation signals - Solid line: Rubber - Dashed line:Plastic - Dotted line: Metal

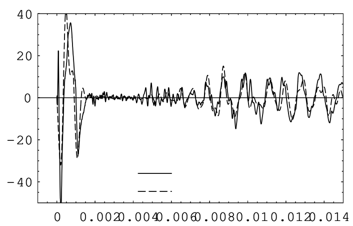

Figure 2: Plastic head - Acceleration - Solid line: Measurement - dashed line: Computation

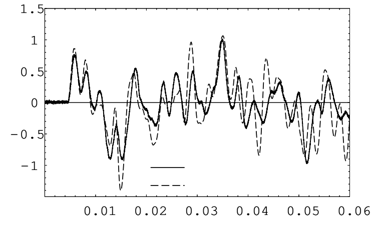

Figure 3: Rubber head - Sound pressure signal - Solid line: Measurement - dashed line: Computation

Plate excited with a small sphere

In this second study, the plate is hit by a small sphere. Several spheres have been used corresponding to several sizes and materials (rubber, polyurethane, wood, plexiglass,...). Acceleration and sound pressure signals have been measured. The accelerometer was located in the corner of the plate and two microphones were located in front of the plate at 11cm and 26 cm respectively from the plate. Unfortunately, in this case, it was not possible to measure the excitation. This could have been done by measuring the acceleration at the impact point on the other side of the plate. However, the model of excitation has been chosen as in formula (3). Such a model is also similar to the recorded excitation measured in [3] for the same type of experiment. Figure 4 presents a sound pressure signal obtained when the plate is hit by a polyurethane sphere. The excitation point is close to the center of the plate and the microphone is 26cm from the plate. For polyurethane, 104 modes have been taken into account which corresponds to a maximum frequency of 2kHz.

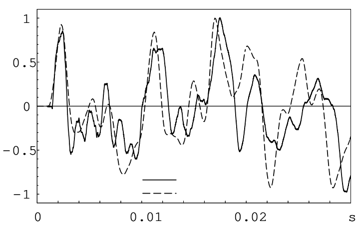

Figure 4: Sound pressure signal - Solid line: Measurement - dashed line: Computation

Conclusion

The aim of this paper was to show some examples of comparison between numerical and experimental results in the case of a plate excited by a short-duration force. One application of this study is Psychomechanics. In this case, the computed sounds are synthesised and submitted to listeners in order to compare them with recorded sounds.

References

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